Centroid of a Composite Area

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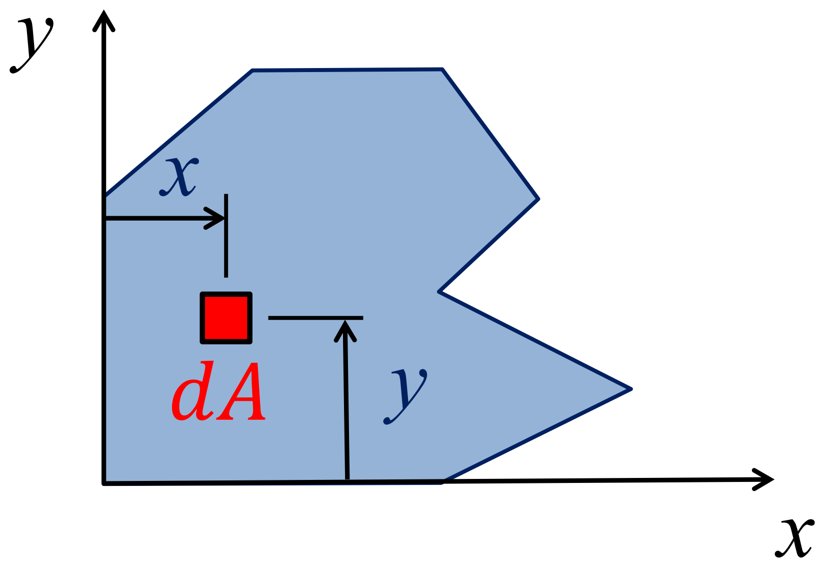
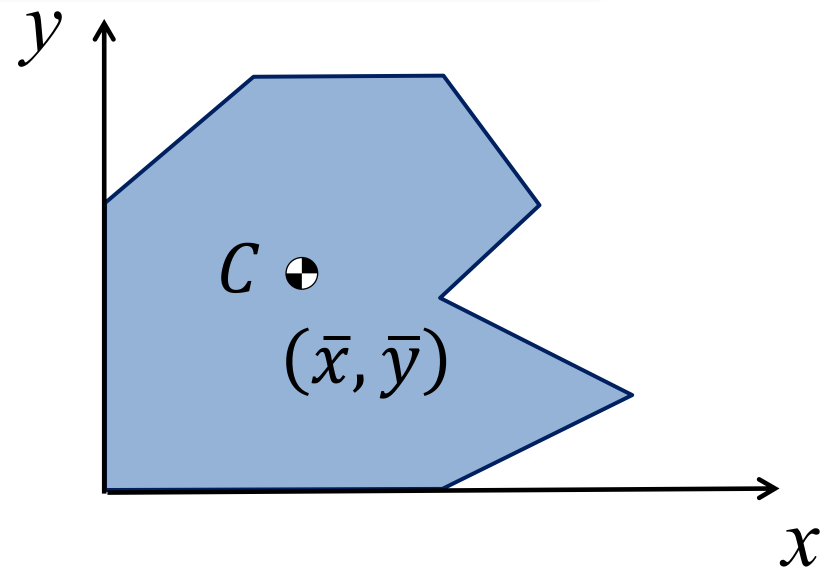
Recall the Definition of the Centroid of an Area





$$A = \iint dA$$

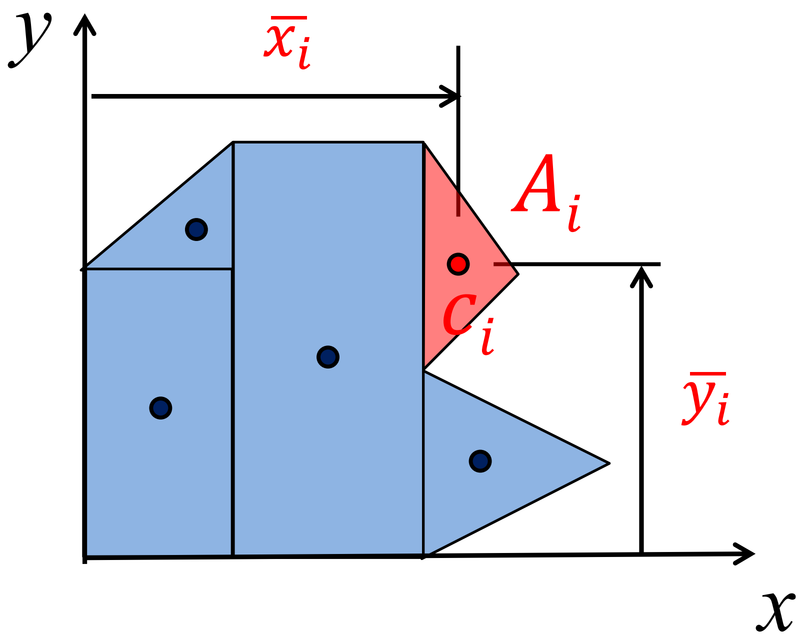
$$\bar{x} = \frac{\iint x dA}{A}$$

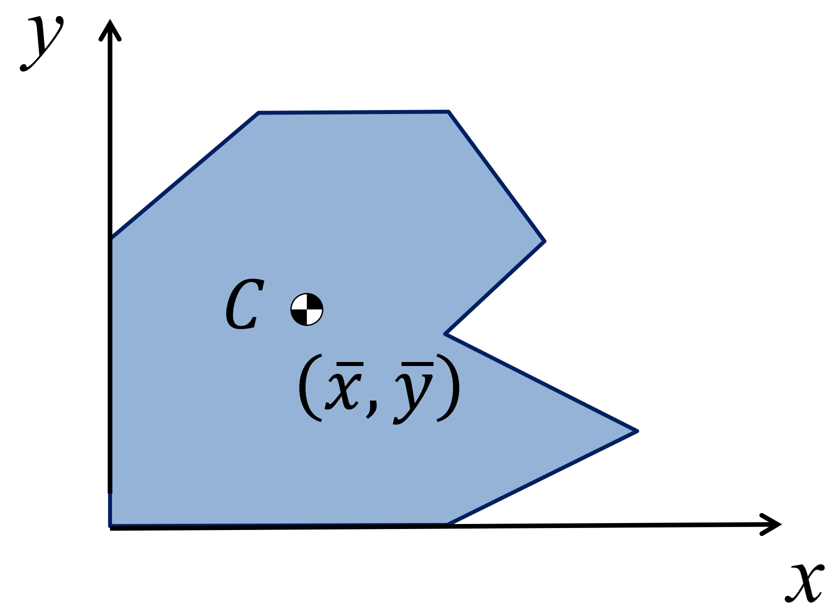
First moment of the area about the y axis

$$\bar{y} = \frac{\iint y dA}{A}$$

First moment of the area about the x axis

If We Can Divide the Area into Simple Shapes With Known Centroid





$$A = \sum A_i$$

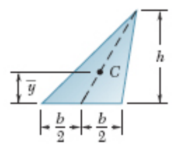
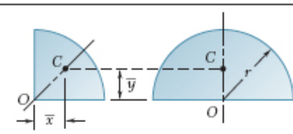
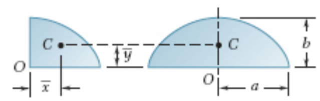
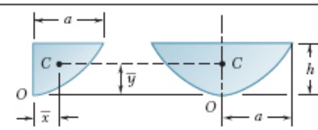
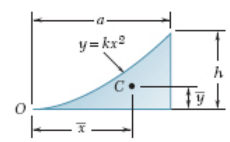
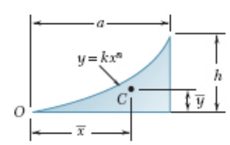
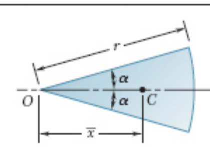
$$\bar{x} = \frac{\sum \bar{x}_i A_i}{A}$$

First moment of the area about the *y* axis

$$\bar{y} = \frac{\sum \bar{y}_i A_i}{A}$$

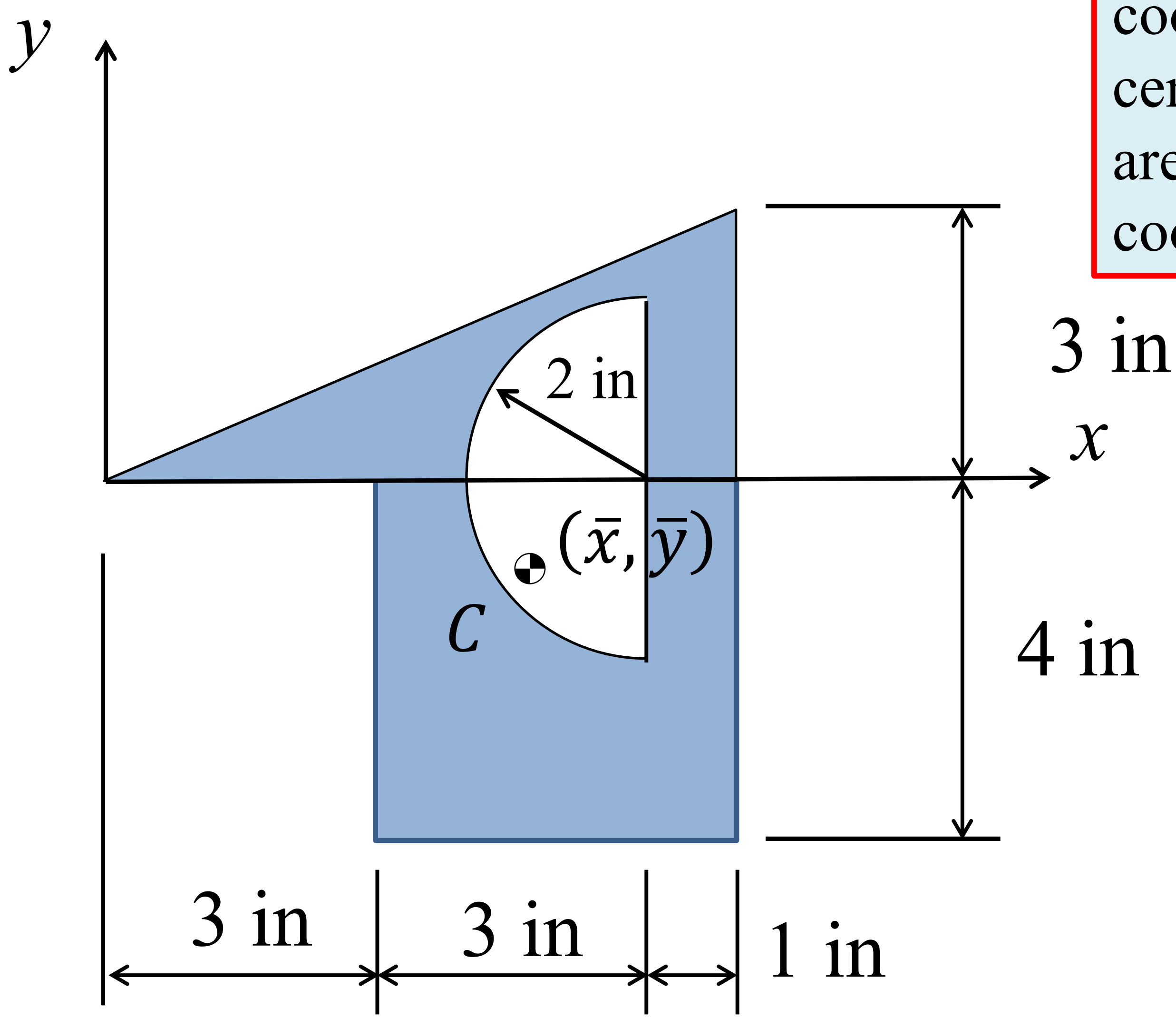
First moment of the area about the *x* axis

Tabulated Centroids of Common Areas Can be Found in the Textbook

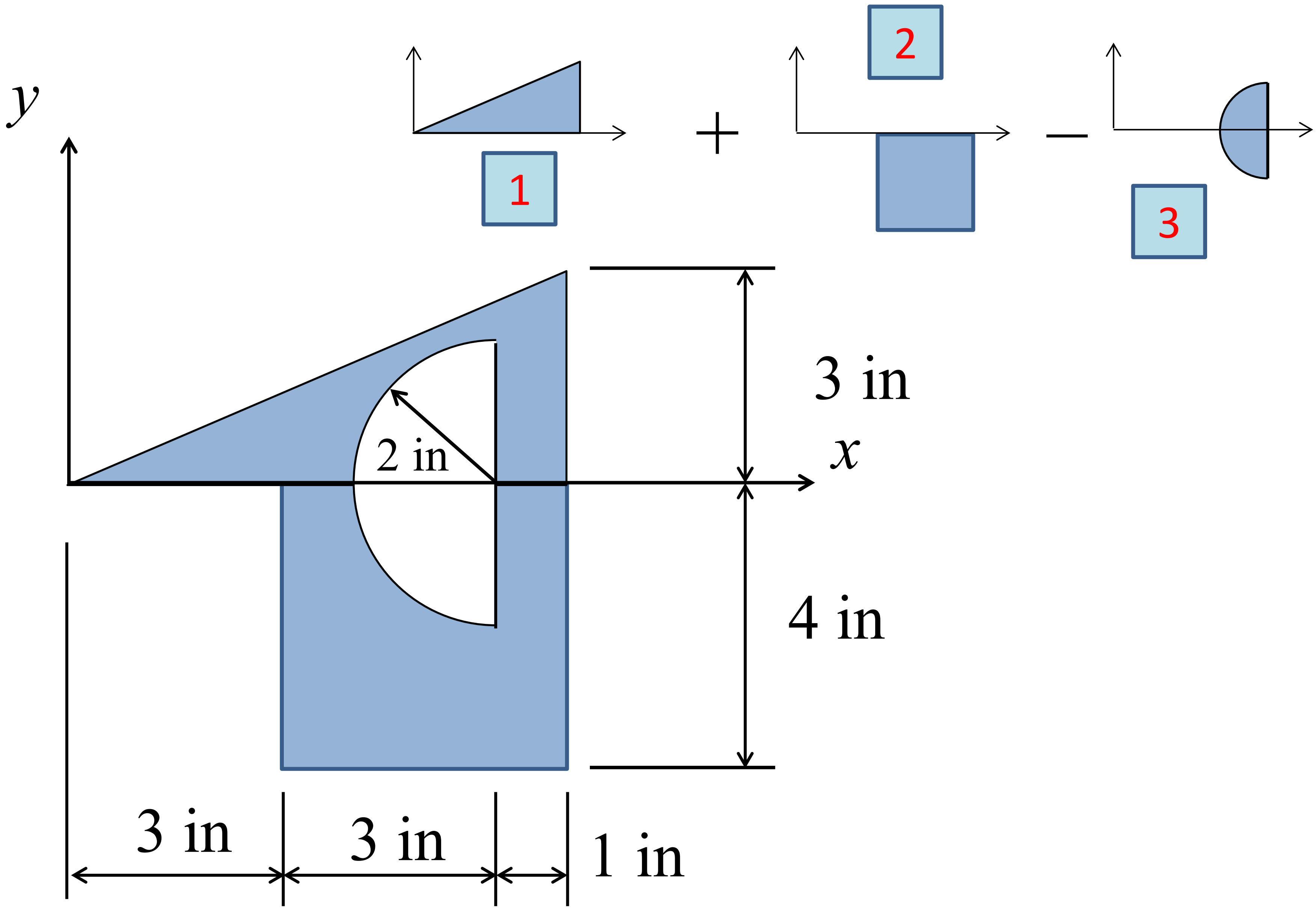
Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Example Problem

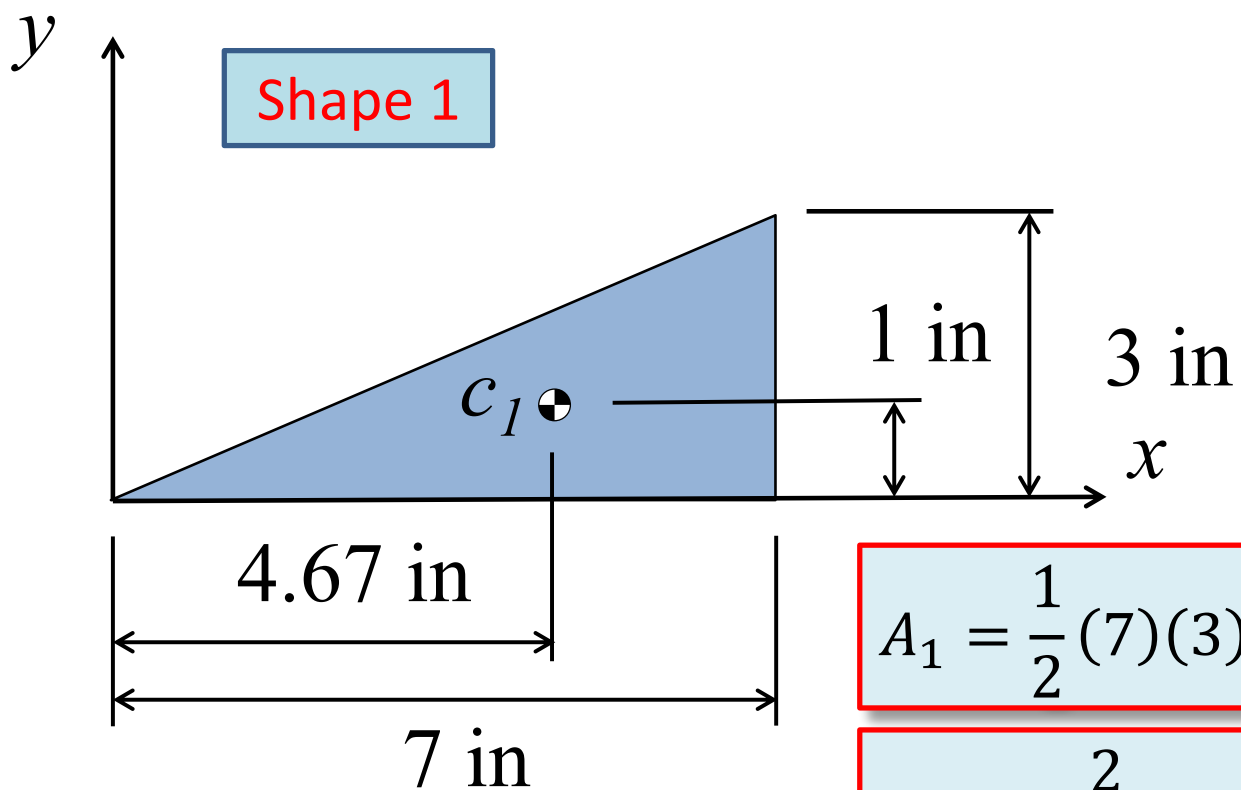
Find the x and y coordinates of the centroid of the shaded area with respect to the coordinate axes shown.



Divide Area into Simple Composite Shapes



Find Area and Location of Centroid of Each Shape Relative to Reference Coordinate Axes

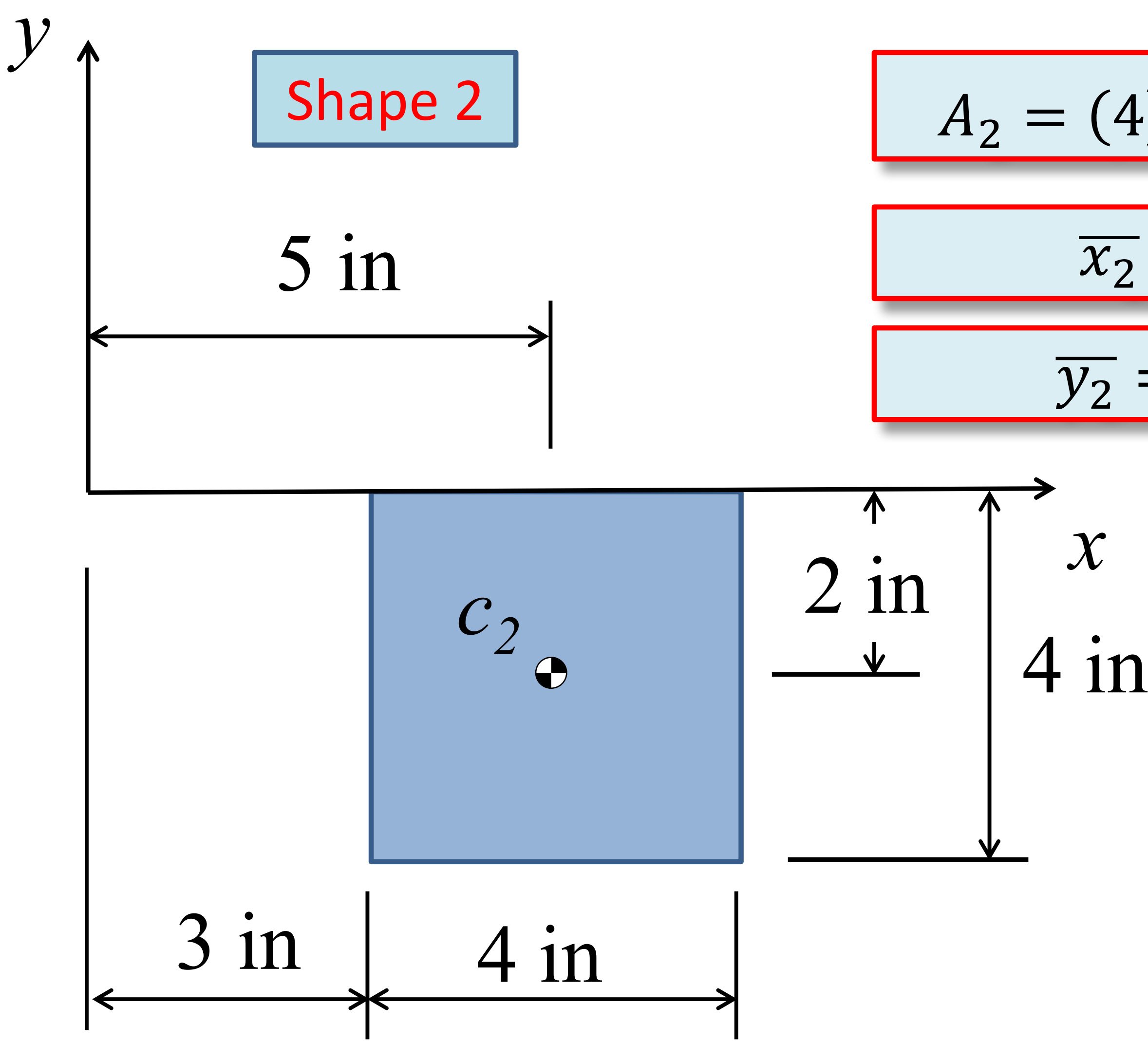


$$A_1 = \frac{1}{2}(7)(3) = 10.5\ \text{in}^2$$

$$\overline{x_1} = \frac{2}{3}(7) = 4.67\ \text{in}$$

$$\overline{y_1} = \frac{1}{3}(3) = 1.0\ \text{in}$$

Find Area and Location of Centroid of Each Shape Relative to Reference Coordinate Axes



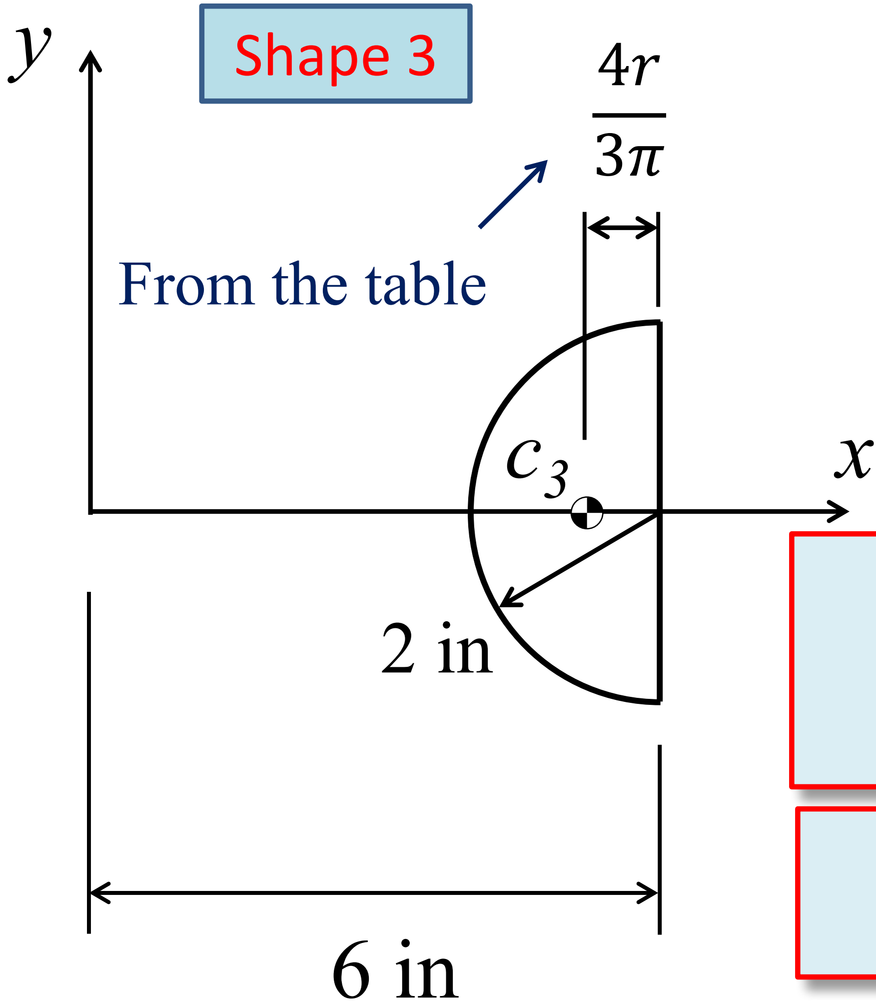
$$A_2 = (4)(4) = 16 \text{ in}^2$$

$$\overline{x_2} = 5.0 \text{ in}$$

$$\overline{y_2} = -2.0 \text{ in}$$

Find Area and Location of Centroid of Each Shape Relative to Reference Coordinate Axes

Shape 3



$$A_3 = -\frac{1}{2}\pi r^2 = -\frac{1}{2}\pi(2^2)$$
$$= -6.2832 \text{ in}^2$$

$$\overline{x_3} = 6 - \frac{4(2)}{3\pi} = 5.1512 \text{ in}$$

$$\overline{y_3} = 0$$

Find the x Coordinate of the Centroid

$\overline{x_1} = 4.67 \text{ in}$

$\overline{x_2} = 5.0 \text{ in}$

$\overline{x_3} = 5.1512 \text{ in}$

$A_1 = 10.5 \text{ in}^2$

$A_2 = 16 \text{ in}^2$

$A_3 = -6.2832 \text{ in}^2$

$$A = \sum A_i = 10.5 + 16 - 6.2832 = 20.2168 \text{ in}^2$$

$$\sum \overline{x_i}\, A_i = (4.67)(10.5) + (5.0)(16) + (5.1512)(-6.2832) = 96.635 \text{ in}^3$$

$$\bar{x} = \frac{\sum \overline{x_i}\, A_i}{A} = \frac{96.635 \text{ in}^3}{20.2168 \text{ in}^2} = 4.78 \text{ in}$$

Find the *y* Coordinate of the Centroid

$\overline{y_1} = 1.0 \text{ in}$

$\overline{y_2} = -2.0 \text{ in}$

$\overline{y_3} = 0$

$A_1 = 10.5 \text{ in}^2$

$A_2 = 16 \text{ in}^2$

$A_3 = -6.2832 \text{ in}^2$

$$A = \sum A_i = 10.5 + 16 - 6.2832 = 20.2168 \text{ in}^2$$

$$\sum \bar{y}_i A_i = (1.0)(10.5) + (-2.0)(16) + (0)(-6.2832) = -21.5 \text{ in}^3$$

$$\bar{y} = \frac{\sum \bar{y}_i A_i}{A} = \frac{-21.5 \text{ in}^3}{20.2168 \text{ in}^2} = -1.06 \text{ in}$$

Coordinates of the Centroid