Virtual Work Truss Example

Temperature and Fabrication Errors

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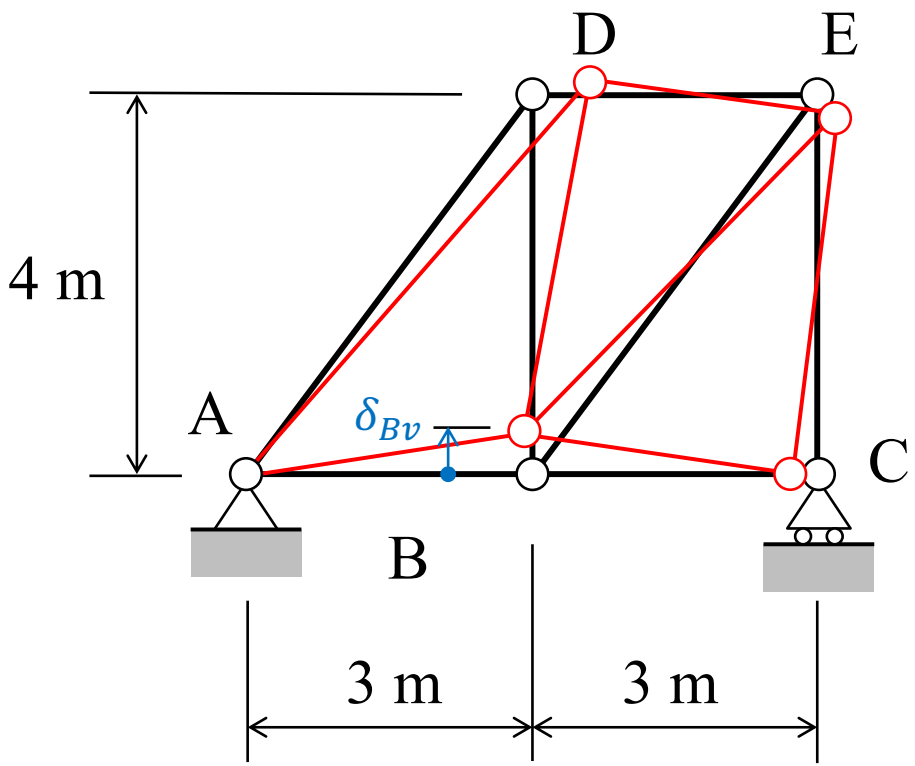
Example Using the Principle of Virtual Work

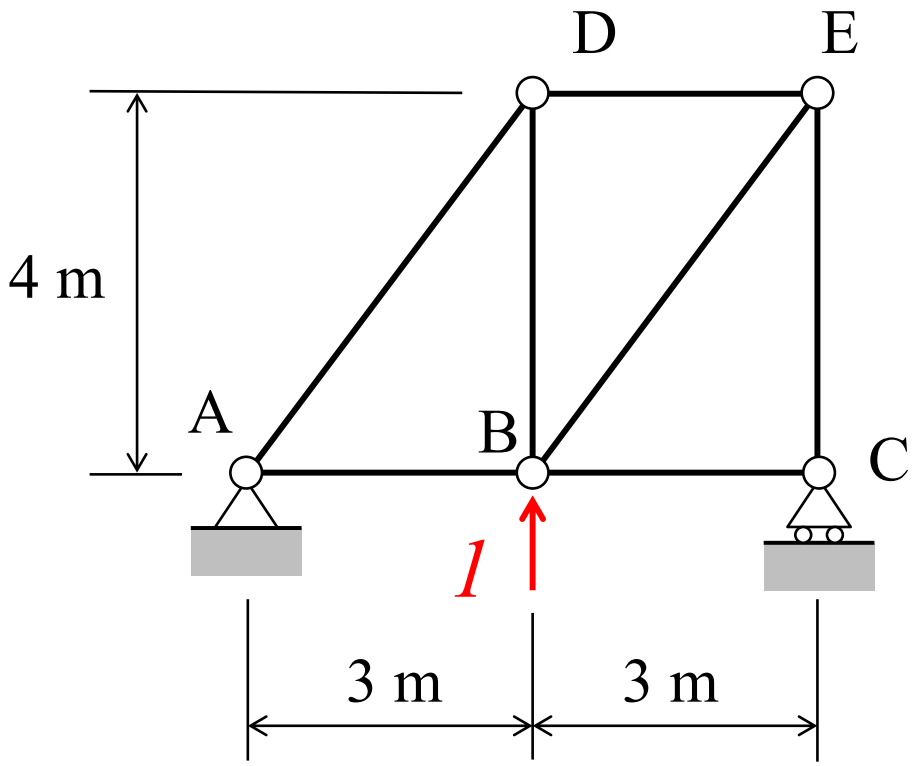


Consider the idealized truss structure from the previous example. Truss members AD and DE increase in temperature 40°C. Member CE decreases in temperature 30°C. In addition, member BE is fabricated 0.5 cm too short.

For all truss members use:
$A = 25$ cm^2
$E = 210$ GPa
$\alpha_i = 12\times10^{-6}$/°C

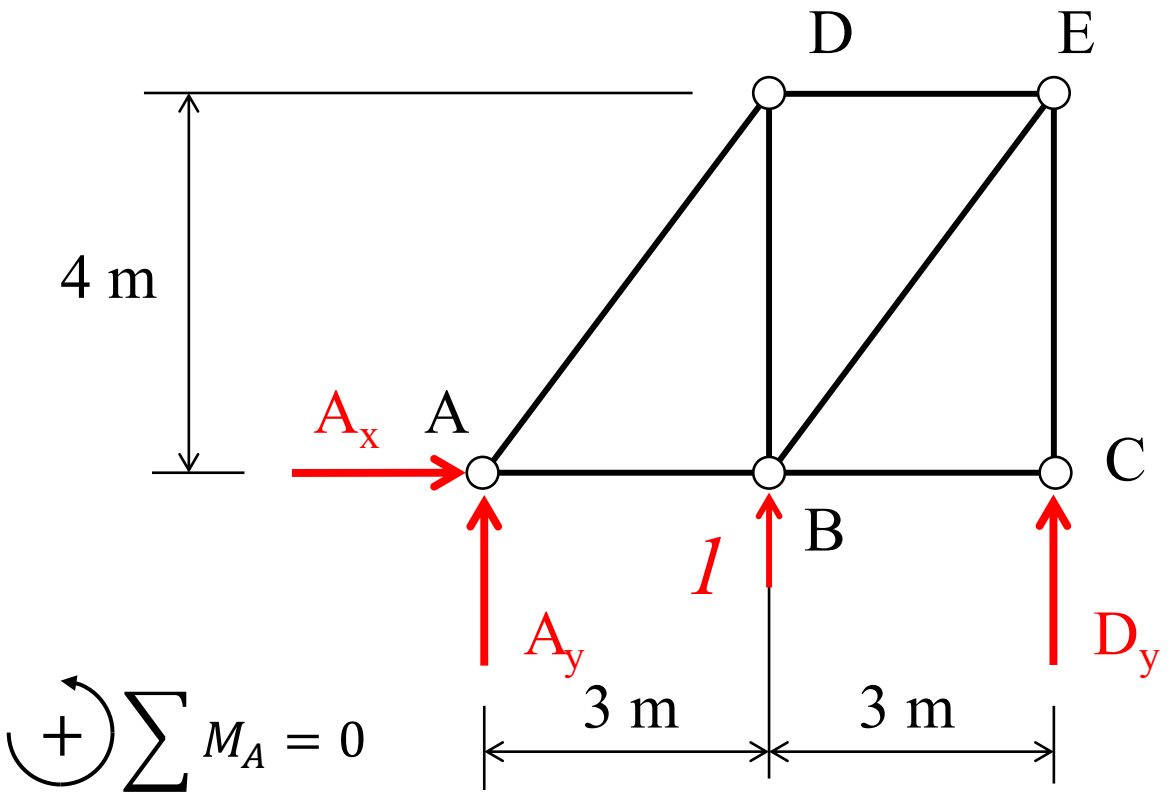
Find the vertical displacement of point B using the Principle of Virtual Work

Virtual System to Measure δ_{Bv}



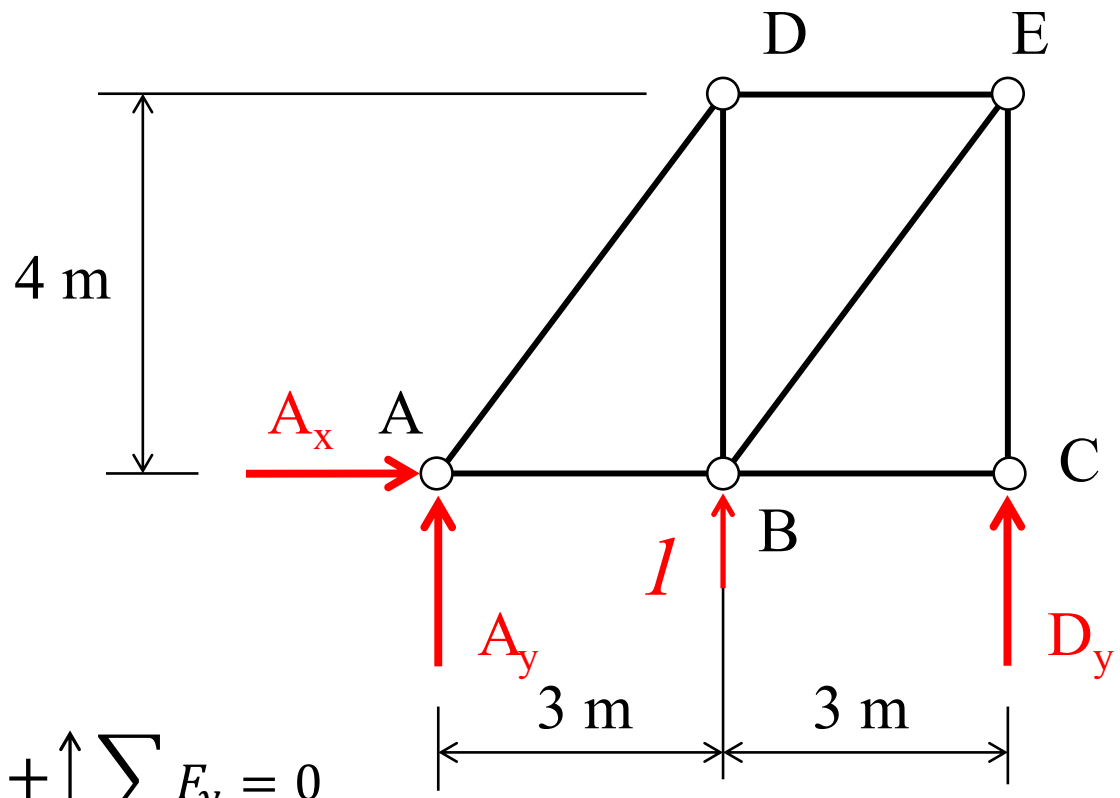
1. Remove all loads (if any) from the structure;
2. Apply a unit, dimensionless virtual load **in-line** with the real displacement, δ_{Bv}, that we want to find;
3. Perform a truss analysis to find all truss member virtual axial forces, F_{Qi}

Find Support Reactions



$D_y = -0.5$

Find Support Reactions

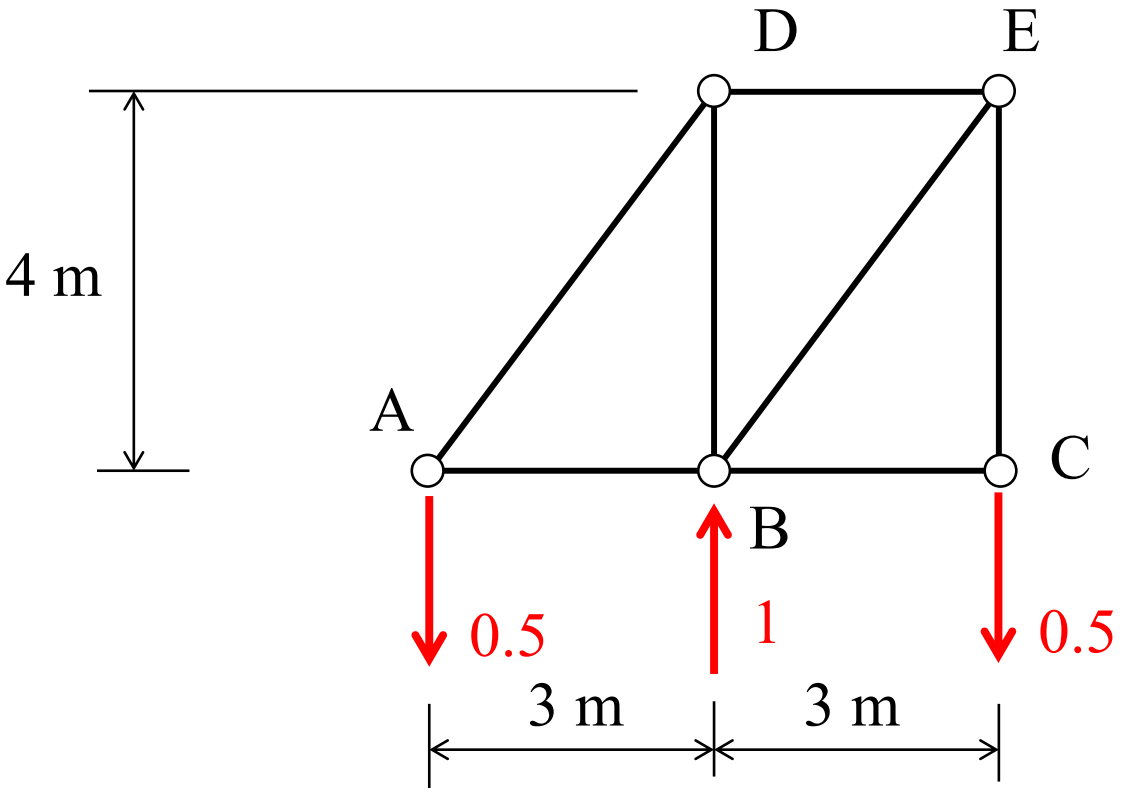


$$+\uparrow \sum F_y = 0$$

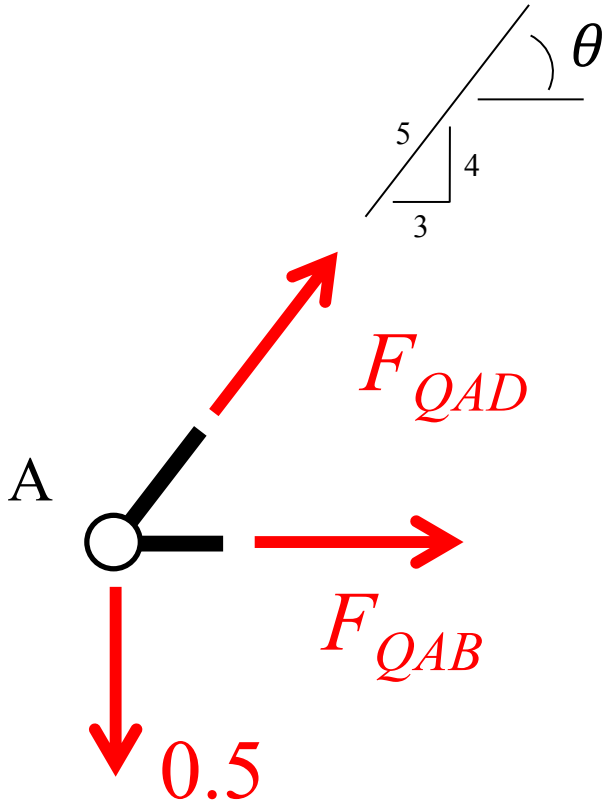
$$\xrightarrow{+} \sum F_x = 0$$

$A_y = -0.5$

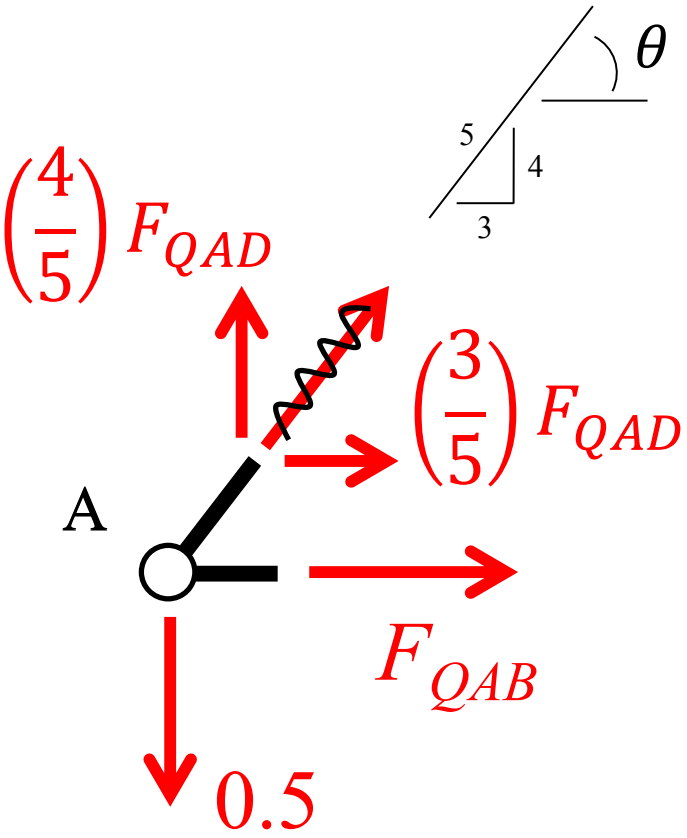
Virtual System Support Reactions



FBD of Joint A



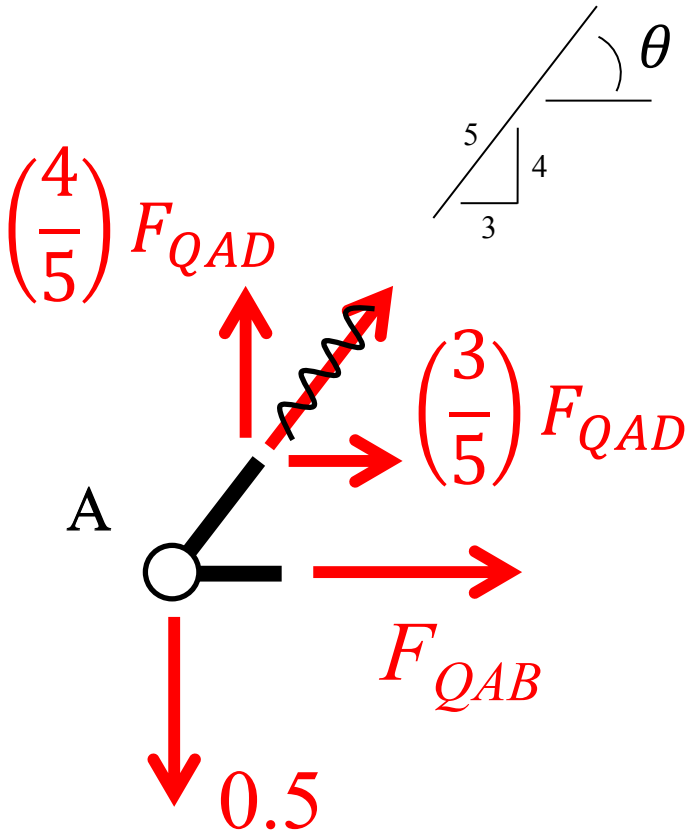
FBD of Joint A



$$+\uparrow \sum F_y = 0$$

$F_{QAD} = 0.625$

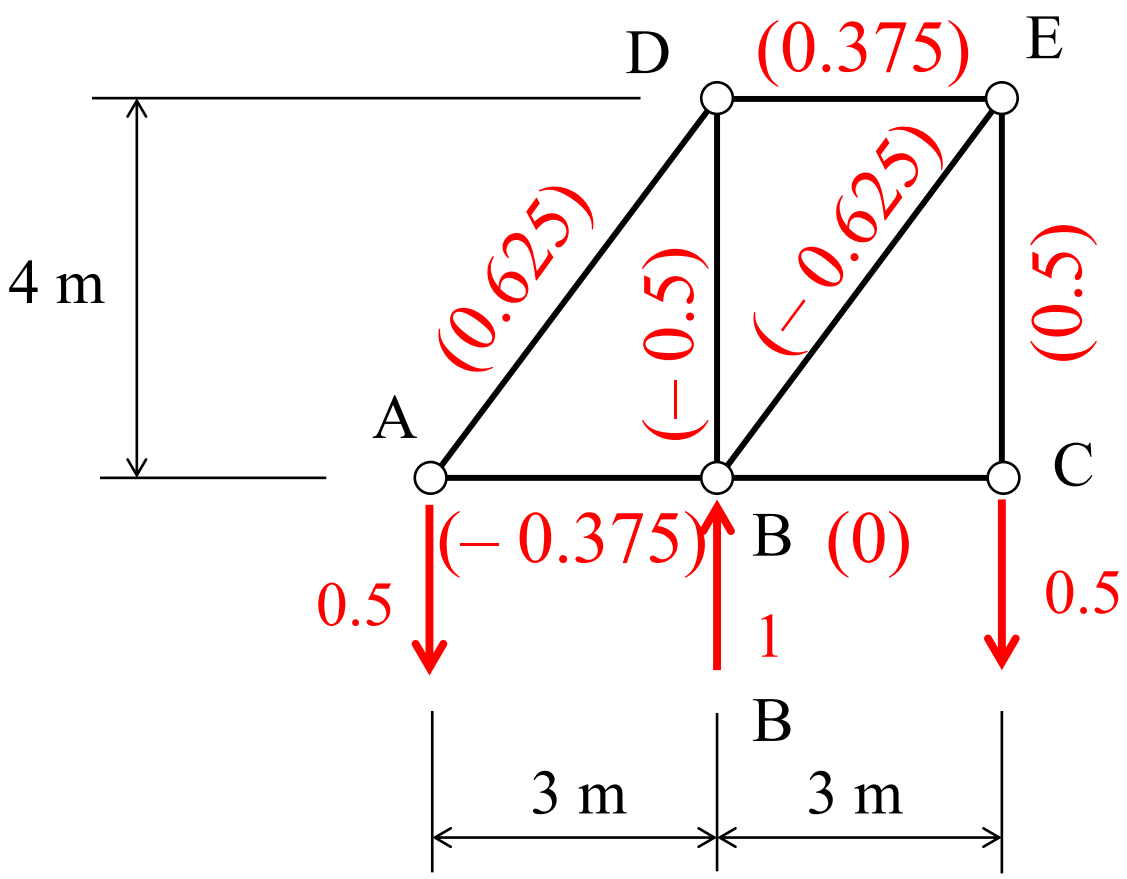
FBD of Joint A



$$\xrightarrow{+} \sum F_x = 0$$

$F_{QAB} = -0.375$

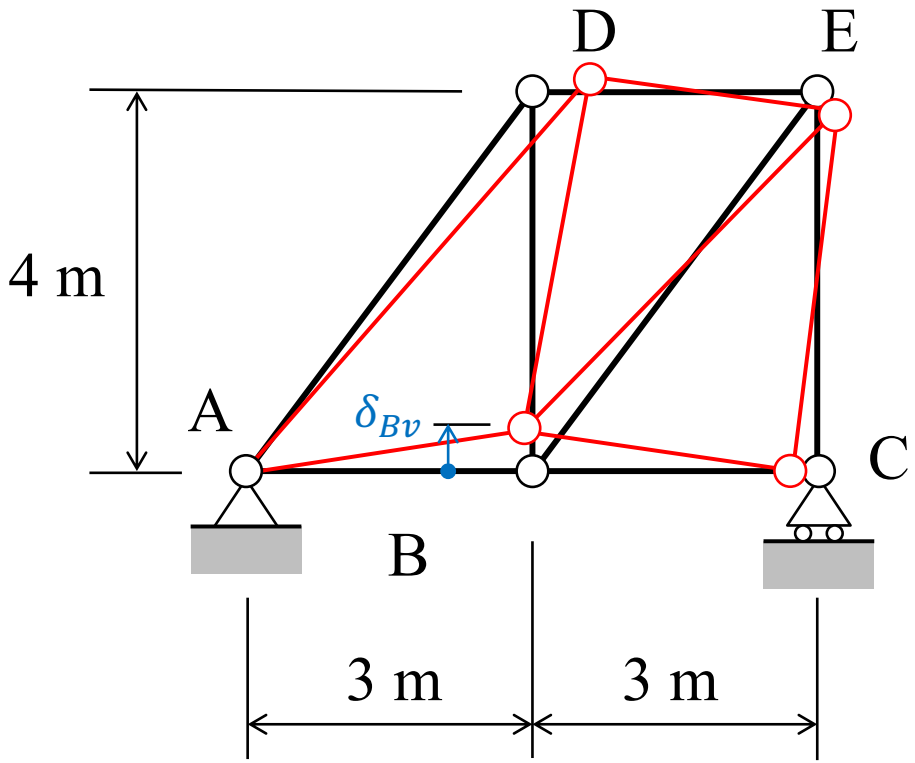
Virtual System Results on a FBD of the Entire Truss



Virtual truss member forces, F_{Qi}

Tension is Positive

Step 2 – Use the Principle of Virtual Work to Find δ_{Bv}



For all truss members use:
$A = 25\ \text{cm}^2$
$E = 210\ \text{GPa}$
$\alpha_i = 12\times10^{-6}/°\text{C}$

From Step 1– virtual analysis

$$1 \cdot \delta_{Bv} = \sum_{i=1}^{n_T} F_{Qi}\alpha_i \Delta T_i L_i + \sum_{i=1}^{n_{fabr}} F_{Qi}\Delta L_{ifabr}$$

Use a Table to Organize Virtual Work Calculations

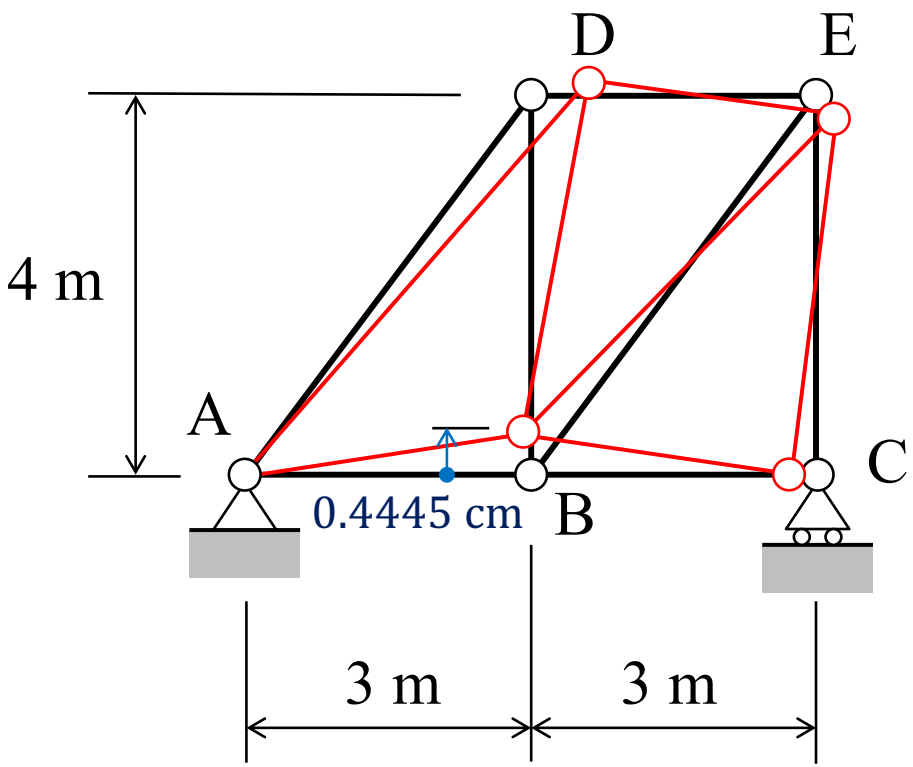
Member	$\alpha\times10^6$ (/°C)	ΔT (°C)	Δ_{fabr} (cm)	L (m)	F_Q	U_Q (cm)
AD	12	40	0	5	0.625	0.15
AB	12	0	0	3	– 0.375	0
BD	12	0	0	4	– 0.5	0
DE	12	40	0	3	0.375	0.054
BE	12	0	–0.5	5	– 0.625	0.3125
BC	12	0	0	3	0	0
CE	12	–30	0	4	0.5	– 0.072
Total						**0.4445**

Sample Calculations

$$F_{QAD} \propto_{AD} \Delta T_{AD} L_{AD} = 0.625(12\times10^{-6}/°\text{C})(40°\text{C})(5\text{ m})\left(\frac{100\text{ cm}}{\text{m}}\right) = 0.15\text{ cm}$$

$$F_{QBE}\Delta L_{fabrAD} = -0.625(-0.5) = 0.3125\text{ cm}$$

Results for δ_{Bv}



Positive sign indicates that deflection is in the same direction of the virtual force

$\delta_{Bv} = 0.4445$ cm upward