Three-Dimensional Equilibrium of a Point Example

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General procedure for the Analysis of Bodies in Static Equilibrium

- Choose the free body to isolate;
- Draw a **Free Body Diagram (FBD)** of the body;
  - Isolate the body from all of its surroundings,
  - Magnitudes and directions of all known and unknown forces acting on the body should be included and clearly indicated,
  - Indicate dimensions on the FBD,
- Write the **equations of equilibrium** and solve the equations for the unknown quantities.

Scalar Equations of Static Equilibrium for Concurrent Force Systems

$$\vec{R} = \sum \vec{F_i} = \vec{0}$$

Two-dimensional (planar) body with concurrent forces

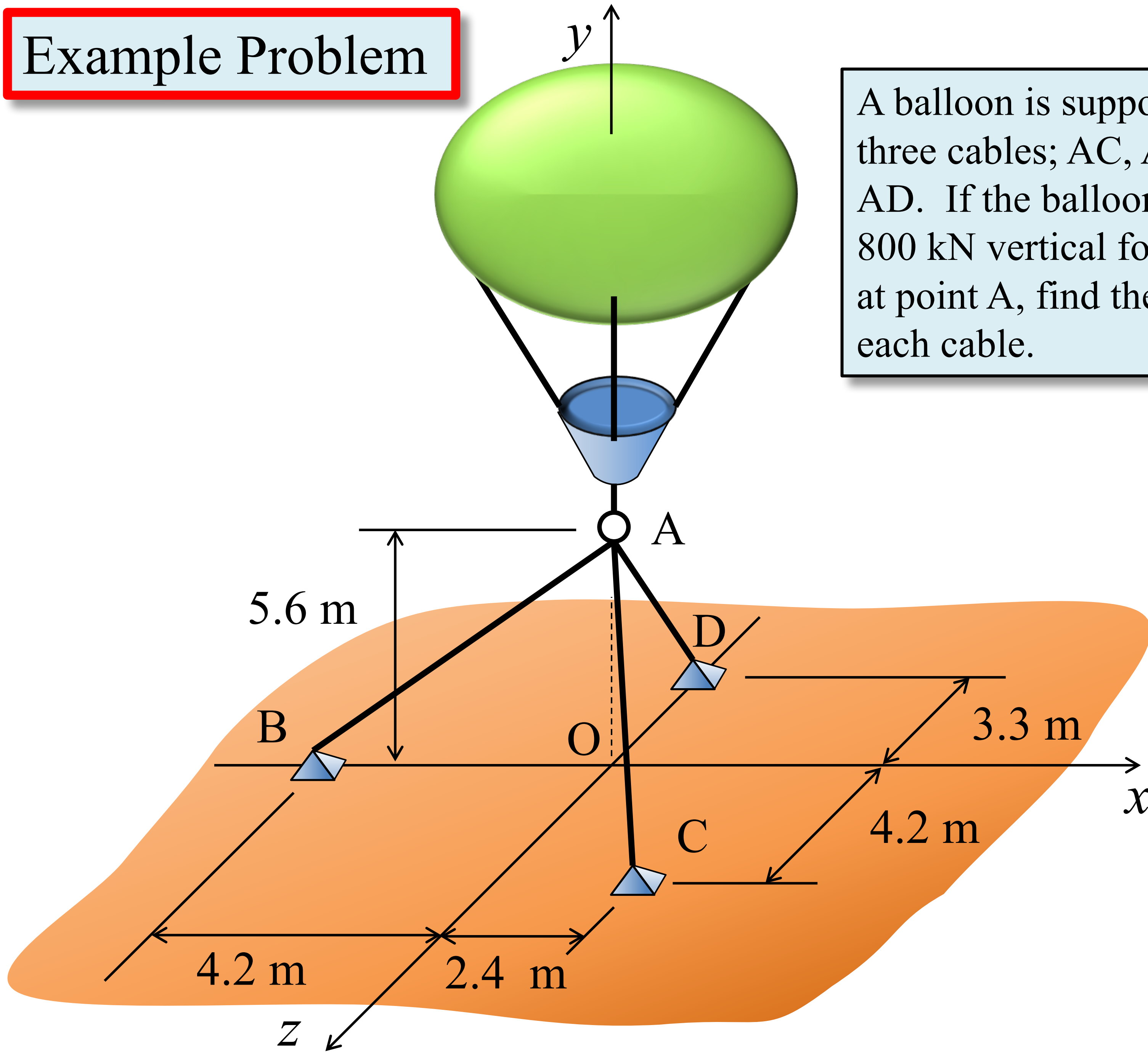
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

Three-dimensional body with concurrent forces

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

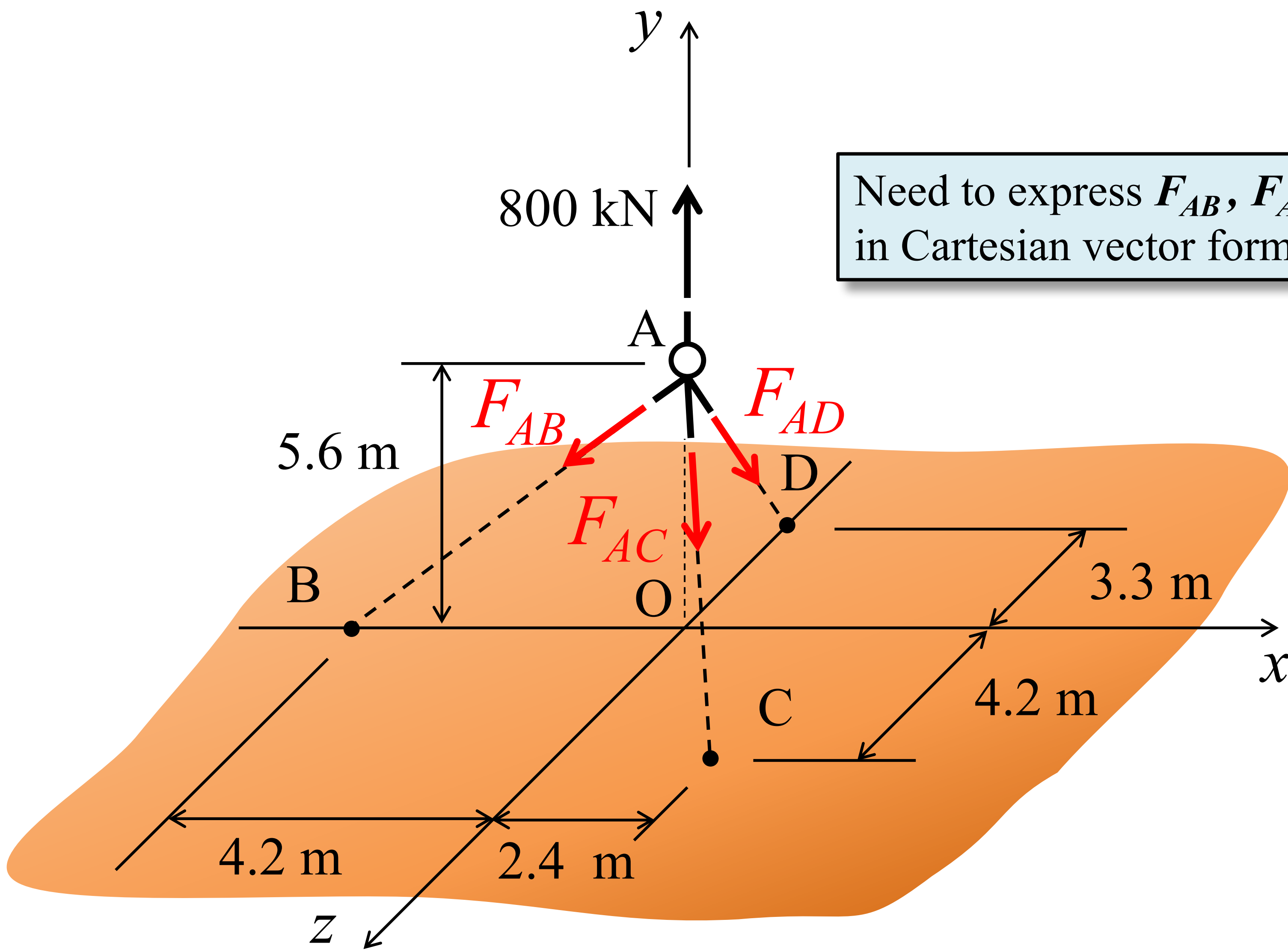
Example Problem

A balloon is supported by three cables; AC, AB, and AD. If the balloon exerts an 800 kN vertical force upward at point A, find the tension in each cable.



Free Body Diagram of Point A

Need to express $\boldsymbol{F_{AB}}$, $\boldsymbol{F_{AC}}$, and $\boldsymbol{F_{AD}}$ in Cartesian vector form



Express Force F_{AB} in Cartesian Vector Form

Coordinates of Point B: (– 4.2, 0, 0)

Coordinates of Point A: (0, 5.60, 0)

$$d_x = x_B - x_A = -4.2 - 0 = -4.2 \text{ m}$$

$$d_y = y_B - y_A = 0 - 5.60 = -5.6 \text{ m}$$

$$d_z = z_B - z_A = 0 - 0 = 0$$

Tip minus Tail

$$d = \sqrt{(-4.2)^2 + (-5.6)^2 + (0)^2} = 7.0 \text{ m}$$

$$F_{ABx} = \left(\frac{(x_B - x_A)}{d}\right) F_{AB} = \left(\frac{-4.2}{7.0}\right) F_{AB} = -0.60 F_{AB}$$

$$F_{ABy} = \left(\frac{(y_B - y_A)}{d}\right) F_{AB} = \left(\frac{-5.6}{7.0}\right) F_{AB} = -0.80 F_{AB}$$

$$F_{ABz} = \left(\frac{(z_B - z_A)}{d}\right) F_{AB} = \left(\frac{0}{7.0}\right) F_{AB} = 0$$

$$\boldsymbol{F_{AB}} = -0.60 F_{AB}\hat{\imath} - 0.80 F_{AB}\hat{\jmath}$$

Express Force F_{AC} in Cartesian Vector Form

Coordinates of Point C: (2.4, 0, 4.20)

Coordinates of Point A: (0, 5.60, 0)

$$d_x = x_B - x_A = 2.4 - 0 = 2.4 \text{ m}$$

$$d_y = y_B - y_A = 0 - 5.60 = -5.6 \text{ m}$$

$$d_z = z_B - z_A = 4.2 - 0 = 4.2 \text{ m}$$

Tip minus Tail

$$d = \sqrt{(2.4)^2 + (-5.6)^2 + (4.2)^2} = 7.4 \text{ m}$$

$$F_{ACx} = \left(\frac{(x_B - x_A)}{d}\right) F_{AC} = \left(\frac{2.4}{7.4}\right) F_{AC} = 0.324 F_{AC}$$

$$F_{ACy} = \left(\frac{(y_B - y_A)}{d}\right) F_{AC} = \left(\frac{-5.6}{7.4}\right) F_{AC} = -0.757 F_{AC}$$

$$F_{ACz} = \left(\frac{(z_B - z_A)}{d}\right) F_{AC} = \left(\frac{4.2}{7.4}\right) F_{AC} = 0.568 F_{AC}$$

$$\boldsymbol{F_{AC}} = 0.324 F_{AC}\hat{\imath} - 0.757 F_{AC}\hat{\jmath} + 0.568 F_{AC}\hat{k}$$

Express Force F_{AD} in Cartesian Vector Form

Coordinates of Point D: (0, 0, – 3.3)

Coordinates of Point A: (0, 5.60, 0)

$$d_x = x_D - x_A = 0 - 0 = 0$$

$$d_y = y_D - y_A = 0 - 5.6 = -5.6 \text{ m}$$

$$d_z = z_D - z_A = -3.3 - 0 = -3.3 \text{ m}$$

Tip minus Tail

$$d = \sqrt{(0)^2 + (-5.6)^2 + (-3.3)^2} = 6.4 \text{ m}$$

$$F_{ADx} = \left(\frac{(x_B - x_A)}{d}\right) F_{AD} = \left(\frac{0}{6.4}\right) F_{AD} = 0$$

$$F_{ADy} = \left(\frac{(y_B - y_A)}{d}\right) F_{AD} = \left(\frac{-5.6}{6.4}\right) F_{AD} = -0.862 F_{AD}$$

$$F_{ADz} = \left(\frac{(z_B - z_A)}{d}\right) F_{AD} = \left(\frac{-3.3}{6.4}\right) F_{AD} = -0.508 F_{AD}$$

$$\boldsymbol{F_{AD}} = -0.862 F_{AD}\hat{\jmath} - 0.508 F_{AD}\hat{k}$$

Forces in Cartesian Vector Form

$$\boldsymbol{F}_{\boldsymbol{AB}} = -0.60F_{AB}\hat{\imath} - 0.80F_{AB}\hat{\jmath}$$

$$\boldsymbol{F}_{\boldsymbol{AC}} = 0.324F_{AC}\hat{\imath} - 0.757F_{AC}\hat{\jmath} + 0.568F_{AC}\hat{k}$$

$$\boldsymbol{F}_{\boldsymbol{AD}} = -0.862F_{AD}\hat{\jmath} - 0.508F_{AD}\hat{k}$$

$$\boldsymbol{F}_{\boldsymbol{balloon}} = 800\hat{\jmath}\ \text{kN}$$

Equilibrium Equations

$$\sum F_x = 0$$

1 $$-0.60F_{AB} + 0.324F_{AC} = 0$$

$$\sum F_y = 0$$

2 $$-0.80F_{AB} - 0.757F_{AC} - 0.862F_{AD} + 800 = 0$$

$$\sum F_z = 0$$

3 $$0.568F_{AC} - 0.508F_{AD} = 0$$

Solve Equilibrium Equations

Equation 1

$$F_{AB} = \frac{-0.324}{-0.60} F_{AC}$$

$$F_{AB} = 0.540 F_{AC}$$

Equation 3

$$F_{AD} = \frac{-0.568}{-0.508} F_{AC}$$

$$F_{AD} = 1.118\, F_{AC}$$

Equation 2

$$-0.80F_{AB} - 0.757F_{AC} - 0.862F_{AD} + 800 = 0$$

$$-0.80(0.540F_{AC}) - 0.757F_{AC} - 0.862(1.118\, F_{AC}) + 800 = 0$$

$$F_{AC} = 371.6 \text{ kN}$$

$$F_{AB} = 0.540(371.6)$$

$$F_{AB} = 200.7 \text{ kN}$$

$$F_{AD} = 1.118(371.6)$$

$$F_{AD} = 415.4 \text{ kN}$$

Free Body Diagram of Point A Showing Results