Friction Problems

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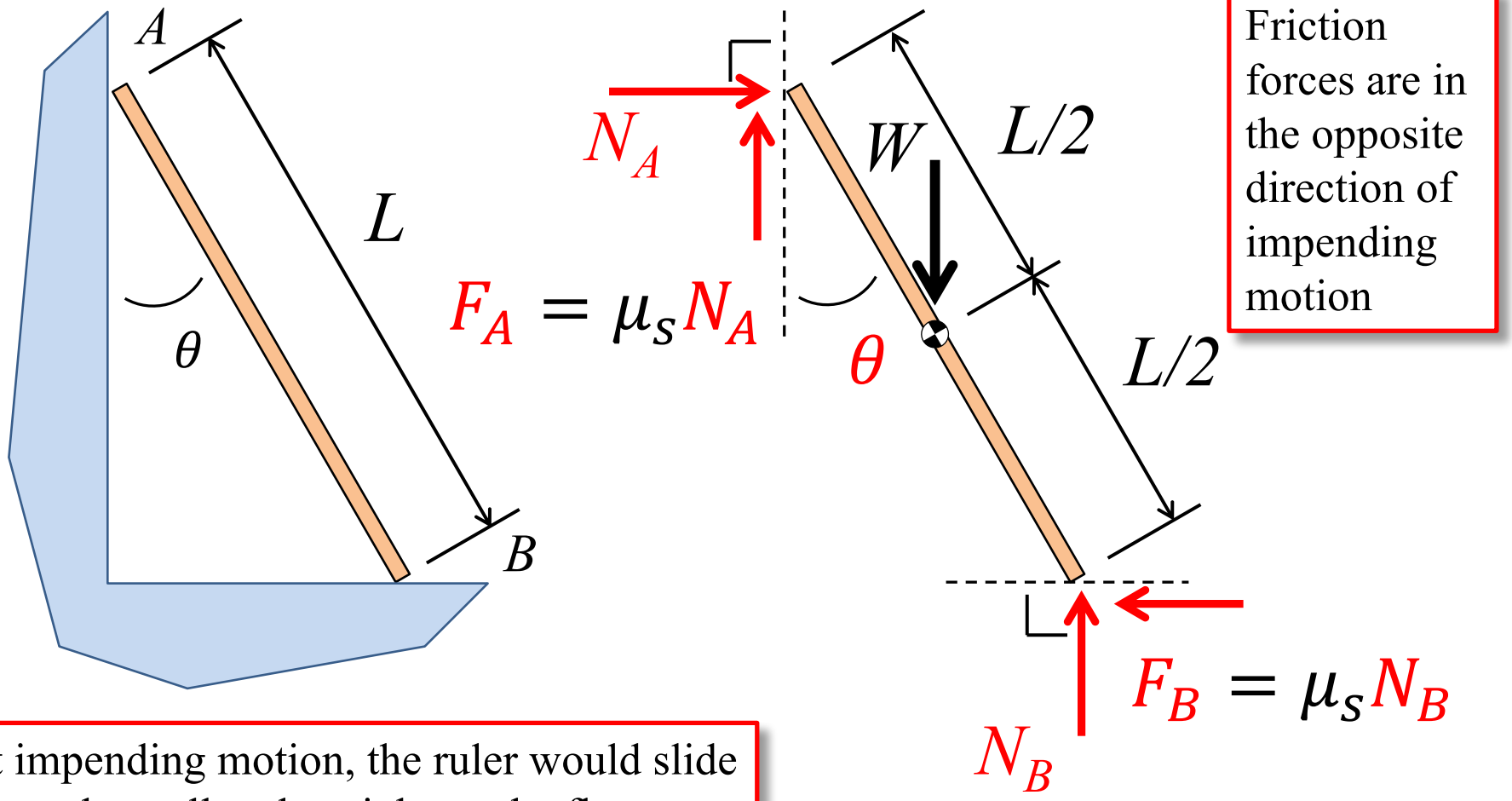
A ruler is leaning on a wall. How much can the ruler be inclined, before it just starts to move, if the coefficient of static friction between the ruler and the wall and the ruler and the floor are both 0.35?

Free-Body Diagram of the Ruler at Impending Motion

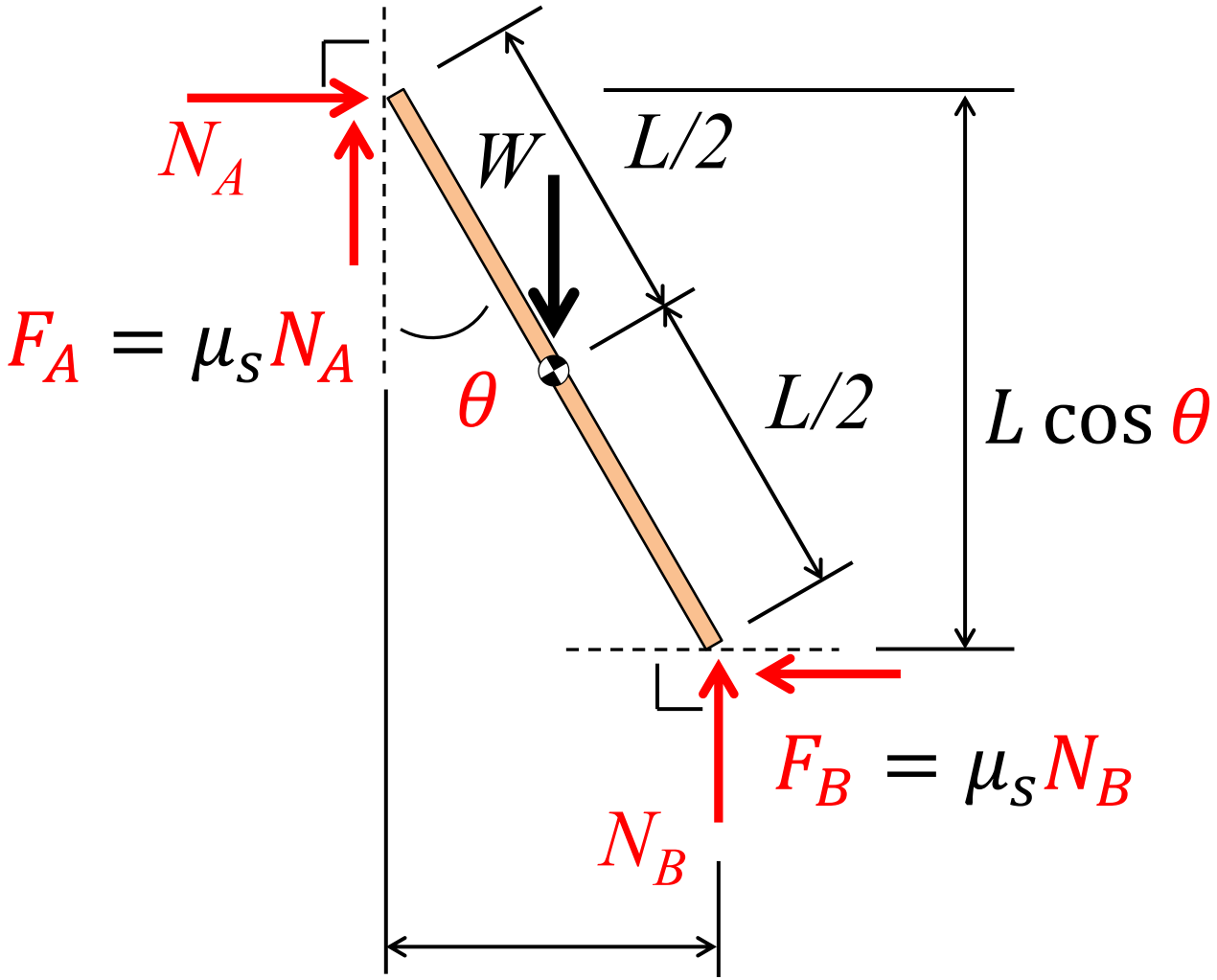


Friction forces are in the opposite direction of impending motion

At impending motion, the ruler would slide down the wall and to right on the floor

3 Unknowns – N_A, N_B, θ

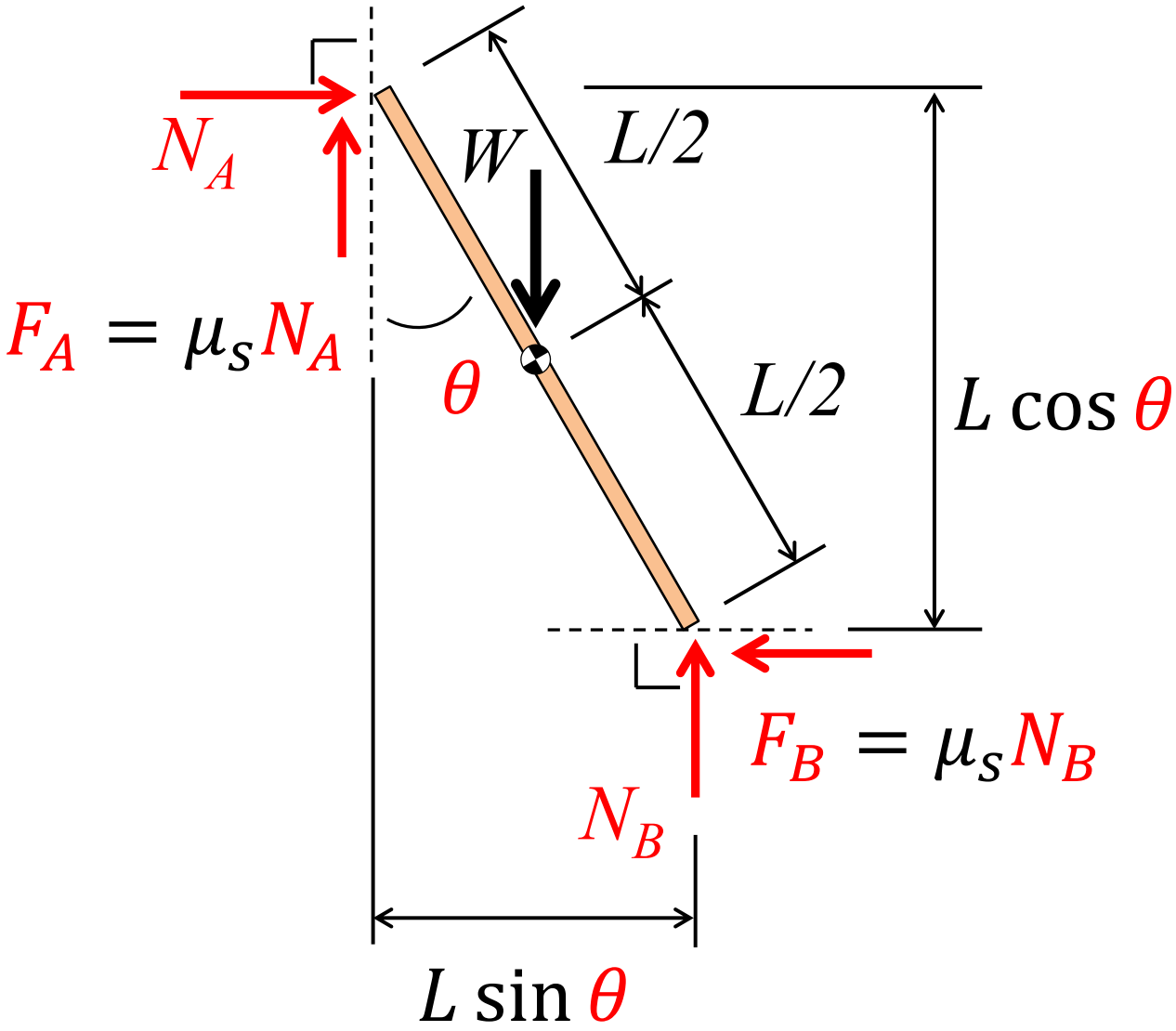
Equations of Equilibrium



$$\circlearrowleft + \sum M_A = 0$$

$$-W\left(\frac{L}{2}\sin\theta\right) + N_B(L\sin\theta) - \mu_s N_B(L\cos\theta) = 0$$

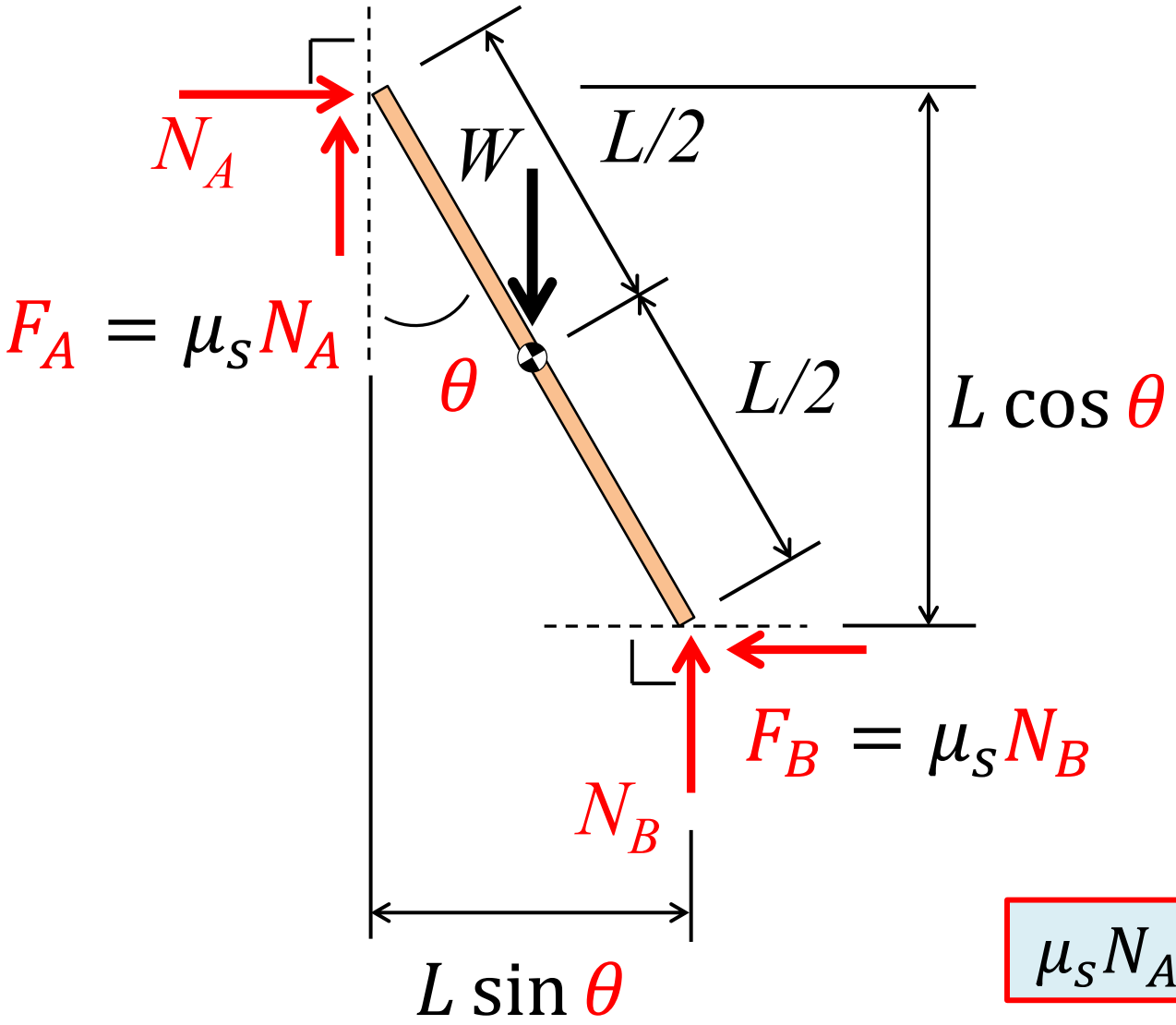
Equations of Equilibrium



$$\xrightarrow{+} \sum F_x = 0$$

$$N_A - \mu_s N_B = 0$$

Equations of Equilibrium



$$+\uparrow \sum F_y = 0$$

$$\mu_s N_A - W + N_B = 0$$

Solve Equations of Equilibrium

1. $-W\left(\frac{L}{2}\sin\theta\right) + N_B(L\sin\theta) - \mu_s N_B(L\cos\theta) = 0$

2. $N_A - \mu_s N_B = 0 \rightarrow N_A = \mu_s N_B$

3. $\mu_s N_A - W + N_B = 0 \rightarrow \mu_s(\mu_s N_B) - W + N_B = 0$

$$N_B = \frac{W}{(\mu_s^2 + 1)}$$

$$-W\left(\frac{L}{2}\sin\theta\right) + \left(\frac{W}{(\mu_s^2+1)}\right)(L\sin\theta) - \mu_s\left(\frac{W}{(\mu_s^2+1)}\right)(L\cos\theta) = 0$$

Solve Equations of Equilibrium

$$-W\left(\frac{L}{2}\sin\theta\right)+\left(\frac{W}{(\mu_s^2+1)}\right)(L\sin\theta)-\mu_s\left(\frac{W}{(\mu_s^2+1)}\right)(L\cos\theta)=0$$

Divide both sides by $WL\cos\theta$ (recall that $\frac{\sin\theta}{\cos\theta}=\tan\theta$)

$$-\frac{1}{2}\tan\theta+\left(\frac{1}{(\mu_s^2+1)}\right)\tan\theta-\left(\frac{\mu_s}{(\mu_s^2+1)}\right)=0$$

$$\left[-\frac{1}{2}+\left(\frac{1}{(\mu_s^2+1)}\right)\right]\tan\theta=\left(\frac{\mu_s}{(\mu_s^2+1)}\right)$$

$\mu_s = 0.35$

$$\tan\theta=\left(\frac{0.3118}{0.3909}\right)=0.7977$$

$$\theta=\tan^{-1}(0.7977\ )=38.6°$$