Chapter 27

Magnetic Fields

A Brief History of Magnetism

- 13th century BC
  - Chinese used a compass
    - Uses a magnetic needle
    - Probably an invention of Arabic or Indian origin
- 800 BC
  - Greeks
    - Discovered magnetite (Fe_3O_4) attracts pieces of iron

A Brief History of Magnetism, 2

- 1269
  - Pierre de Maricourt found that the direction of a needle near a spherical natural magnet formed lines that encircled the sphere
  - The lines also passed through two points diametrically opposed to each other
  - He called the points poles

A Brief History of Magnetism, 3

- 1600
  - William Gilbert
    - Expanded experiments with magnetism to a variety of materials
    - Suggested the Earth itself was a large permanent magnet

A Brief History of Magnetism, 4

- 1819
  - Hans Christian Oersted
    - Discovered the relationship between electricity and magnetism
    - An electric current in a wire deflected a nearby compass needle

A Brief History of Magnetism, final

- 1820's
  - Faraday and Henry
    - Further connections between electricity and magnetism
    - A changing magnetic field creates an electric field
  - Maxwell
    - A changing electric field produces a magnetic field

Magnetic Poles

- Every magnet, regardless of its shape, has two poles
  - Called north and south poles
  - Poles exert forces on one another
    - Similar to the way electric charges exert forces on each other
    - Like poles repel each other
      - N-N or S-S
    - Unlike poles attract each other
      - N-S

Magnetic Poles, cont.

- The poles received their names due to the way a magnet behaves in the Earth's magnetic field
- If a bar magnet is suspended so that it can move freely, it will rotate
  - The magnetic north pole points toward the Earth's north geographic pole
    - This means the Earth's north geographic pole is a magnetic south pole
    - Similarly, the Earth's south geographic pole is a magnetic north pole

Magnetic Poles, final

- The force between two poles varies as the inverse square of the distance between them
- A single magnetic pole has never been isolated
  - In other words, magnetic poles are always found in pairs
  - All attempts so far to detect an isolated magnetic pole has been unsuccessful
    - No matter how many times a permanent magnetic is cut in two, each piece always has a north and south pole

Magnetic Fields

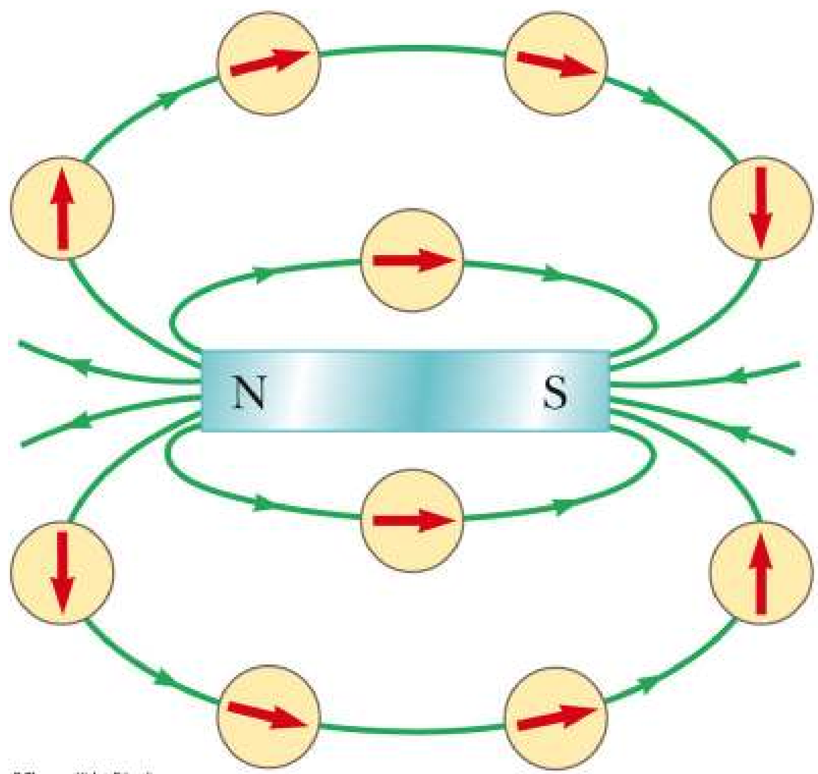
- Reminder: an electric field surrounds any electric charge
- The region of space surrounding any *moving* electric charge also contains a magnetic field
- A magnetic field also surrounds a magnetic substance making up a permanent magnet

Magnetic Fields, cont.

- A vector quantity
- Symbolized by $\vec{\mathbf{B}}$
- Direction is given by the direction a north pole of a compass needle points in that location
- Magnetic field lines can be used to show how the field lines, as traced out by a compass, would look

Magnetic Field Lines, Bar Magnet Example

- The compass can be used to trace the field lines
- The lines outside the magnet point from the North pole to the South pole
- Use the active figure to trace the field lines



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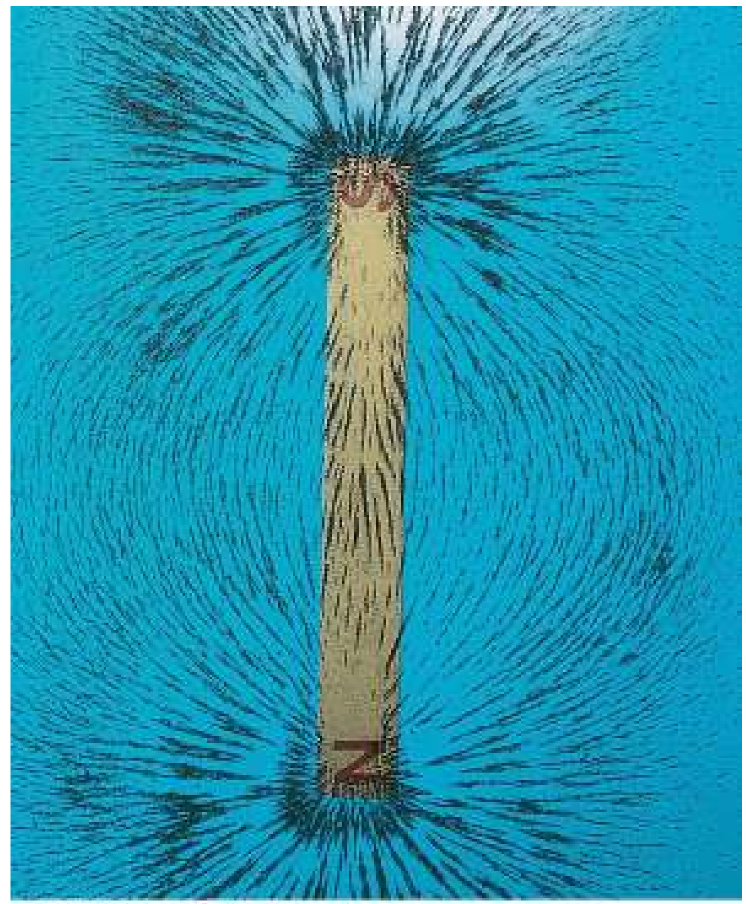
ACTIVE FIGURE

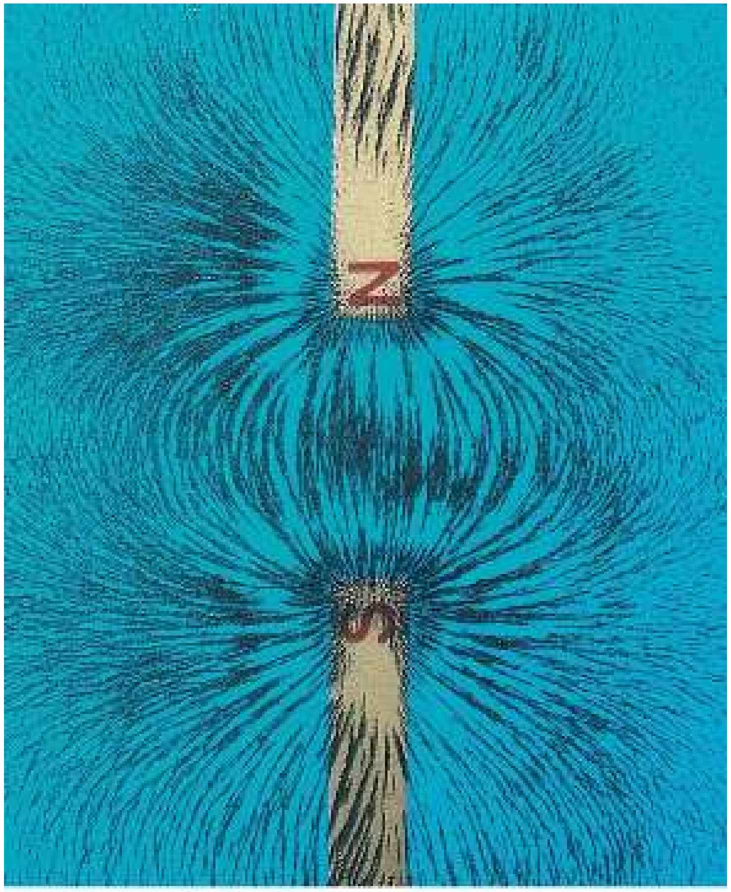
Magnetic Field Lines, Bar Magnet

- Iron filings are used to show the pattern of the electric field lines
- The direction of the field is the direction a north pole would point



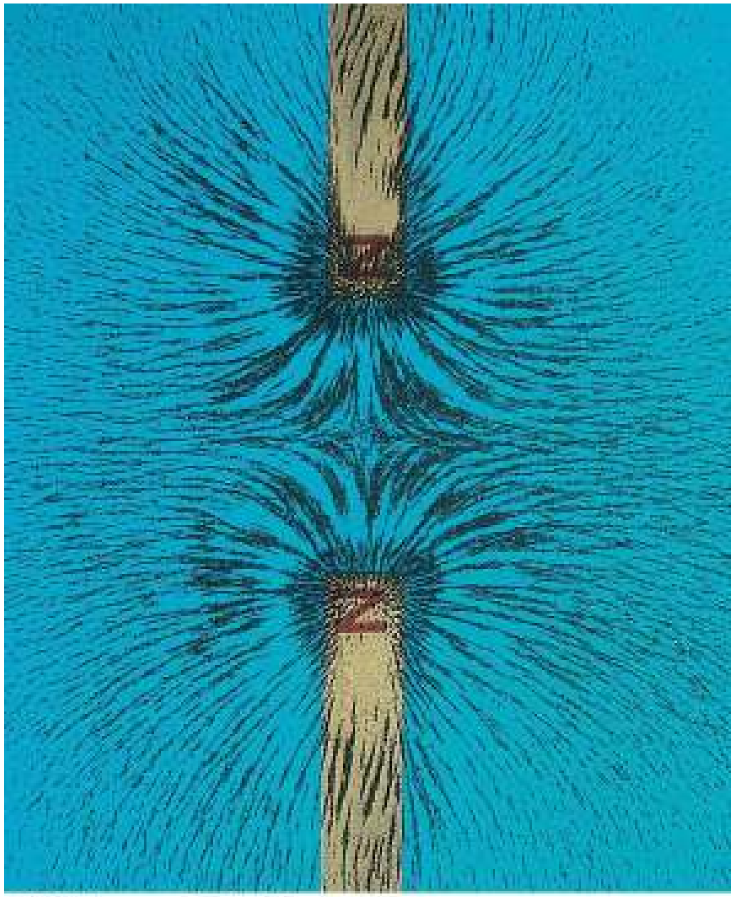
Magnetic Field Lines, Unlike Poles

- Iron filings are used to show the pattern of the electric field lines
- The direction of the field is the direction a north pole would point
  - Compare to the electric field produced by an electric dipole



Magnetic Field Lines, Like Poles

- Iron filings are used to show the pattern of the electric field lines
- The direction of the field is the direction a north pole would point
  - Compare to the electric field produced by like charges



Definition of Magnetic Field

- The magnetic field at some point in space can be defined in terms of the magnetic force, $\vec{\mathbf{F}}_B$
- The magnetic force will be exerted on a charged particle moving with a velocity, $\vec{\mathbf{v}}$
  - Assume (for now) there are no gravitational or electric fields present

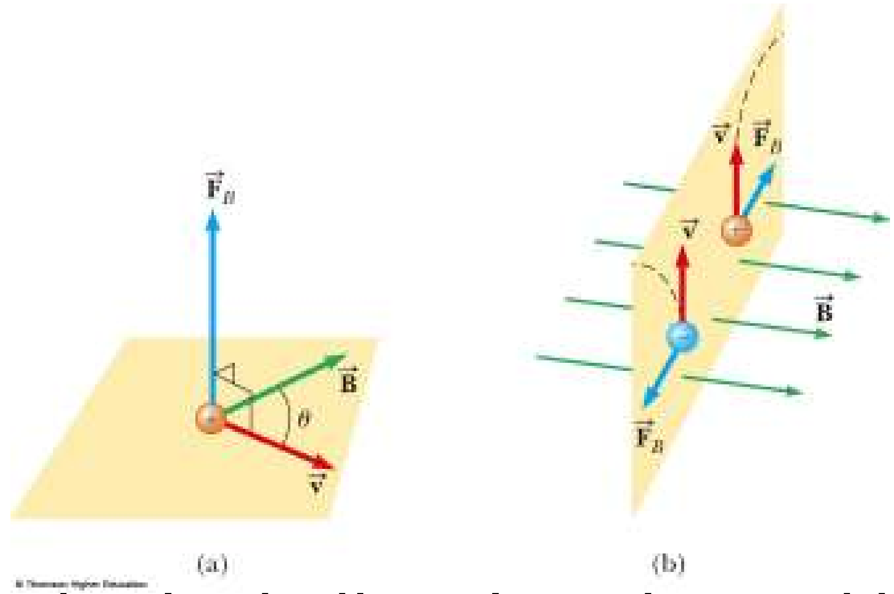
Force on a Charge Moving in a Magnetic Field

- The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge, q, and to the speed, v, of the particle
- When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero
- When the particle's velocity vector makes any angle $\theta \neq 0$ with the field, the force acts in a direction perpendicular to both the velocity and the field

F_B on a Charge Moving in a Magnetic Field, final

- The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction
- The magnitude of the magnetic force is proportional to sin θ, where θ is the angle the particle's velocity makes with the direction of the magnetic field

More About Direction



- $\vec{F}_B$ is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$
- Oppositely directed forces exerted on oppositely charged particles will cause the particles to move in opposite directions

Force on a Charge Moving in a Magnetic Field, Formula

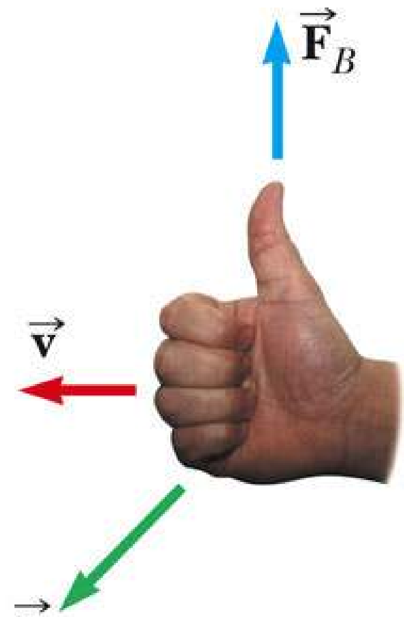
- The properties can be summarized in a vector equation:

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

  - $\vec{\mathbf{F}}_B$ is the magnetic force
  - q is the charge
  - $\mathbf{v}$ is the velocity of the moving charge
  - $\vec{\mathbf{B}}$ is the magnetic field

Direction: Right-Hand Rule #1

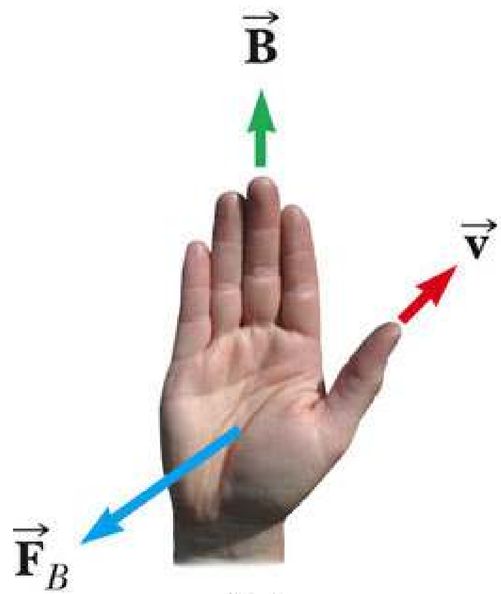
- The fingers point in the direction of $\vec{\mathbf{v}}$
- $\vec{\mathbf{B}}$ comes out of your palm
  - Curl your fingers in the direction of $\vec{\mathbf{B}}$
- The thumb points in the direction of $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ which is the direction of $\vec{\mathbf{F}}_B$



(a)

Direction: Right-Hand Rule #2

- Alternative to Rule #1
- Thumb is in the direction of $\vec{\mathbf{v}}$
- Fingers are in the direction of $\vec{\mathbf{B}}$
- Palm is in the direction of $\vec{\mathbf{F}}_B$
  - On a positive particle
  - You can think of this as your hand pushing the particle



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More About Magnitude of F

- The magnitude of the magnetic force on a charged particle is $F_B = |q|\, v\, B \sin \theta$
  - θ is the smaller angle between v and B
  - F_B is zero when the field and velocity are parallel or antiparallel
    - $\theta = 0$ or 180^o
  - F_B is a maximum when the field and velocity are perpendicular
    - $\theta = 90^o$

Differences Between Electric and Magnetic Fields

- Direction of force
  - The electric force acts along the direction of the electric field
  - The magnetic force acts perpendicular to the magnetic field
- Motion
  - The electric force acts on a charged particle regardless of whether the particle is moving
  - The magnetic force acts on a charged particle only when the particle is in motion

More Differences Between Electric and Magnetic Fields

- Work
  - The electric force does work in displacing a charged particle
  - The magnetic force associated with a steady magnetic field does no work when a particle is displaced
    - This is because the force is perpendicular to the displacement

Work in Fields, cont.

- The kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone
- When a charged particle moves with a given velocity through a magnetic field, the field can alter the direction of the velocity, but not the speed or the kinetic energy

Units of Magnetic Field

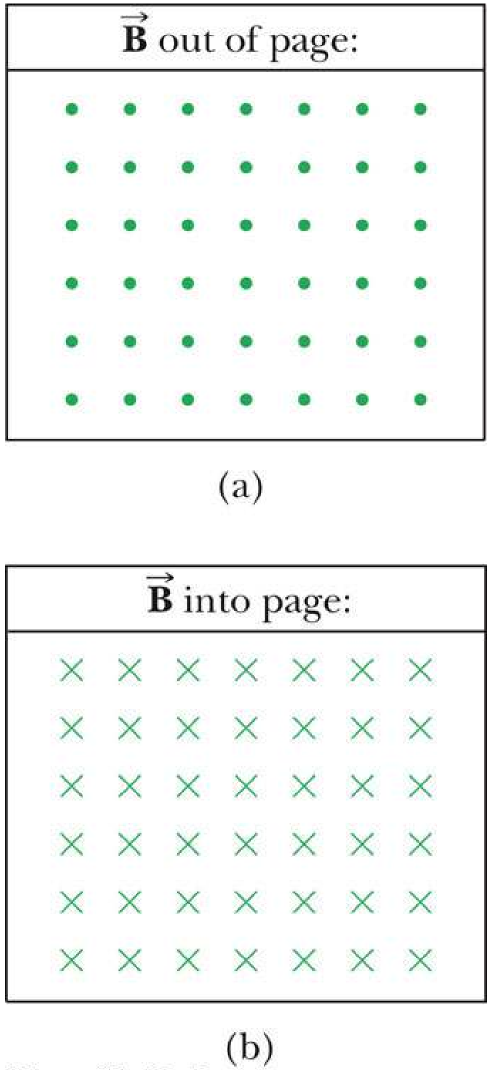
- The SI unit of magnetic field is the tesla (T)

$$T = \frac{Wb}{m^2} = \frac{N}{C \cdot (m/s)} = \frac{N}{A \cdot m}$$

  - Wb is a weber
- A non-SI commonly used unit is a gauss (G)
  - 1 T = 10^4 G

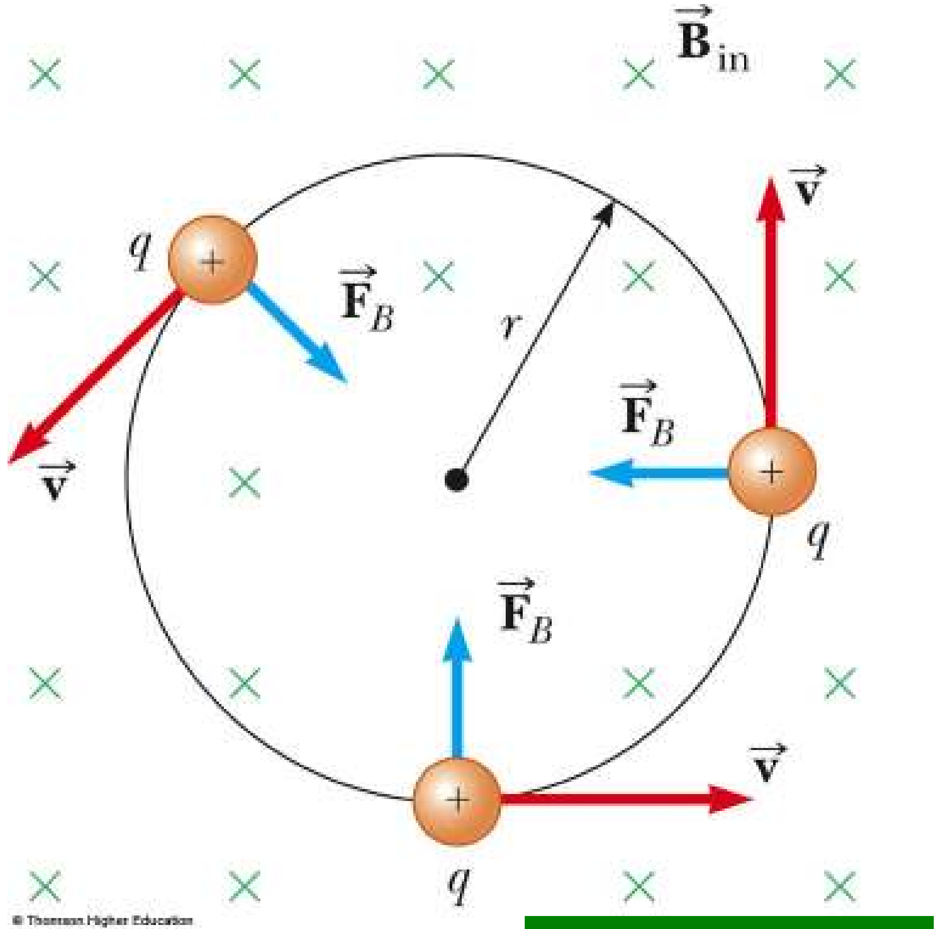
Notation Notes

- When vectors are perpendicular to the page, dots and crosses are used
  - The dots represent the arrows coming out of the page
  - The crosses represent the arrows going into the page



Charged Particle in a Magnetic Field

- Consider a particle moving in an external magnetic field with its velocity perpendicular to the field
- The force is always directed toward the center of the circular path
- The magnetic force causes a centripetal acceleration, changing the direction of the velocity of the particle
- Use the active figure to change the parameters of the particle and observe the motion



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ACTIVE FIGURE

Force on a Charged Particle

- Equating the magnetic and centripetal forces:

$$F_B = qvB = \frac{mv^2}{r}$$

- Solving for r:

$$r = \frac{mv}{qB}$$

  - r is proportional to the linear momentum of the particle and inversely proportional to the magnetic field

More About Motion of Charged Particle

- The angular speed of the particle is

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

  - The angular speed, ω, is also referred to as the **cyclotron frequency**

- The period of the motion is

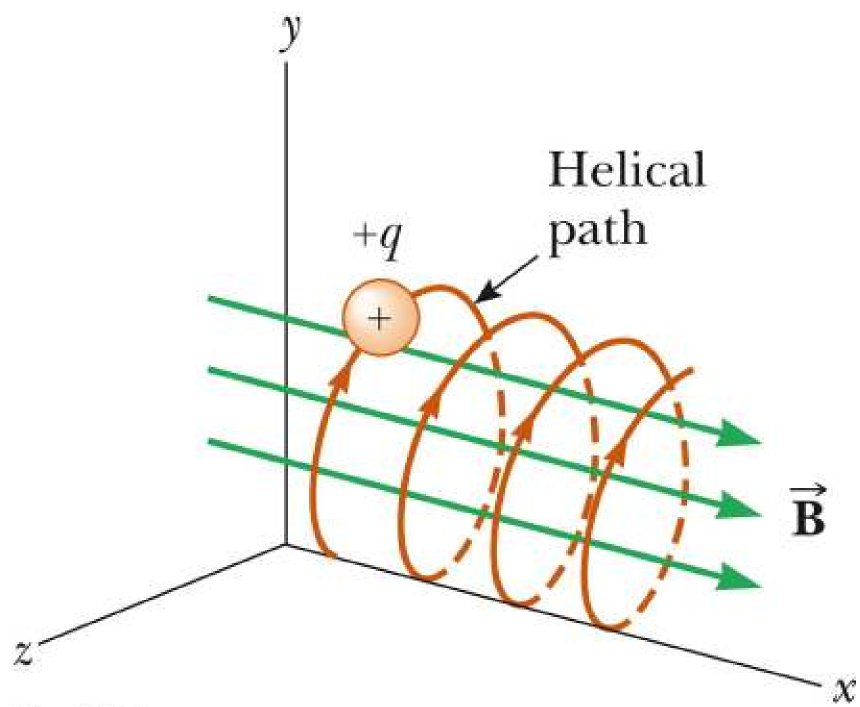
$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

Motion of a Particle, General

- If a charged particle moves in a magnetic field at some arbitrary angle with respect to the field, its path is a helix
- Same equations apply, with

$$v_{\perp} = \sqrt{v_y^2 + v_z^2}$$

- Use the active figure to vary the initial velocity and observe the resulting motion



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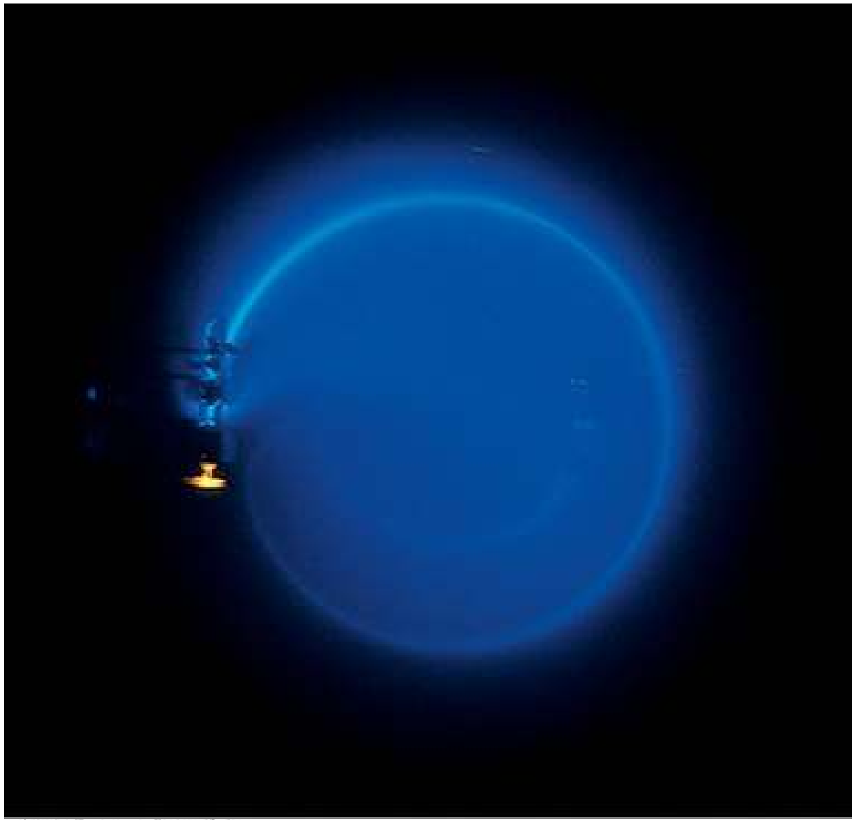
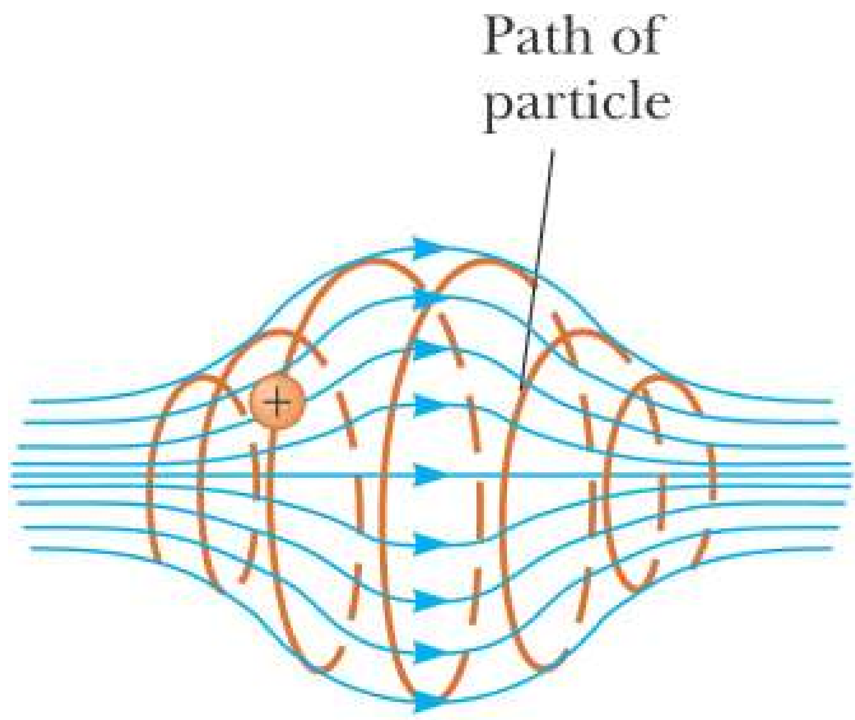
Bending of an Electron Beam

- Electrons are accelerated from rest through a potential difference
- The electrons travel in a curved path
- Conservation of energy will give *v*
- Other parameters can be found



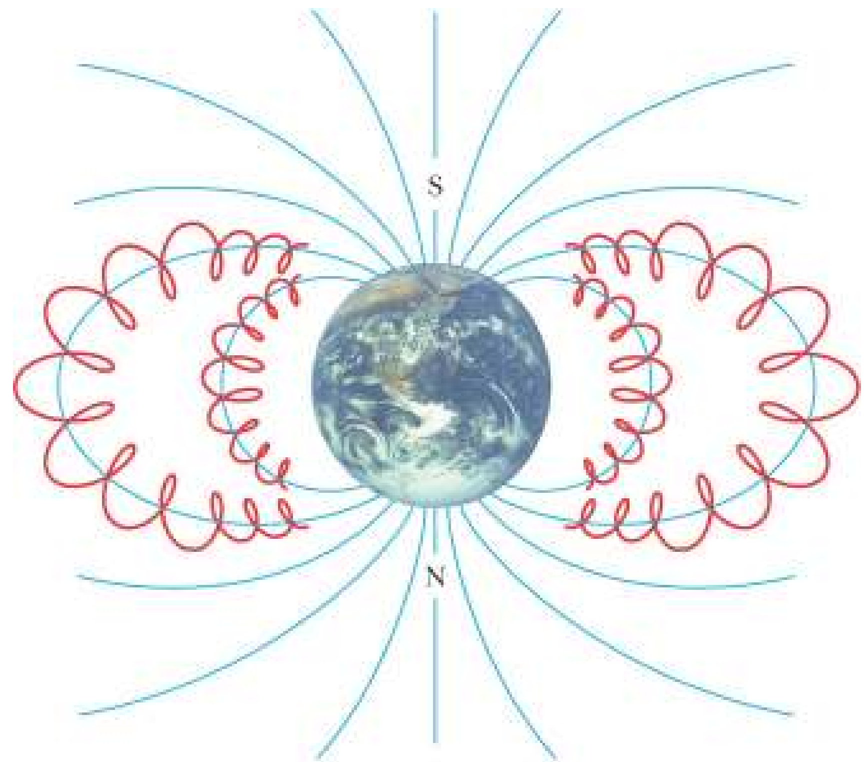
Particle in a Nonuniform Magnetic Field

- The motion is complex
- For example, the particles can oscillate back and forth between two positions
- This configuration is known as a *magnetic bottle*



Van Allen Radiation Belts

- The Van Allen radiation belts consist of charged particles surrounding the Earth in doughnut-shaped regions
- The particles are trapped by the Earth's magnetic field
- The particles spiral from pole to pole
  - May result in Auroras

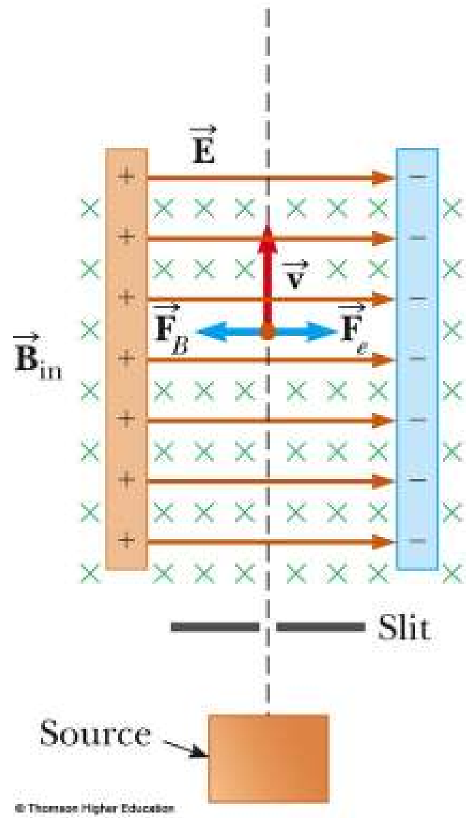


Charged Particles Moving in Electric and Magnetic Fields

- In many applications, charged particles will move in the presence of both magnetic and electric fields
- In that case, the total force is the sum of the forces due to the individual fields
- In general: $\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

Velocity Selector

- Used when all the particles need to move with the same velocity
- A uniform electric field is perpendicular to a uniform magnetic field
- Use the active figure to vary the fields to achieve the straight line motion



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Velocity Selector, cont.

- When the force due to the electric field is equal but opposite to the force due to the magnetic field, the particle moves in a straight line
- This occurs for velocities of value

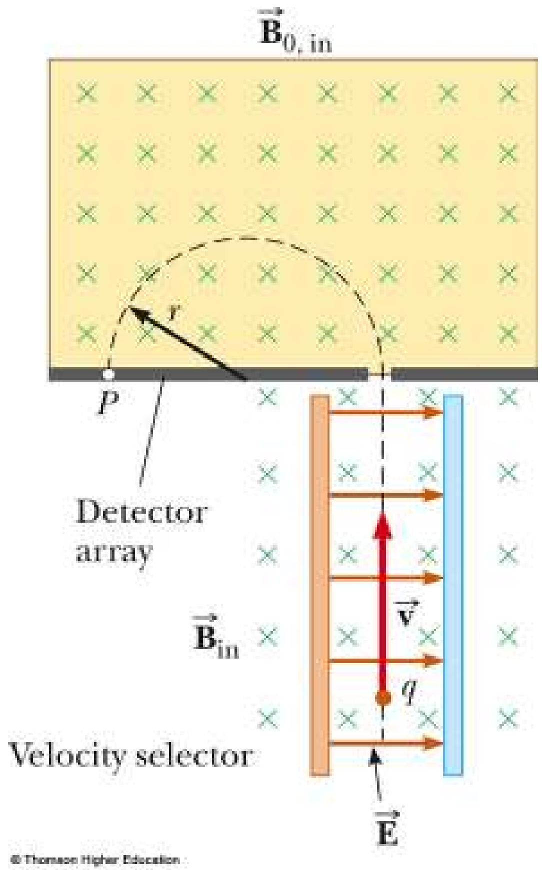
  $v = E / B$

Velocity Selector, final

- Only those particles with the given speed will pass through the two fields undeflected
- The magnetic force exerted on particles moving at speed greater than this is stronger than the electric field and the particles will be deflected to the left
- Those moving more slowly will be deflected to the right

Mass Spectrometer

- A mass spectrometer separates ions according to their mass-to-charge ratio
- A beam of ions passes through a velocity selector and enters a second magnetic field
- Use the active figure to see where the particles strike the detector array



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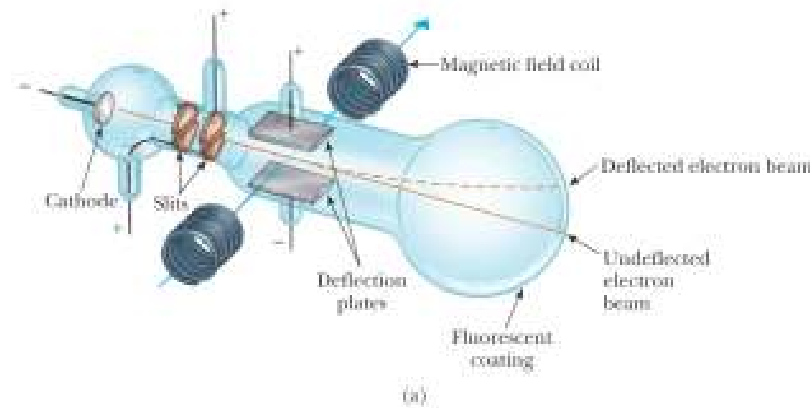
Mass Spectrometer, cont.

- After entering the second magnetic field, the ions move in a semicircle of radius r before striking a detector at P
- If the ions are positively charged, they deflect to the left
- If the ions are negatively charged, they deflect to the right

Thomson's *e*/*m* Experiment

- Electrons are accelerated from the cathode
- They are deflected by electric and magnetic fields
- The beam of electrons strikes a fluorescent screen
- e/m was measured

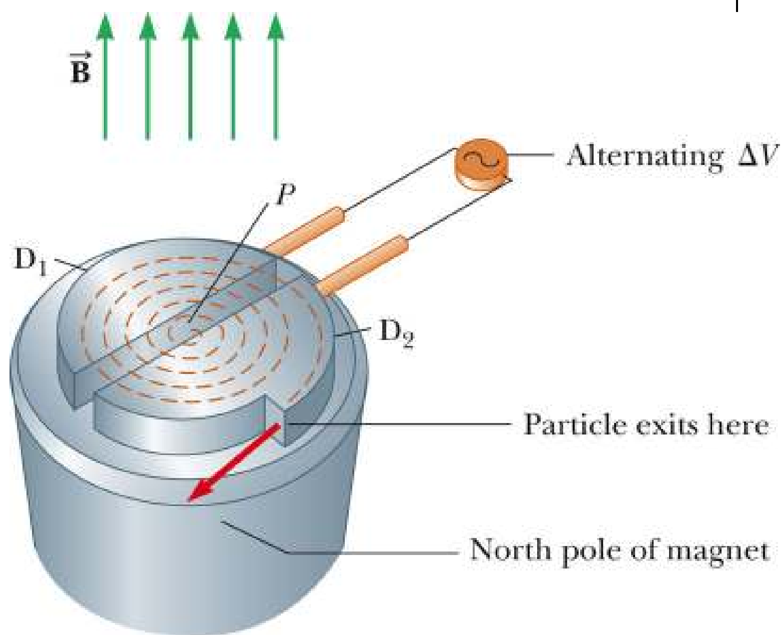


Cyclotron

- A **cyclotron** is a device that can accelerate charged particles to very high speeds
- The energetic particles produced are used to bombard atomic nuclei and thereby produce reactions
- These reactions can be analyzed by researchers

Cyclotron, 2

- D_1 and D_2 are called *dees* because of their shape
- A high frequency alternating potential is applied to the dees
- A uniform magnetic field is perpendicular to them



(a)

(b)

Cyclotron, 3

- A positive ion is released near the center and moves in a semicircular path
- The potential difference is adjusted so that the polarity of the dees is reversed in the same time interval as the particle travels around one dee
- This ensures the kinetic energy of the particle increases each trip

Cyclotron, final

- The cyclotron's operation is based on the fact that T is independent of the speed of the particles and of the radius of their path

$$K = \frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$$

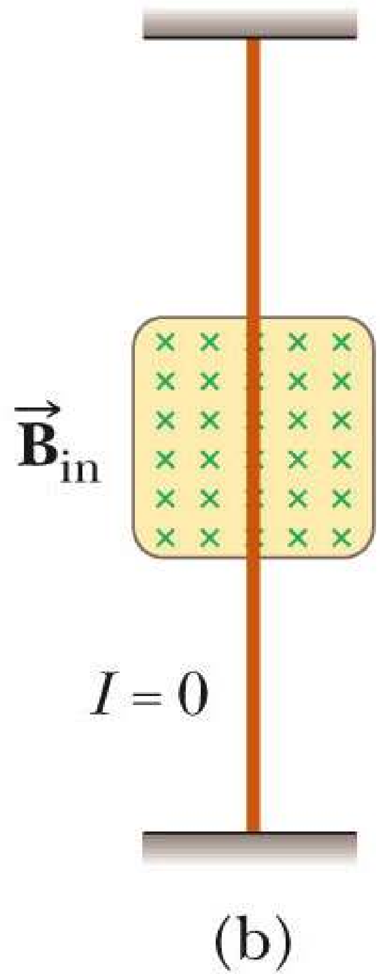
- When the energy of the ions in a cyclotron exceeds about 20 MeV, relativistic effects come into play

Magnetic Force on a Current Carrying Conductor

- A force is exerted on a current-carrying wire placed in a magnetic field
  - The current is a collection of many charged particles in motion
- The direction of the force is given by the right-hand rule

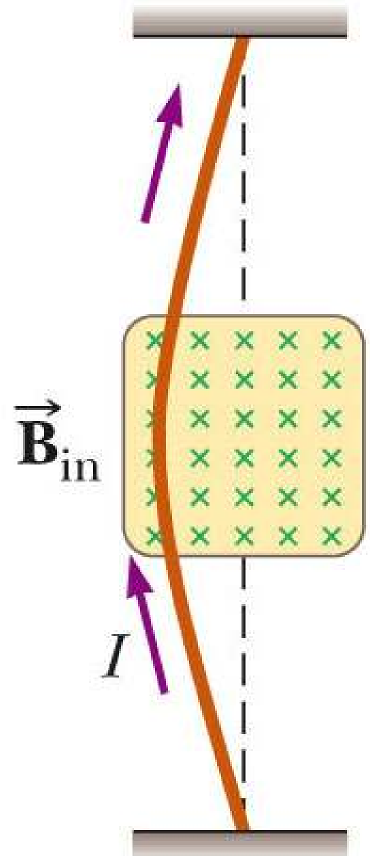
Force on a Wire

- In this case, there is no current, so there is no force
- Therefore, the wire remains vertical



Force on a Wire (2)

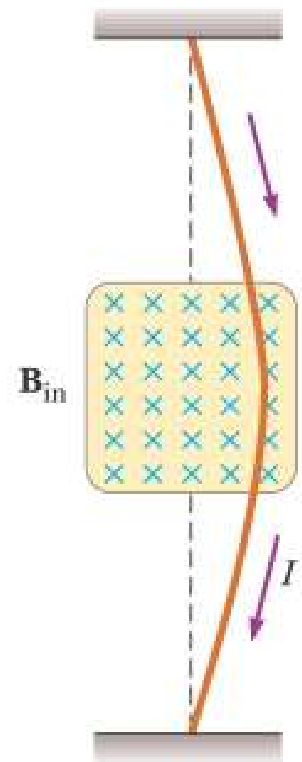
- The magnetic field is into the page
- The current is up the page
- The force is to the left



(c)

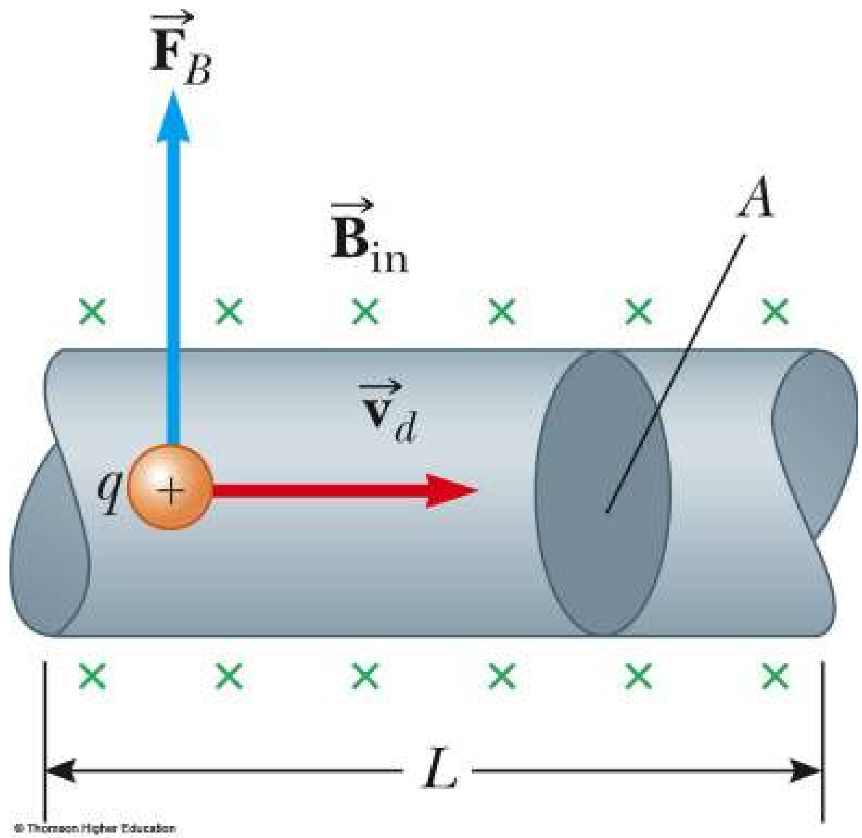
Force on a Wire, (3)

- The magnetic field is into the page
- The current is down the page
- The force is to the right



Force on a Wire, equation

- The magnetic force is exerted on each moving charge in the wire
  - $\vec{\mathbf{F}} = q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}$
- The total force is the product of the force on one charge and the number of charges
  - $\vec{\mathbf{F}} = \left(q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}\right) nAL$



Force on a Wire, (4)

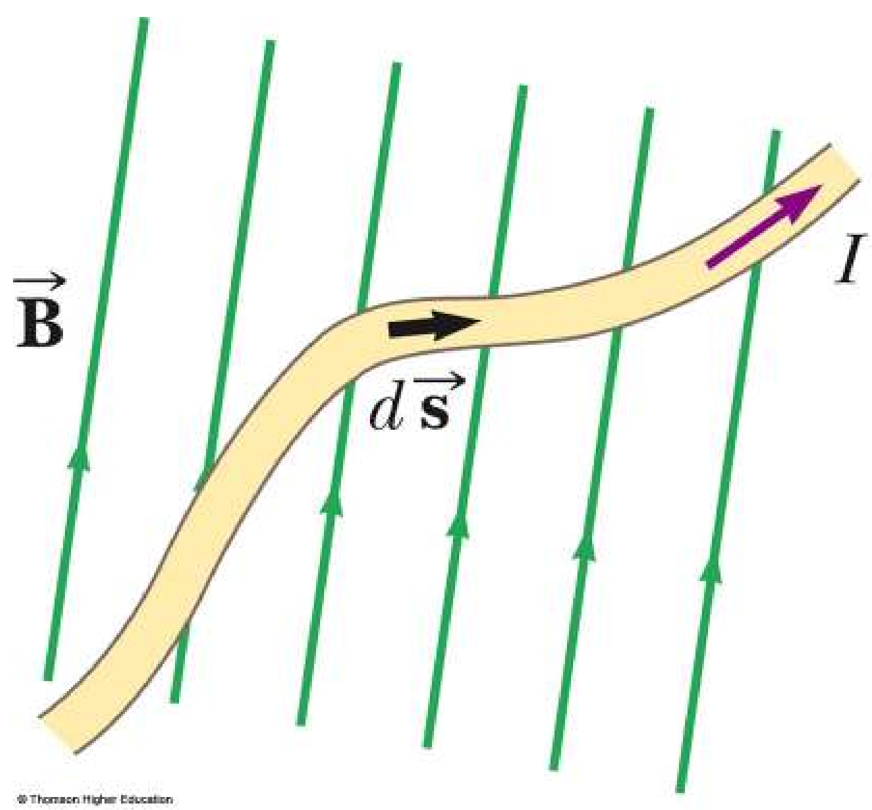
- In terms of the current, this becomes

$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$$

  - I is the current
  - $\vec{\mathbf{L}}$ is a vector that points in the direction of the current
    - Its magnitude is the length L of the segment
  - $\vec{\mathbf{B}}$ is the magnetic field

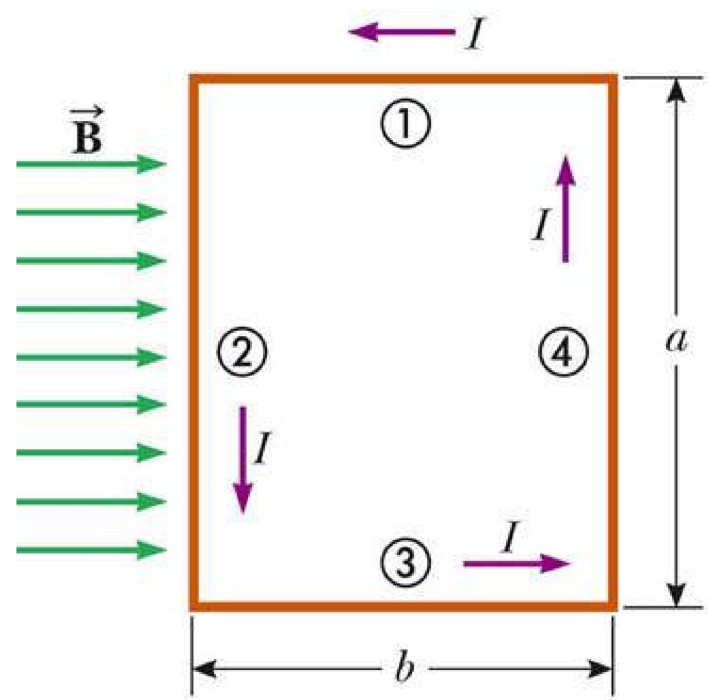
Force on a Wire, Arbitrary Shape

- Consider a small segment of the wire, $d\vec{\mathbf{s}}$
- The force exerted on this segment is
  $$d\vec{\mathbf{F}}_B = I\, d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$
- The total force is
  $$\vec{\mathbf{F}}_B = I \int_{\mathbf{a}}^{b} d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$



Torque on a Current Loop

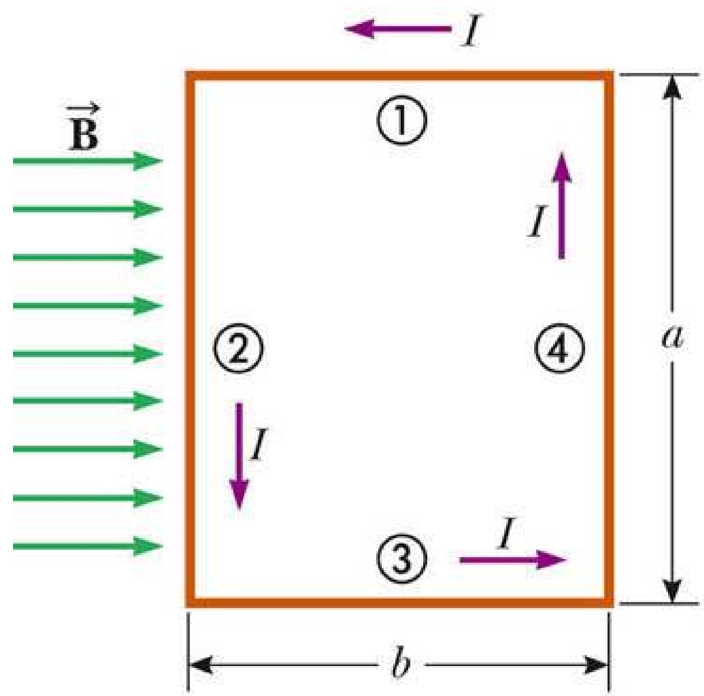
- The rectangular loop carries a current I in a uniform magnetic field
- No magnetic force acts on sides 1 & 3
  - The wires are parallel to the field and $\vec{\mathbf{L}} \times \vec{\mathbf{B}} = 0$



(a)

Torque on a Current Loop, 2

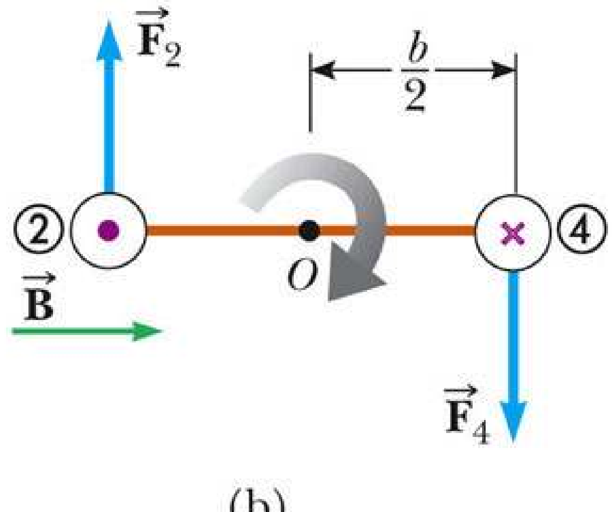
- There is a force on sides 2 & 4 since they are perpendicular to the field
- The magnitude of the magnetic force on these sides will be:
  - $F_2 = F_4 = I\,a\,B$
- The direction of F_2 is out of the page
- The direction of F_4 is into the page



(a)

Torque on a Current Loop, 3

- The forces are equal and in opposite directions, but not along the same line of action
- The forces produce a torque around point *O*



Torque on a Current Loop, Equation

- The maximum torque is found by:

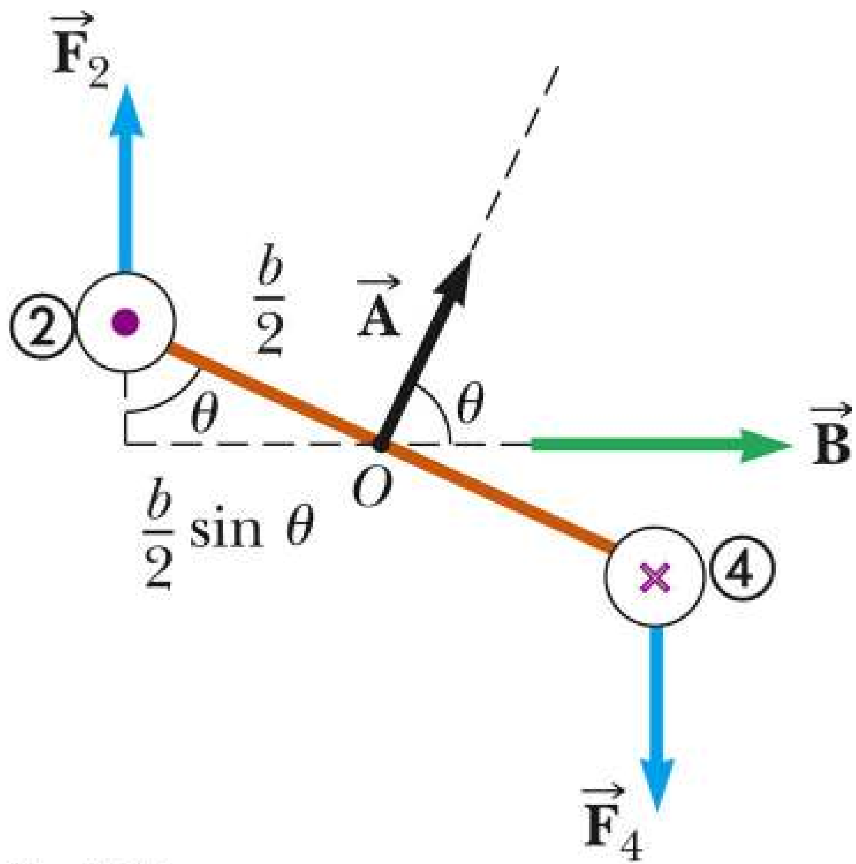
$$\tau_{max} = F_2 \frac{b}{2} + F_4 \frac{b}{2} = (I\, aB) \frac{b}{2} + (I\, aB) \frac{b}{2}$$

$$= I\, abB$$

- The area enclosed by the loop is *ab*, so $\tau_{max} = IAB$
  - This maximum value occurs only when the field is parallel to the plane of the loop

Torque on a Current Loop, General

- Assume the magnetic field makes an angle of $\theta < 90^o$ with a line perpendicular to the plane of the loop
- The net torque about point *O* will be $\tau = IAB \sin \theta$
- Use the active figure to vary the initial settings and observe the resulting motion



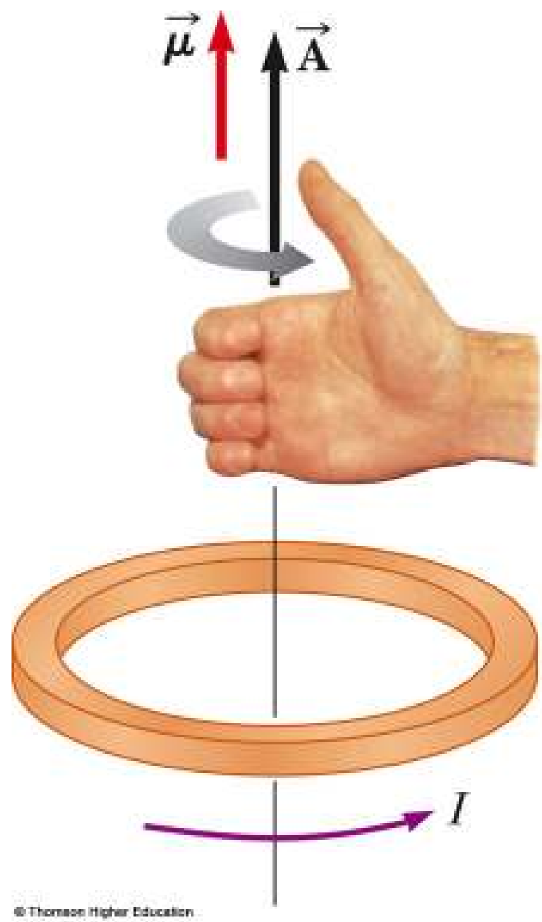
PLAY ACTIVE FIGURE

Torque on a Current Loop, Summary

- The torque has a maximum value when the field is perpendicular to the normal to the plane of the loop
- The torque is zero when the field is parallel to the normal to the plane of the loop
- $\vec{\tau} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ where $\vec{\mathbf{A}}$ is perpendicular to the plane of the loop and has a magnitude equal to the area of the loop

Direction

- The right-hand rule can be used to determine the direction of $\vec{\mathbf{A}}$
- Curl your fingers in the direction of the current in the loop
- Your thumb points in the direction of $\vec{\mathbf{A}}$



Magnetic Dipole Moment

- The product $I\vec{\mathbf{A}}$ is defined as the **magnetic dipole moment**, $\vec{\mu}$, of the loop
  - Often called the magnetic moment
- SI units: A · m^2
- Torque in terms of magnetic moment:

$$\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$$

  - Analogous to $\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$ for electric dipole

Potential Energy

- The potential energy of the system of a magnetic dipole in a magnetic field depends on the orientation of the dipole in the magnetic field:

  $$U = -\vec{\mu} \cdot \vec{\mathbf{B}}$$

  - $U_{min} = -\mu B$ and occurs when the dipole moment is in the same direction as the field
  - $U_{max} = +\mu B$ and occurs when the dipole moment is in the direction opposite the field

Hall Effect

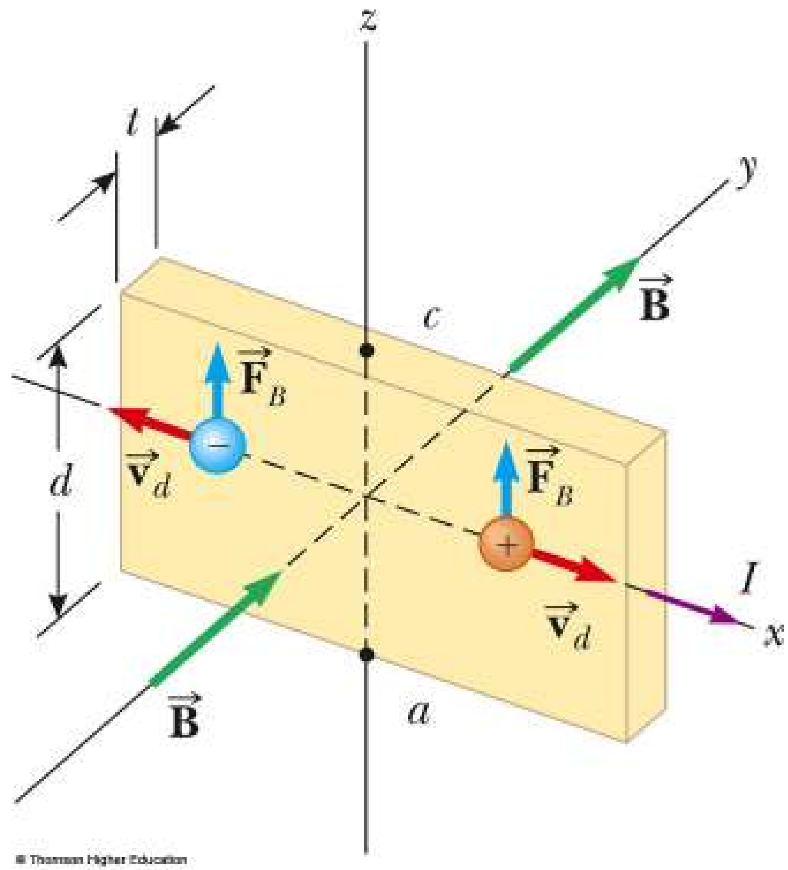
- When a current carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field
- This phenomena is known as the Hall effect
- It arises from the deflection of charge carriers to one side of the conductor as a result of the magnetic forces they experience

Hall Effect, cont.

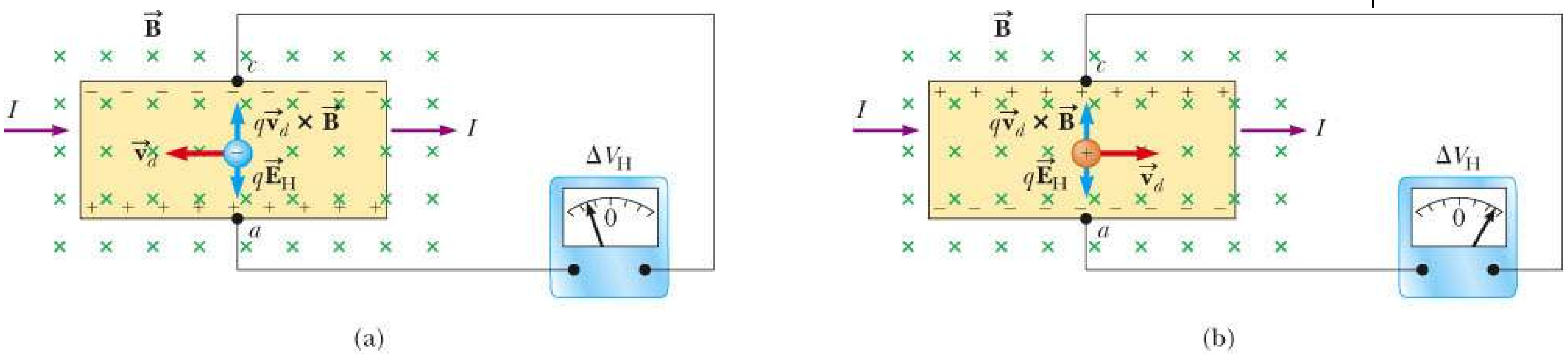
- The Hall effect gives information regarding the sign of the charge carriers and their density
- It can also be used to measure magnetic fields

Hall Voltage

- This shows an arrangement for observing the Hall effect
- The Hall voltage is measured between points *a* and *c*



Hall Voltage, cont



- When the charge carriers are negative, the upper edge of the conductor becomes negatively charged
  - c is at a lower potential than a
- When the charge carriers are positive, the upper edge becomes positively charged
  - c is at a higher potential than a

Hall Voltage, final

- $\Delta V_H = E_H d = v_d B d$
  - d is the width of the conductor
  - v_d is the drift velocity
  - If B and d are known, v_d can be found
- $$\Delta V_H = \frac{I B}{nqt} = \frac{R_H I B}{t}$$
  - $R_H = 1 / nq$ is called the Hall coefficient
  - A properly calibrated conductor can be used to measure the magnitude of an unknown magnetic field