Moment of Inertia of a Composite Steel Beam

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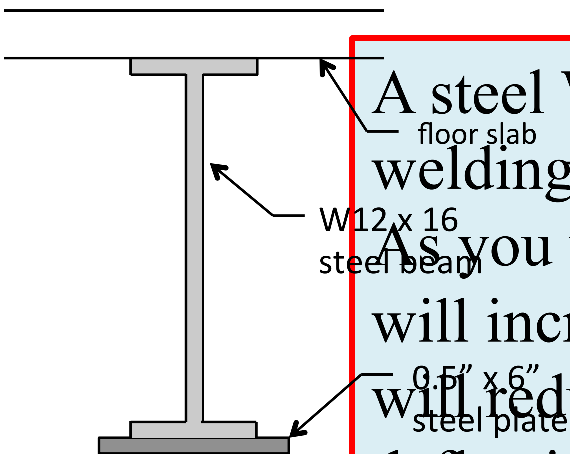
Example Problem

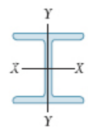
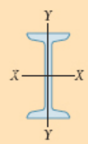
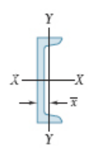
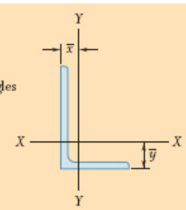


A steel W section supporting a floor slab is reinforced by welding a steel plate to its bottom flange as shown above. As you will learn in CE 112, the addition of the steel plate will increase the moment of inertia of the W shape which will reduce the maximum bending stress and decrease the deflection of the beam.

For the composite section find:

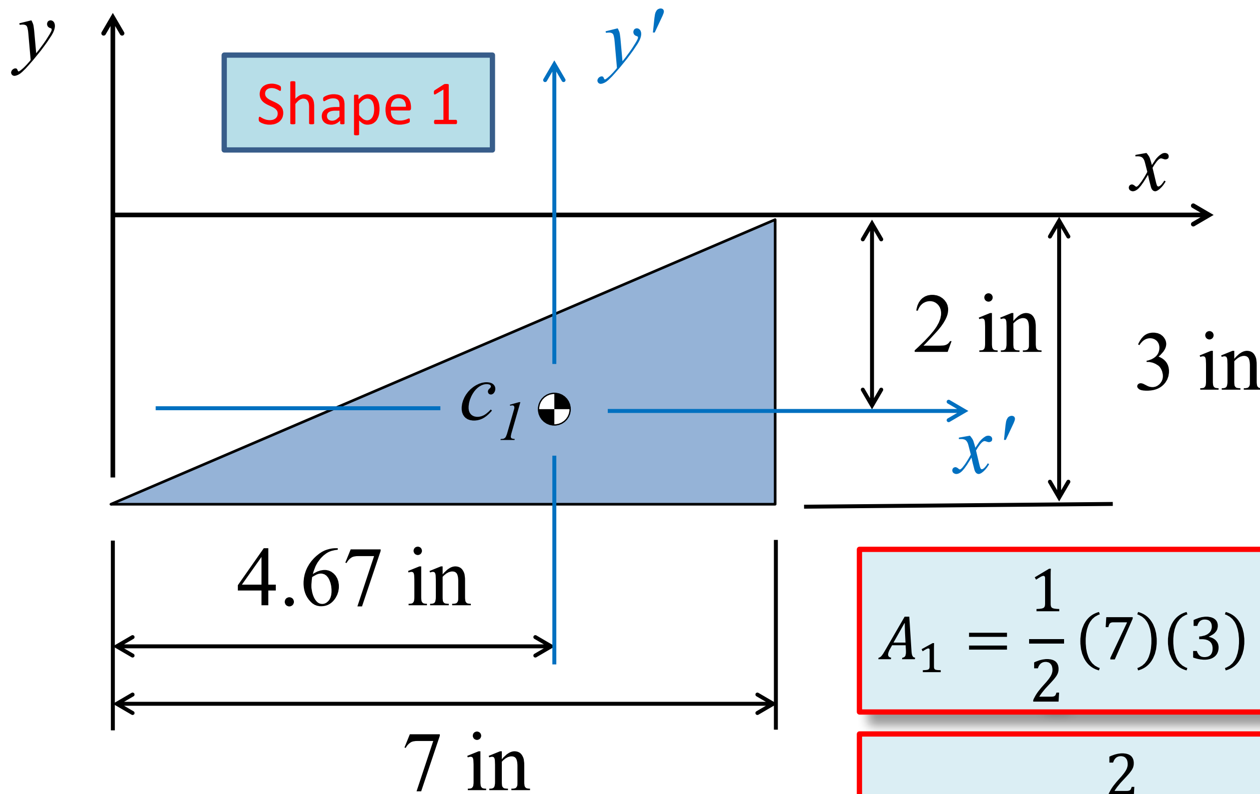
1. The centroid of the composite section;
2. The moment of inertia about the centroid of the composite section.

Tabulated Centroidal Moments of Inertia Can be Found in the Textbook

	Designation	Area in²	Depth in.	Width in.	Axis X–X: $\bar{I}_x$, in⁴	Axis X–X: $\bar{k}_x$, in.	Axis X–X: $\bar{y}$, in.	Axis Y–Y: $\bar{I}_y$, in⁴	Axis Y–Y: $\bar{k}_y$, in.	Axis Y–Y: $\bar{x}$, in.
W Shapes (Wide-Flange Shapes) 	W18 × 76†	22.3	18.2	11.0	1330	7.73		152	2.61	
	W16 × 57	16.8	16.4	7.12	758	6.72		43.1	1.60	
	W14 × 38	11.2	14.1	6.77	385	5.87		26.7	1.55	
	W8 × 31	9.12	8.00	8.00	110	3.47		37.1	2.02	
S Shapes (American Standard Shapes) 	S18 × 54.7†	16.0	18.0	6.00	801	7.07		20.7	1.14	
	S12 × 31.8	9.31	12.0	5.00	217	4.83		9.33	1.00	
	S10 × 25.4	7.45	10.0	4.66	123	4.07		6.73	0.950	
	S6 × 12.5	3.66	6.00	3.33	22.0	2.45		1.80	0.702	
C Shapes (American Standard Channels) 	C12 × 20.7†	6.08	12.0	2.94	129	4.61		3.86	0.797	0.698
	C10 × 15.3	4.48	10.0	2.60	67.3	3.87		2.27	0.711	0.634
	C8 × 11.5	3.37	8.00	2.26	32.5	3.11		1.31	0.623	0.572
	C6 × 8.2	2.39	6.00	1.92	13.1	2.34		0.687	0.536	0.512
Angles 	L6 × 6 × 1‡	11.0			35.4	1.79	1.86	35.4	1.79	1.86
	L4 × 4 × $\frac{1}{2}$	3.75			5.52	1.21	1.18	5.52	1.21	1.18
	L3 × 3 × $\frac{1}{4}$	1.44			1.23	0.926	0.836	1.23	0.926	0.836
	L6 × 4 × $\frac{1}{2}$	4.75			17.3	1.91	1.98	6.22	1.14	0.981
	L5 × 3 × $\frac{1}{2}$	3.75			9.43	1.58	1.74	2.55	0.824	0.746
	L3 × 2 × $\frac{1}{4}$	1.19			1.09	0.953	0.980	0.390	0.569	0.487

Divide Area into Simple Composite Shapes

Find the Area, Location of Centroid, and the Centroidal Moment of Inertia of Each Shape



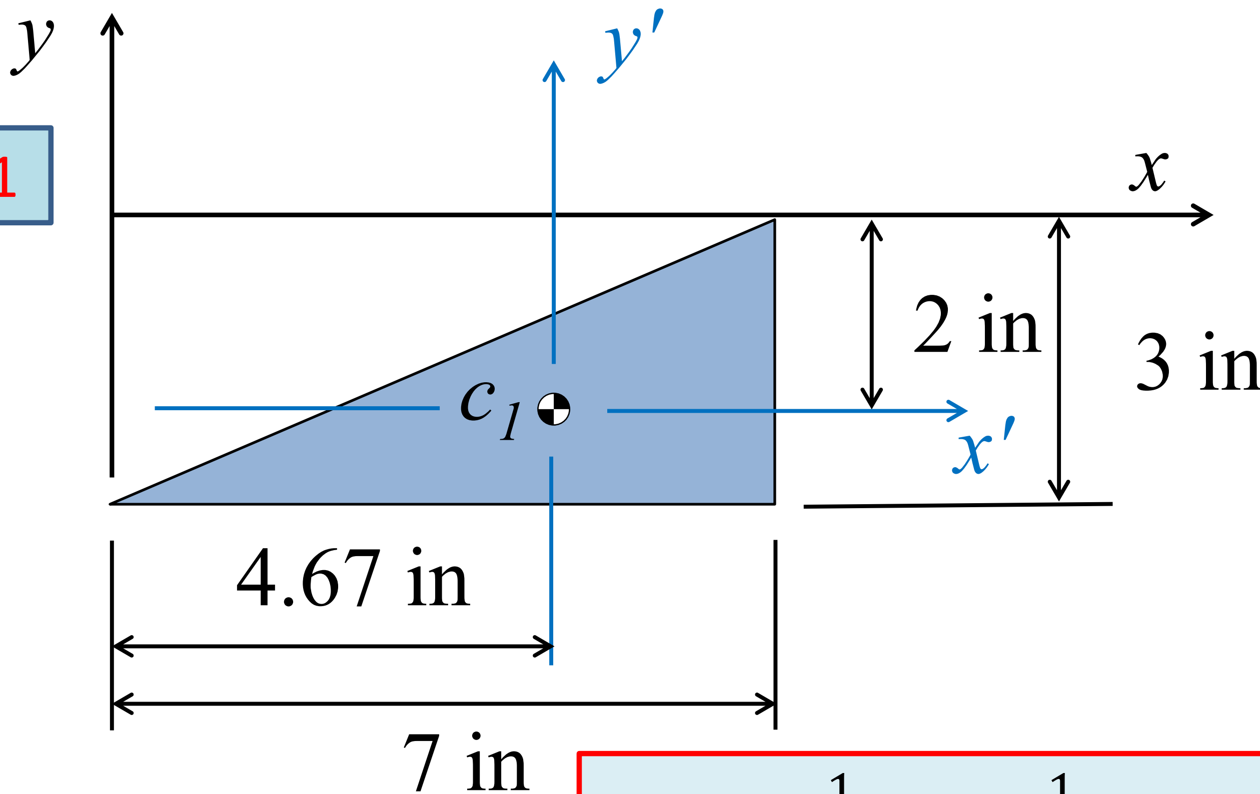
$$A_1 = \frac{1}{2}(7)(3) = 10.5 \text{ in}^2$$

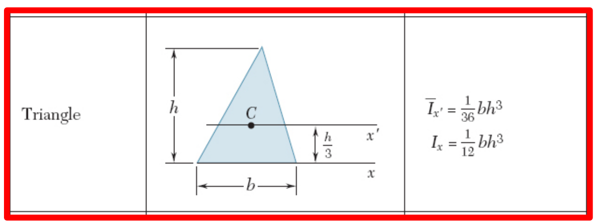
$$d_{x1} = \frac{2}{3}(7) = 4.67 \text{ in}$$

$$d_{y1} = -\frac{2}{3}(3) = -2.0 \text{ in}$$

Find the Area, Location of Centroid, and the Centroidal Moment of Inertia of Each Shape

Shape 1

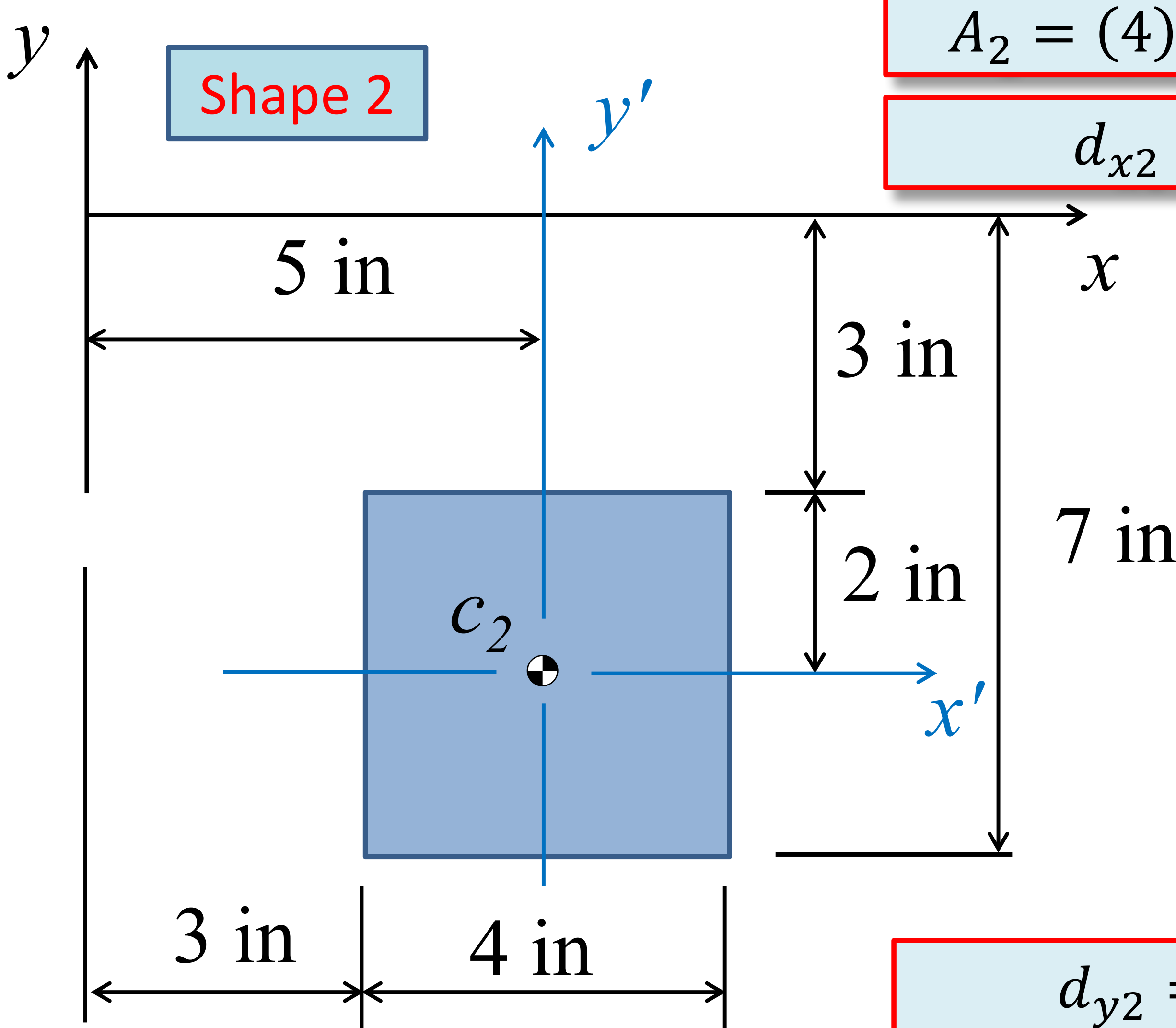




$$\bar{I}_{x1'} = \frac{1}{36}bh^3 = \frac{1}{36}(7)(3^3) = 5.25 \text{ in}^4$$

$$\bar{I}_{y1'} = \frac{1}{36}hb^3 = \frac{1}{36}(3)(7^3) = 28.5833 \text{ in}^4$$

Find Area and Location of Centroid of Each Shape Relative to Reference Coordinate Axes

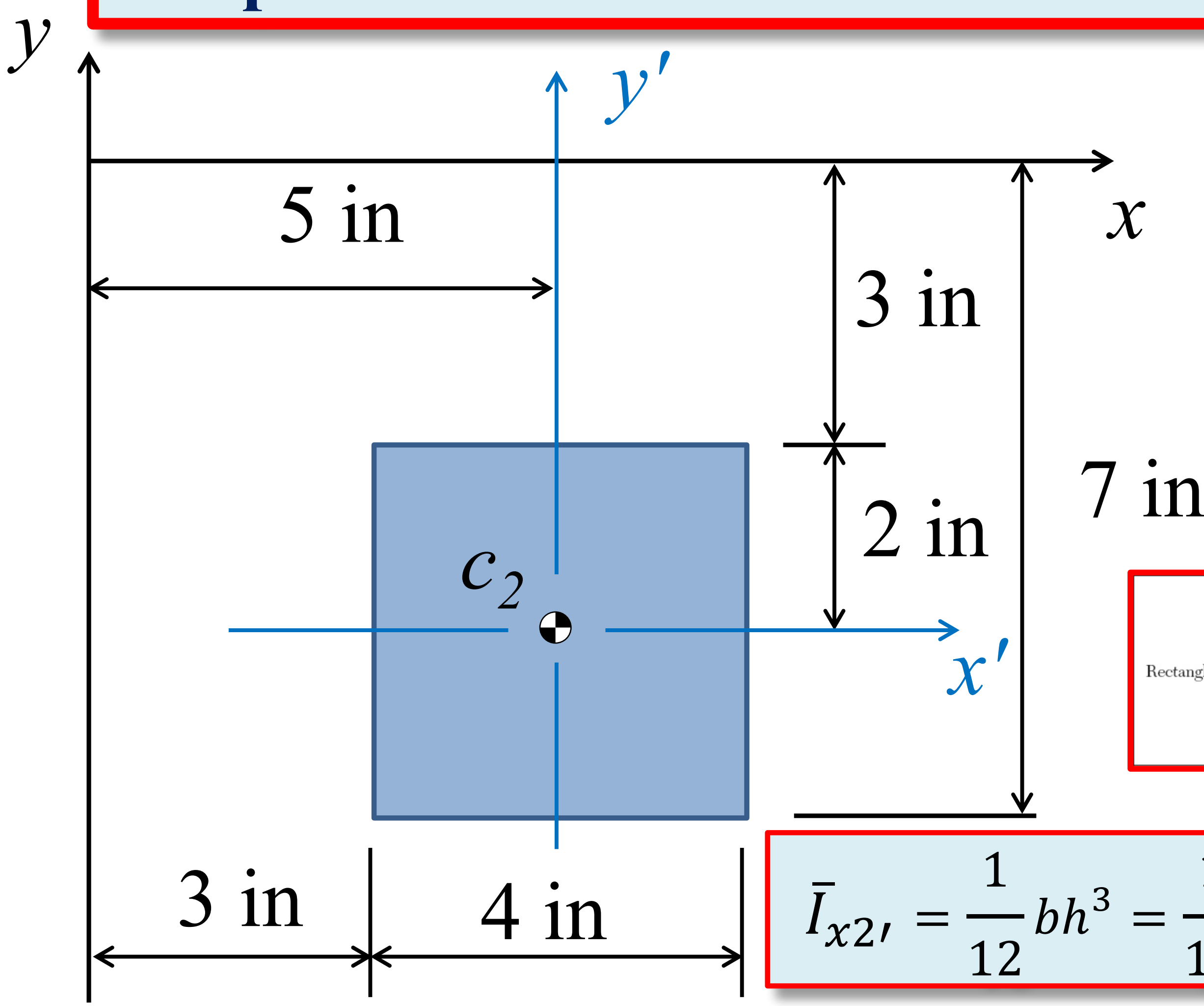


$$A_2 = (4)(4) = 16\ \text{in}^2$$

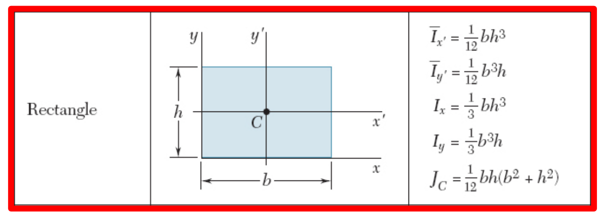
$$d_{x2} = 5.0\ \text{in}$$

$$d_{y2} = -5.0\ \text{in}$$

Find Area and Location of Centroid of Each Shape Relative to Reference Coordinate Axes



Shape 2

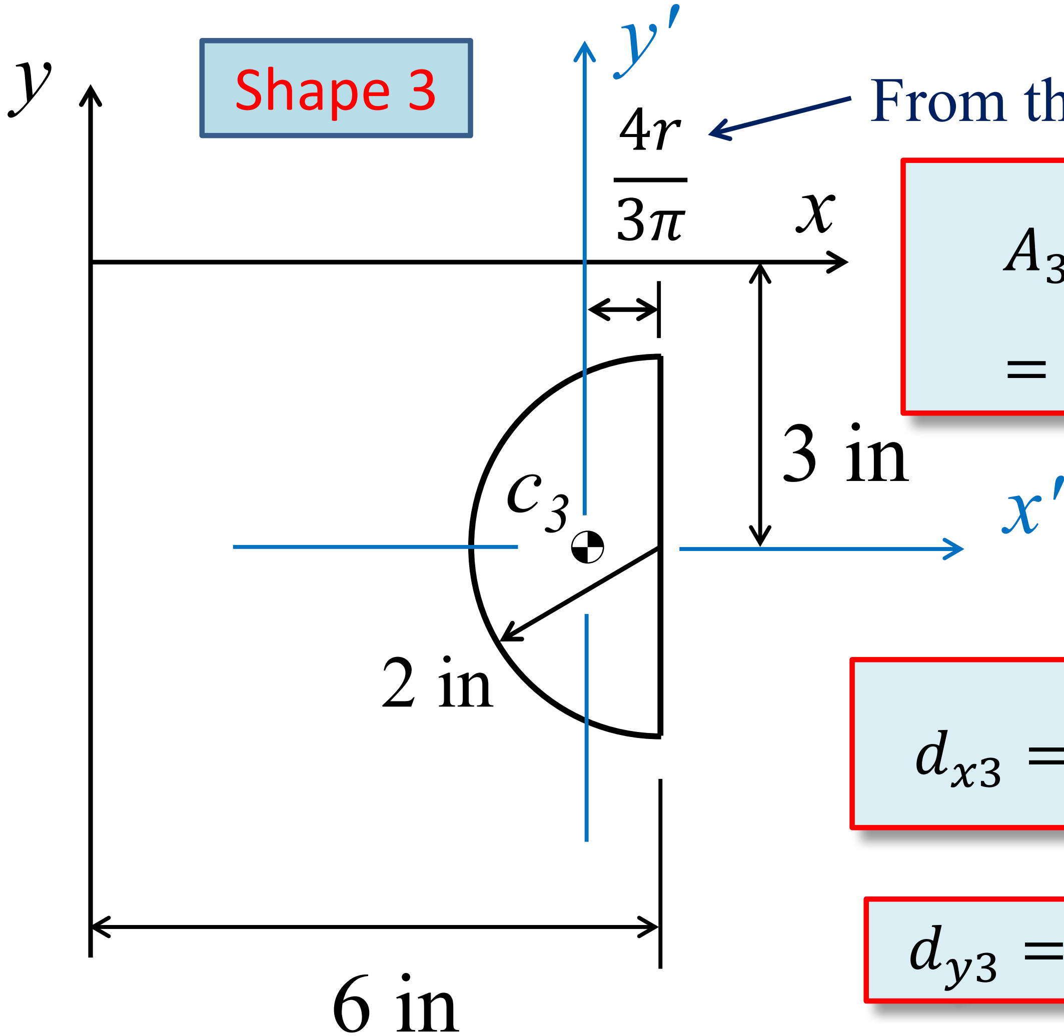


$$\bar{I}_{x2'} = \frac{1}{12}bh^3 = \frac{1}{12}(4)(4^3) = 21.333 \text{ in}^4$$

$$\bar{I}_{y2'} = \frac{1}{12}hb^3 = \frac{1}{12}(4)(4^3) = 21.333 \text{ in}^4$$

Find Area and Location of Centroid of Each Shape Relative to Reference Coordinate Axes

Shape 3



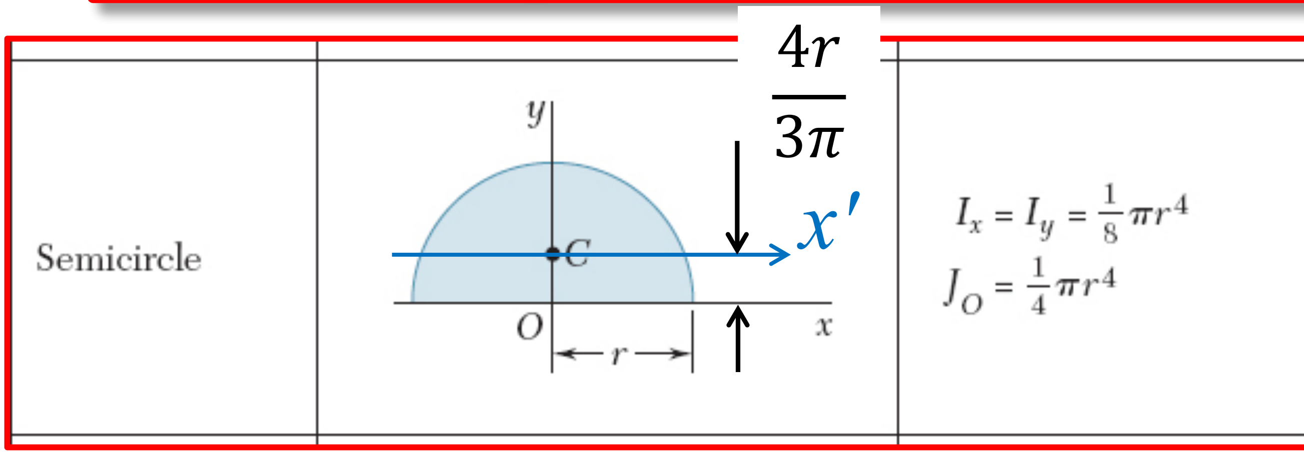
From the table

$$A_3 = -\frac{1}{2}\pi r^2 = -\frac{1}{2}\pi(2^2)$$
$$= -6.2832 \text{ in}^2$$

$$d_{x3} = 6 - \frac{4(2)}{3\pi} = 5.1512 \text{ in}$$

$$d_{y3} = -3 \text{ in}$$

Use Parallel Axis Theorem to Complete Table for Semicircle



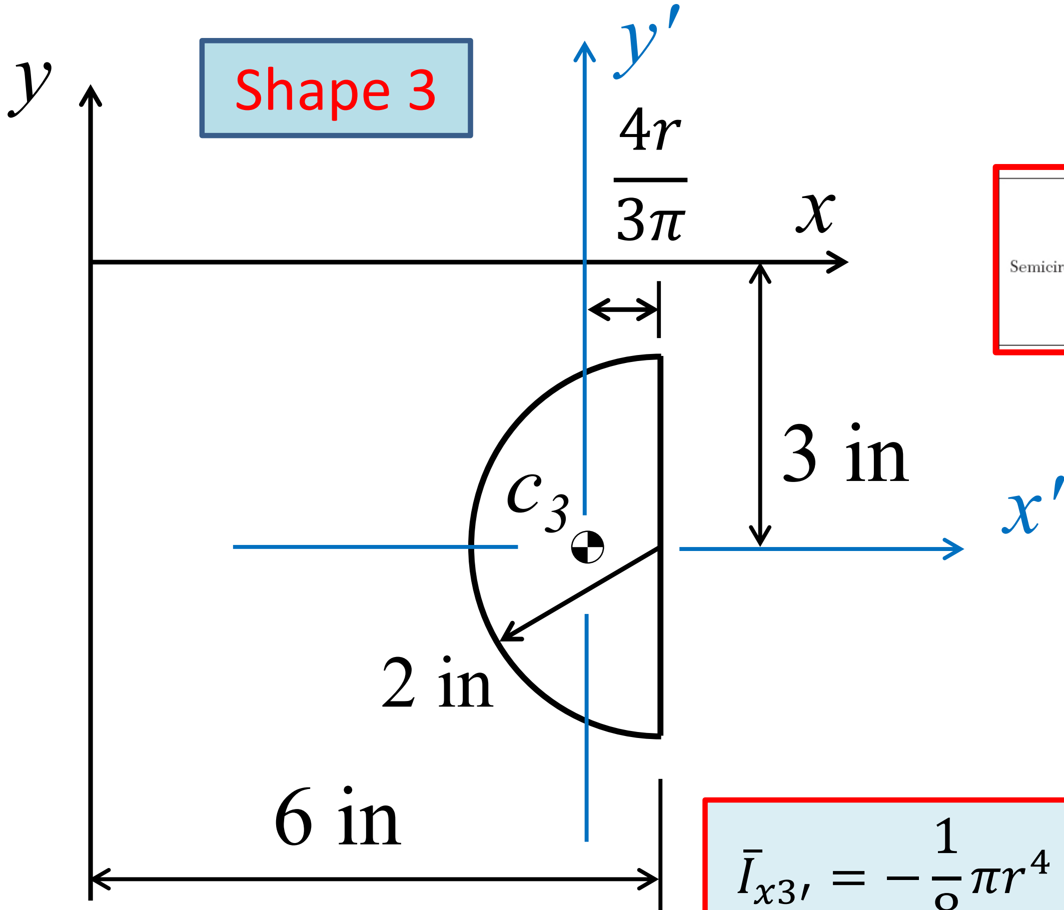
$$I_x = \bar{I}_{x'} + d_y^2 A$$

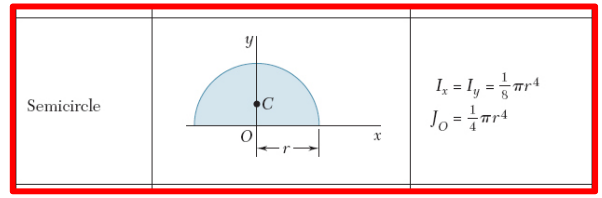
$$\bar{I}_{x'} = I_x - d_y^2 A$$

$$\bar{I}_{x'} = \frac{1}{8}\pi r^4 - \left(\frac{4r}{3\pi}\right)^2 \left(\frac{\pi r^2}{2}\right)$$

$$\bar{I}_{x'} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$$

Find Area and Location of Centroid of Each Shape Relative to Reference Coordinate Axes





$$\bar{I}_{x3'} = -\frac{1}{8}\pi r^4 = -\frac{1}{8}\pi(2^4) = -6.28318 \text{ in}^4$$

$$\bar{I}_{y3'} = -\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 = -\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)(2^4) = -1.75610 \text{ in}^4$$

Find the Moment of Inertia about the x axis

$\bar{I}_{x1'} = 5.25 \text{ in}^4$

$\bar{I}_{x2'} = 21.333 \text{ in}^4$

$\bar{I}_{x3'} = -6.28318 \text{ in}^4$

$d_{y1} = -2.0 \text{ in}$

$d_{y2} = -5.0 \text{ in}$

$d_{y3} = -3 \text{ in}$

$A_1 = 10.5 \text{ in}^2$

$A_2 = 16 \text{ in}^2$

$A_3 = -6.2832 \text{ in}^2$

$$I_x = \sum \bar{I}_{xi'} + \sum d_{yi}^2 A_i$$

Shape	$\bar{I}_{xi'}$	d_{yi}	A_i	$d_{yi}^2 A_i$
1	5.25	–2.0	10.5	42.0
2	21.333	–5.0	16.0	400.0
3	–6.28318	–3.0	-6.28318	–56.5486
$\sum$	20.300			385.4514

$$I_x = 20.30 + 385.4514 = 405.75 \text{ in}^4$$

Find the Moment of Inertia about the *y* axis

$\bar{I}_{y1'} = 28.5833 \text{ in}^4$

$\bar{I}_{y2'} = 21.333 \text{ in}^4$

$\bar{I}_{y3'} = -1.75610 \text{ in}^4$

$d_{x1} = 4.67 \text{ in}$

$d_{x2} = 5.0 \text{ in}$

$d_{x3} = 5.1512 \text{ in}$

$A_1 = 10.5 \text{ in}^2$

$A_2 = 16 \text{ in}^2$

$A_3 = -6.2832 \text{ in}^2$

$$I_y = \sum \bar{I}_{yi'} + \sum d_{xi}^2 A_i$$

Shape	$\bar{I}_{yi'}$	d_{xi}	A_i	$d_{xi}^2 A_i$
1	28.5833	4.67	10.5	919.0274
2	21.3333	5.0	16.0	400.0
3	–1.75610	5.1512	-6.28318	–166.7233
$\sum$	48.1606			1152.3041

$$I_y = 48.1606 + 1152.3041 = 1200.46 \text{ in}^4$$