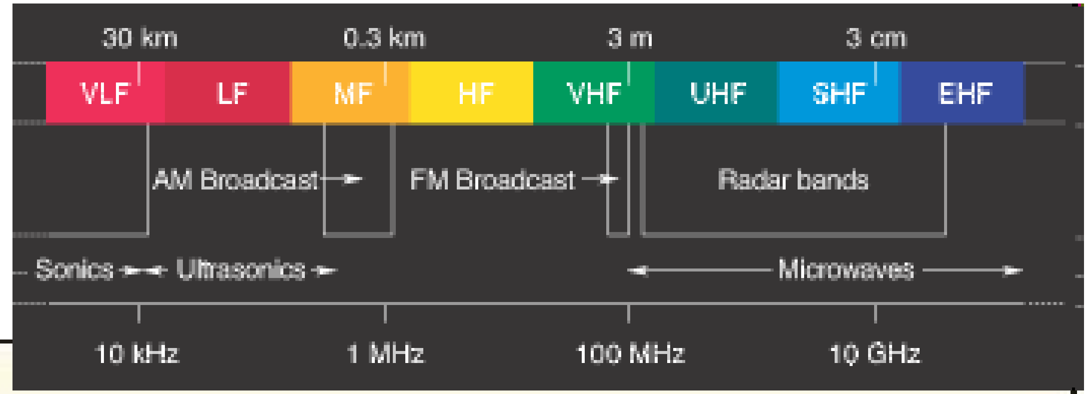
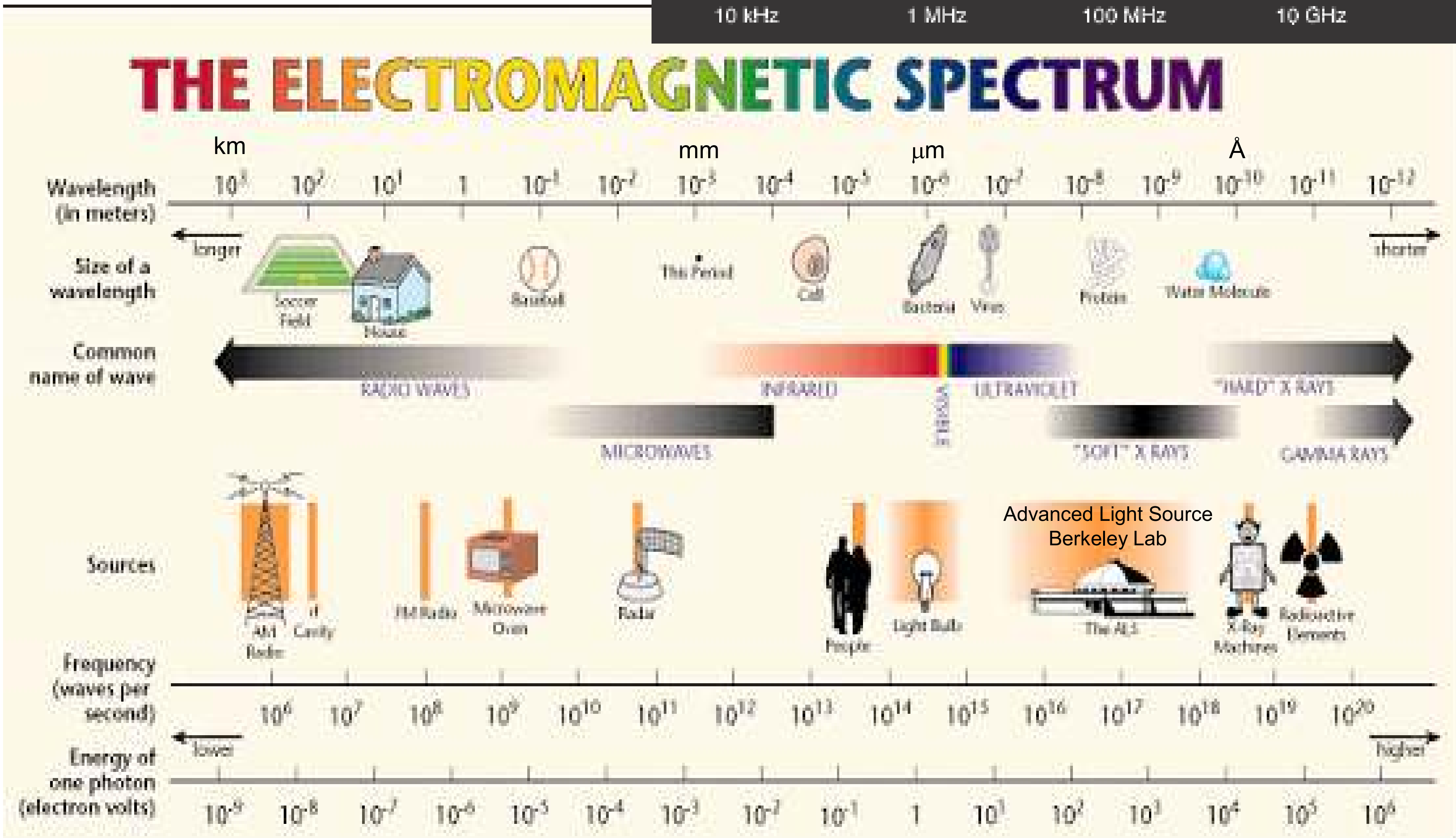
Transmission Line Theory

EE142

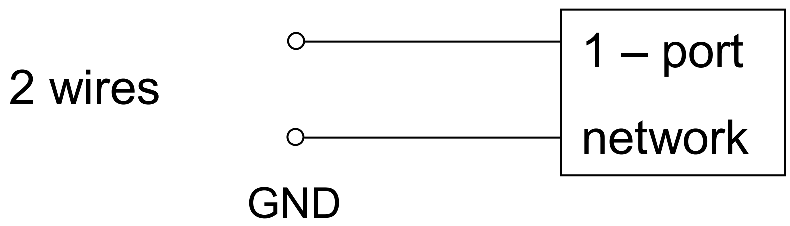
Dr. Ray Kwok

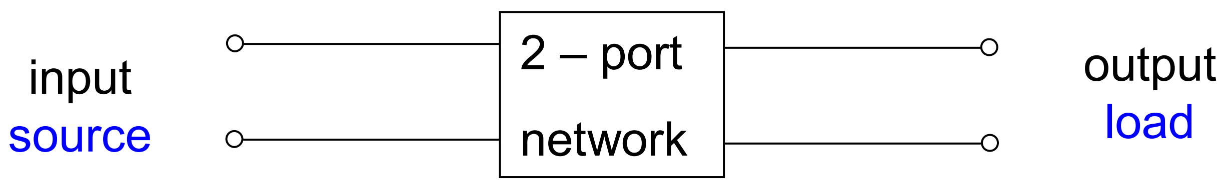
RF Spectrum


30 km
0.3 km
3 m
3 cm
VLF
LF
MF
HF
VHF
UHF
SHF
EHF
AM Broadcast
FM Broadcast
Radar bands
Sonics
Ultrasonics
Microwaves
10 kHz
1 MHz
100 MHz
10 GHz


THE ELECTROMAGNETIC SPECTRUM
km
mm
µm
Å
Wavelength (in meters)
10^3 10^2 10^1 1 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10} 10^{-11} 10^{-12}
longer
shorter
Size of a wavelength
Soccer Field
House
Baseball
This Period
Cell
Bacteria
Virus
Protein
Water Molecule
Common name of wave
RADIO WAVES
INFRARED
VISIBLE
ULTRAVIOLET
"HARD" X RAYS
MICROWAVES
"SOFT" X RAYS
GAMMA RAYS
Sources
AM Radio
rf Cavity
FM Radio
Microwave Oven
Radar
People
Light Bulb
Advanced Light Source
Berkeley Lab
The ALS
X-Ray Machines
Radioactive Elements
Frequency (waves per second)
10^6 10^7 10^8 10^9 10^{10} 10^{11} 10^{12} 10^{13} 10^{14} 10^{15} 10^{16} 10^{17} 10^{18} 10^{19} 10^{20}
lower
higher
Energy of one photon (electron volts)
10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10^1 10^2 10^3 10^4 10^5 10^6

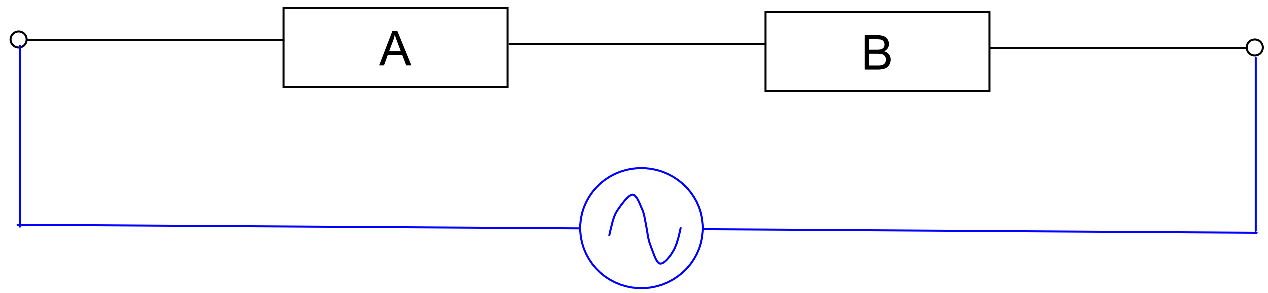
RF / Microwave Circuit



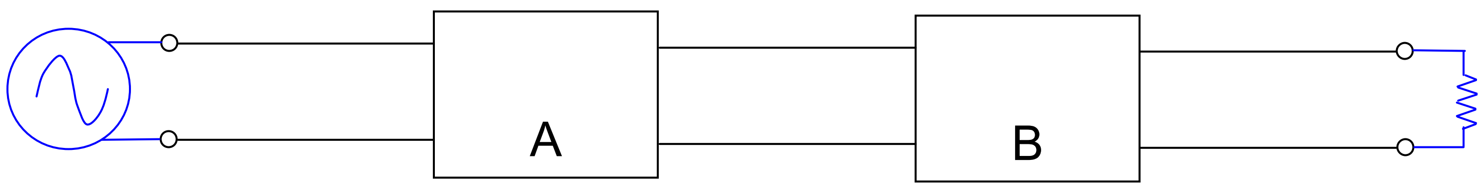


Series connection

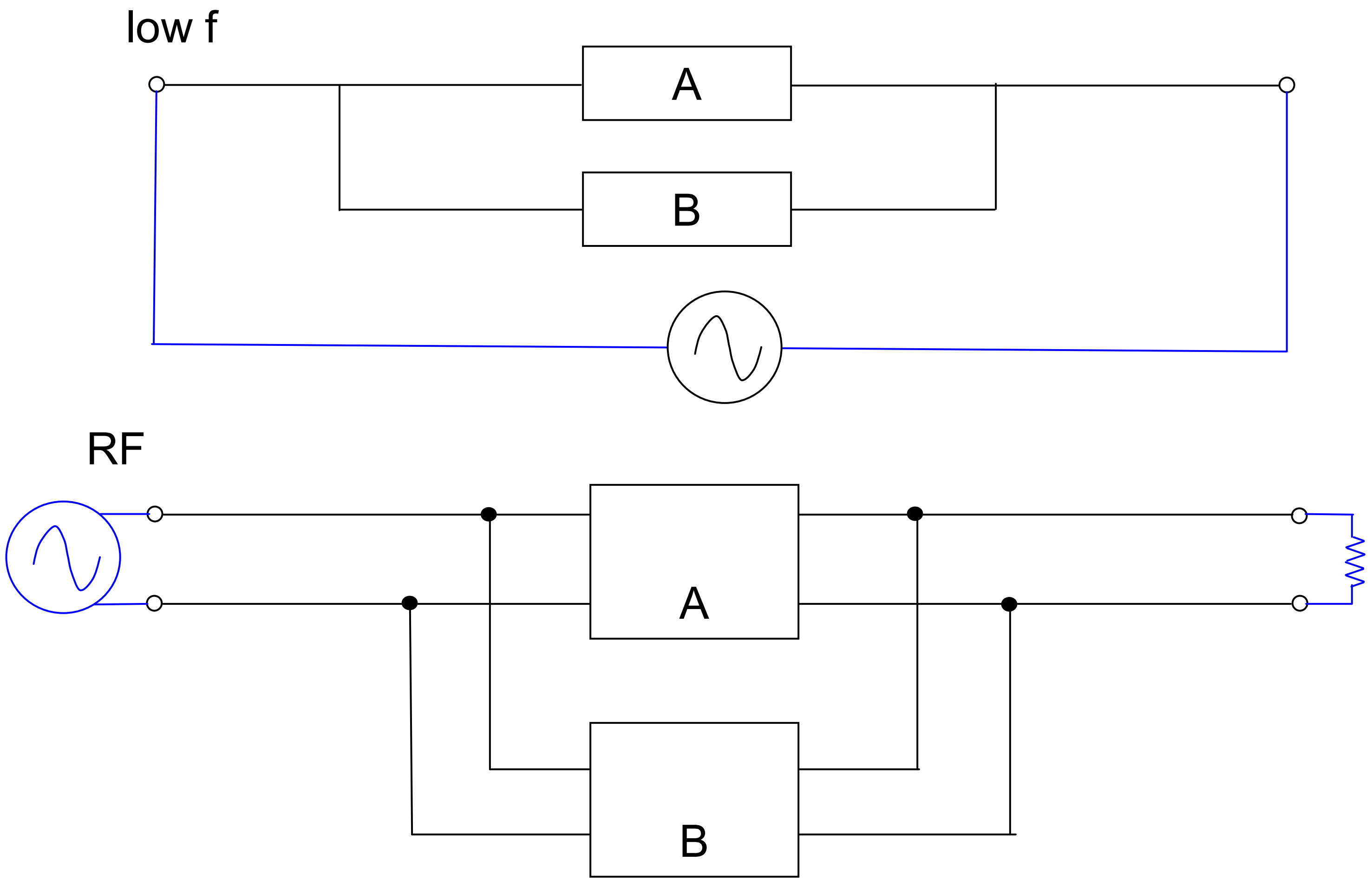
low f



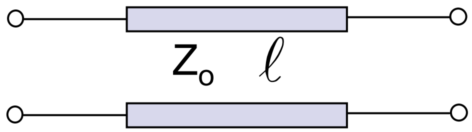
RF



Parallel connection

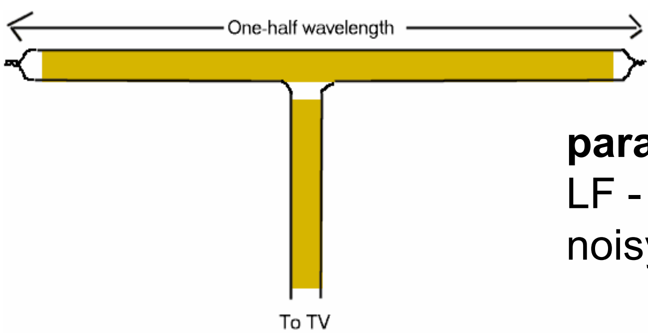


Common transmission lines

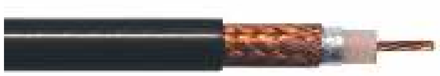


most correct schematic

twisted pair
VLF
lossy & noisy

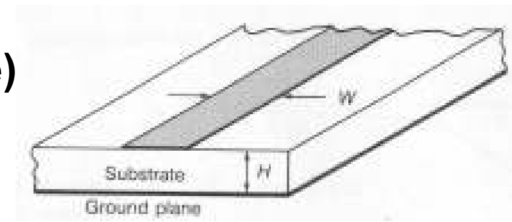


paralllel wire
LF - HF
noisy & lossy



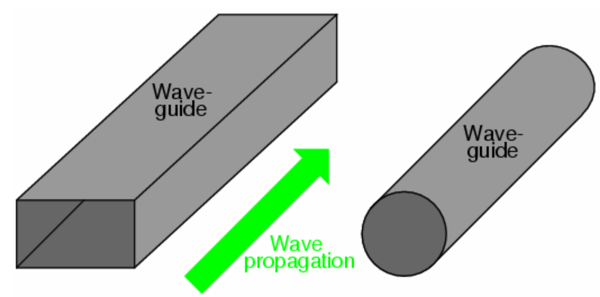
coaxial cable
no distortion
wide freq range

microstrip (line)
no distortion
wide freq range
lowest cost

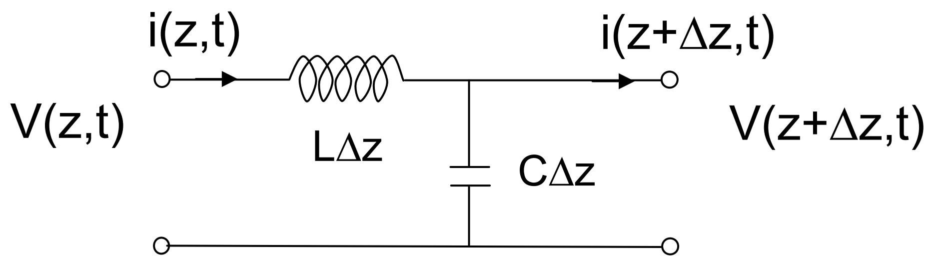


co-planar waveguide
low cost
flip chip access
complex design

waveguide
lowest loss
freq bands



Equivalent circuit



Ideal transmission line

Taylor

Kirchhoff's law: $$V(z,t)-(L\Delta z)\frac{\partial i(z,t)}{\partial t}=V(z+\Delta z,t)\approx V(z,t)+\frac{\partial V(z,t)}{\partial z}\Delta z$$

$$\boxed{-L\frac{\partial i(z,t)}{\partial t}=\frac{\partial V(z,t)}{\partial z}}$$

Q=CV
dQ/dt=i=C dV/dt

Junction rule: $$i(z+\Delta z,t)-i(z,t)=-(C\Delta z)\frac{\partial V(z,t)}{\partial t}\approx\frac{\partial i(z,t)}{\partial z}\Delta z$$

$$\boxed{-C\frac{\partial V(z,t)}{\partial t}=\frac{\partial i(z,t)}{\partial z}}$$

Coupled equations (V – i)

$$-L\frac{\partial i(z,t)}{\partial t}=\frac{\partial V(z,t)}{\partial z}$$

$$-C\frac{\partial V(z,t)}{\partial t}=\frac{\partial i(z,t)}{\partial z}$$

$$-L\frac{\partial^2 i}{\partial t^2}=\frac{\partial^2 V}{\partial t\partial z}=\frac{\partial}{\partial z}\left(-\frac{1}{C}\frac{\partial i}{\partial z}\right)=-\frac{1}{C}\frac{\partial^2 i}{\partial z^2}$$

$$\frac{\partial^2 i}{\partial z^2}=LC\frac{\partial^2 i}{\partial t^2}$$

current wave

similarly

$$-C\frac{\partial^2 V}{\partial t^2}=\frac{\partial^2 i}{\partial t\partial z}=\frac{\partial}{\partial z}\left(-\frac{1}{L}\frac{\partial V}{\partial z}\right)=-\frac{1}{L}\frac{\partial^2 V}{\partial z^2}$$

$$\frac{\partial^2 V}{\partial z^2}=LC\frac{\partial^2 V}{\partial t^2}$$

voltage wave

Wave equation

$$f(x \pm vt) \equiv f(u)$$

reverse / forward traveling wave

$$\frac{\partial f}{\partial x} = f'(u)\frac{\partial u}{\partial x} = f'(u)$$

$$\frac{\partial^2 f}{\partial x^2} = f''(u)\frac{\partial u}{\partial x} = f''(u)$$

$$\frac{\partial f}{\partial t} = f'(u)\frac{\partial u}{\partial t} = \pm v f'(u)$$

$$\frac{\partial^2 f}{\partial t^2} = \pm v f''(u)\frac{\partial u}{\partial t} = v^2 f''(u)$$

$$\boxed{\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

wave equation

note:

$$x \pm vt = \frac{1}{k}\left(\frac{2\pi}{\lambda}x \pm \frac{2\pi}{\lambda}vt\right)$$

$$= \frac{1}{k}(kx \pm 2\pi ft)$$

$$= \pm\frac{1}{k}(\omega t \pm kx)$$

$$f(x \pm vt) = f(\omega t \pm kx)$$

$$f(\omega t - \vec{k}\cdot\vec{r})$$ (3D)

Voltage & Current Waves

$$V(z,t) = V_o^+ e^{j(\omega t - \beta z)} + V_o^- e^{j(\omega t + \beta z)}$$

$$i(z,t) = I_o^+ e^{j(\omega t - \beta z)} - I_o^- e^{j(\omega t + \beta z)}$$

where $\beta = \frac{2\pi}{\lambda}$

$$v = f\lambda = (2\pi f)\left(\frac{\lambda}{2\pi}\right) = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

why "-" ?

$$\frac{\partial i}{\partial z} = -j\beta I_o^+ e^{j(\omega t - \beta z)} - j\beta I_o^- e^{j(\omega t + \beta z)}$$

$$\frac{\partial i}{\partial z} = -C\frac{\partial V}{\partial t} = -C\left[j\omega V_o^+ e^{j(\omega t - \beta z)} + j\omega V_o^- e^{j(\omega t + \beta z)}\right]$$

$$\beta I_o^+ = C\omega V_o^+$$

$$\beta I_o^- = C\omega V_o^-$$

$$V_o^\pm = \frac{\beta}{C\omega} I_o^\pm = \frac{\sqrt{LC}}{C} I_o^\pm = \sqrt{\frac{L}{C}} I_o^\pm \equiv Z_o I_o^\pm$$

$$v = \frac{1}{\sqrt{LC}}$$

$$Z_o = \sqrt{\frac{L}{C}}$$

Fields and circuits

$$\nabla^2 \begin{pmatrix} \vec{E} \\ \vec{H} \end{pmatrix} = \mu\varepsilon \frac{\partial^2}{\partial t^2} \begin{pmatrix} \vec{E} \\ \vec{H} \end{pmatrix}$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\vec{E}(z,t) = \vec{E}_{oi} e^{j(\omega t - \vec{k}_i \cdot \vec{r})} + \vec{E}_{or} e^{j(\omega t - \vec{k}_r \cdot \vec{r})}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\vec{H}(z,t) = \vec{H}_{oi} e^{j(\omega t - \vec{k}_i \cdot \vec{r})} - \vec{H}_{or} e^{j(\omega t - \vec{k}_r \cdot \vec{r})}$$

$$\frac{\partial^2}{\partial z^2} \begin{pmatrix} V \\ i \end{pmatrix} = LC \frac{\partial^2}{\partial t^2} \begin{pmatrix} V \\ i \end{pmatrix}$$

$$v = \frac{1}{\sqrt{LC}}$$

$$V(z,t) = V_o^+ e^{j(\omega t - \beta z)} + V_o^- e^{j(\omega t + \beta z)}$$

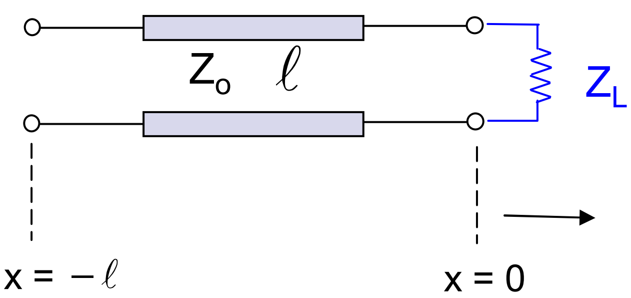
$$Z_o = \sqrt{\frac{L}{C}}$$

$$i(z,t) = I_o^+ e^{j(\omega t - \beta z)} - I_o^- e^{j(\omega t + \beta z)}$$

What is Z_o?

- Characteristic Impedance.
- 50 ohms for most communications system,
- 75 ohms for TV cable.
- Measure 75 ohms with a ohmmeter?
- Two 75Ω cables together (in series) makes a 150Ω cable?
- 75 + 75 = 75 !!!!
- What does Z_o represent?

Reflection at Load



$$V(x) = V_o^+ e^{-j\beta x} + V_o^- e^{j\beta x}$$

$$i(x) = I_o^+ e^{-j\beta x} - I_o^- e^{j\beta x}$$

$$V(0) \equiv V_L = V_o^+ + V_o^-$$ at the load

$$i(0) \equiv I_L = I_o^+ - I_o^- = \frac{1}{Z_o}\left(V_o^+ - V_o^-\right)$$

$$\frac{V_L}{I_L} \equiv Z_L = Z_o\left(\frac{V_o^+ + V_o^-}{V_o^+ - V_o^-}\right)$$

$$Z_L\left(V_o^+ - V_o^-\right) = Z_o\left(V_o^+ + V_o^-\right)$$

$$V_o^+\left(Z_L - Z_o\right) = V_o^-\left(Z_L + Z_o\right)$$

$$\frac{V_o^-}{V_o^+} \equiv \boxed{\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}}$$

$Z_L \neq Z_o$
reflection

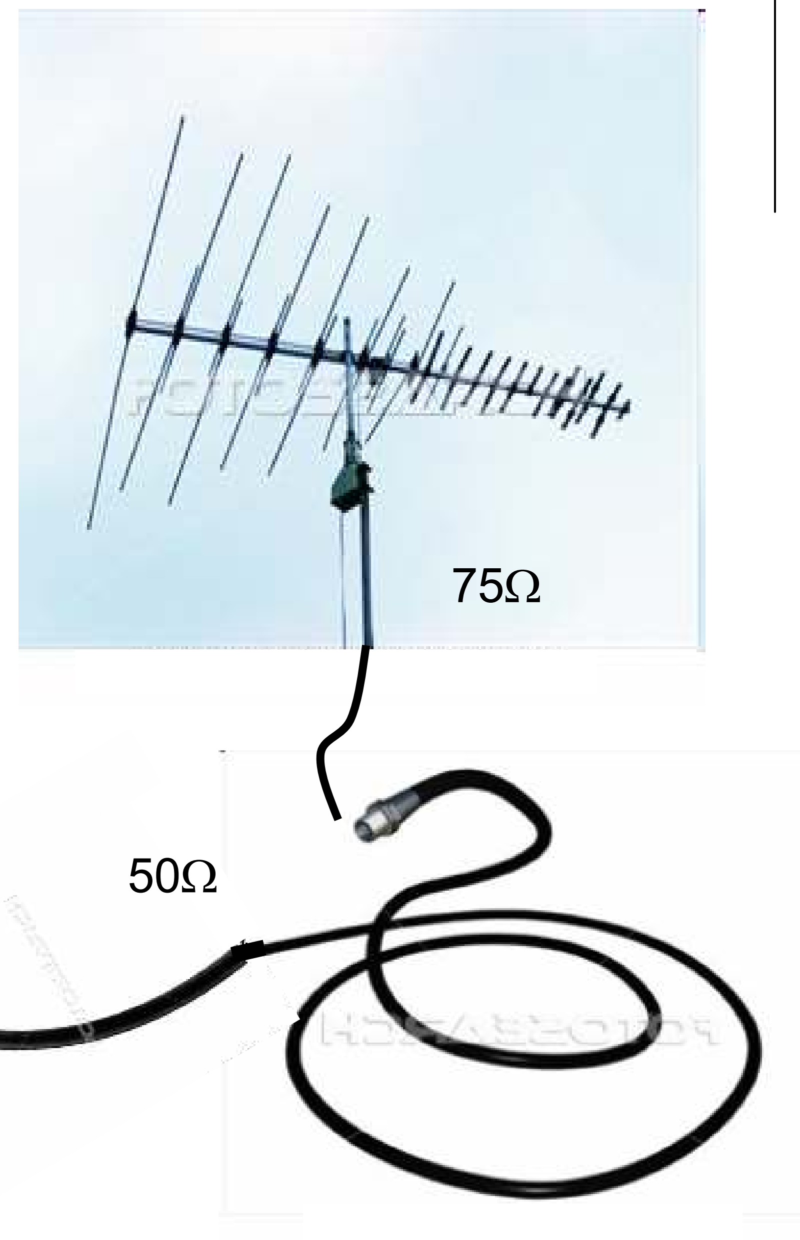
Define normalized impedance

$$\overline{Z} \equiv \frac{Z}{Z_o}$$

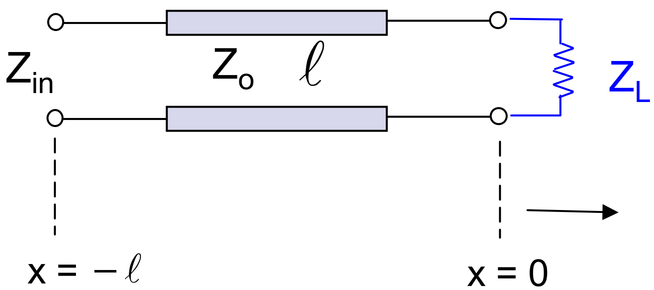
$$\Gamma_L = \frac{\overline{Z}_L - 1}{\overline{Z}_L + 1}$$

Example

does it work?



Impedance at Input



$$V(x) = V_o^+ e^{-j\beta x} + V_o^- e^{j\beta x}$$

$$i(x) = I_o^+ e^{-j\beta x} - I_o^- e^{j\beta x}$$

$$Z_{in} \equiv \frac{V_{in}}{I_{in}} = \frac{V(-\ell)}{i(-\ell)} = \frac{V_o^+ e^{j\beta\ell} + V_o^- e^{-j\beta\ell}}{\frac{1}{Z_o}\left(V_o^+ e^{j\beta\ell} - V_o^- e^{-j\beta\ell}\right)}$$

$$Z_{in} = Z_o\left(\frac{e^{j\beta\ell} + \Gamma_L e^{-j\beta\ell}}{e^{j\beta\ell} - \Gamma_L e^{-j\beta\ell}}\right)$$

$$\overline{Z}_{in} = \frac{e^{-j\beta\ell}\left(\frac{1+j\tan\beta\ell}{1-j\tan\beta\ell}\right) + \left(\frac{\overline{Z}_L - 1}{\overline{Z}_L + 1}\right)e^{-j\beta\ell}}{e^{-j\beta\ell}\left(\frac{1+j\tan\beta\ell}{1-j\tan\beta\ell}\right) - \left(\frac{\overline{Z}_L - 1}{\overline{Z}_L + 1}\right)e^{-j\beta\ell}}$$

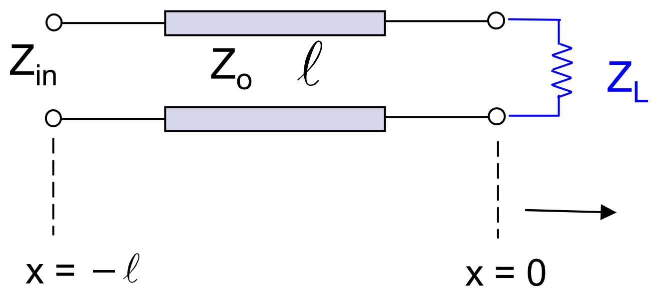
$$\overline{Z}_{in} = \frac{(\overline{Z}_L + 1)(1 + j\tan\beta\ell) + (\overline{Z}_L - 1)(1 - j\tan\beta\ell)}{(\overline{Z}_L + 1)(1 + j\tan\beta\ell) - (\overline{Z}_L - 1)(1 - j\tan\beta\ell)}$$

$$\overline{Z}_{in} = \frac{2(\overline{Z}_L + j\tan\beta\ell)}{2(1 + j\overline{Z}_L \tan\beta\ell)}$$

$$\overline{Z}_{in} = \frac{\overline{Z}_L + j\tan\beta\ell}{1 + j\overline{Z}_L \tan\beta\ell}$$

$$Z_{in} = Z_o\left(\frac{Z_L + jZ_o \tan\beta\ell}{Z_o + jZ_L \tan\beta\ell}\right)$$

Exercise



$Z_o = 50\ \Omega$
$Z_L = 100\Omega$
$Z_{in} = ?$

For length = $\lambda/8$? $\lambda/4$? $\lambda/2$?

What if $Z_o = Z_L = 50\Omega$?
Would the length make any difference?

$$\bar{Z}_{in} = \frac{\bar{Z}_L + j\tan\beta\ell}{1 + j\bar{Z}_L\tan\beta\ell}$$

$$Z_{in} = Z_o\left(\frac{Z_L + jZ_o\tan\beta\ell}{Z_o + jZ_L\tan\beta\ell}\right)$$

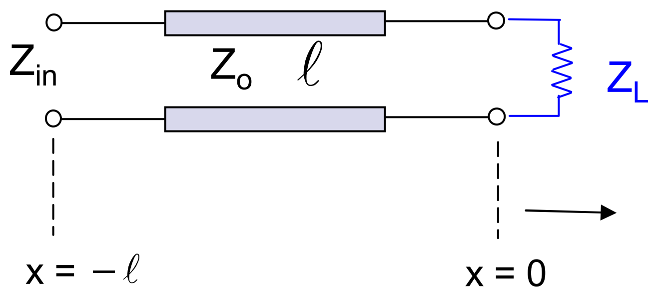
$50\Omega\ (-37^o)$

25Ω

100Ω

$Z_{in}=Z_o=Z_L$

Transmission Line Impedance



$$\bar{Z}_{in} = \frac{\bar{Z}_L + j\tan\beta\ell}{1 + j\bar{Z}_L \tan\beta\ell}$$

$$Z_{in} = Z_o\left(\frac{Z_L + jZ_o \tan\beta\ell}{Z_o + jZ_L \tan\beta\ell}\right)$$

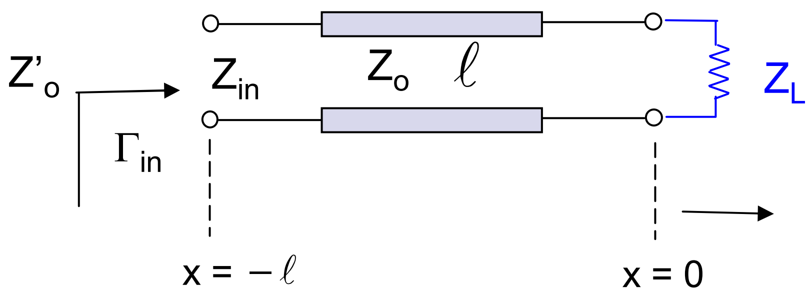
case 1: $\beta\ell = 0$, or $\ell = 0$
$\tan\beta = 0$
$Z_{in} = Z_L$

case 2: $\beta\ell = \pi$, or $\ell = \lambda/2$
$\tan\beta\ell = 0$
$Z_{in} = Z_L$

case 3: $\beta\ell = \pi/2$, or $\ell = \lambda/4$
$\tan\beta\ell \rightarrow \infty$
$Z_{in} = Z_o^2 / Z_L$
Quarter-wave transformer (impedance), real-to-real, complex-to-complex.

note: at low freq, $\beta \rightarrow 0$, $Z_{in} = Z_L$ regardless of line length or line impedance.

Reflection at Input



$$\bar{Z}_{in} = \frac{\bar{Z}_L + j\tan\beta\ell}{1 + j\bar{Z}_L \tan\beta\ell}$$

$$Z_{in} = Z_o\left(\frac{Z_L + jZ_o \tan\beta\ell}{Z_o + jZ_L \tan\beta\ell}\right)$$

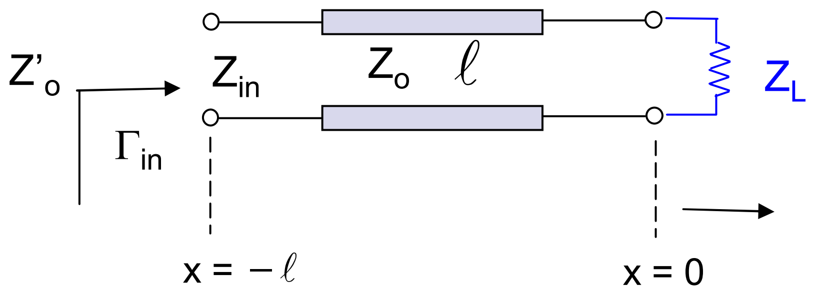
$$\Gamma_{in} = \frac{Z_{in} - Z_o'}{Z_{in} + Z_o'} = \frac{\bar{Z}_{in}' - 1}{\bar{Z}_{in}' + 1}$$

In general

$$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = \frac{\bar{Z}_{in} - 1}{\bar{Z}_{in} + 1}$$

just have to know what Z to use

Exercise



$Z_o = 50\ \Omega$
$Z'_o = 50\ \Omega$
$Z_L = 100\Omega$
Length = $\lambda/8$
$\Gamma_L = ?\quad \Gamma_{in} = ?$
What if Z'_o is 75 Ω?

$$\overline{Z}_{in} = \frac{\overline{Z}_L + j\tan\beta\ell}{1 + j\overline{Z}_L \tan\beta\ell}$$

$$Z_{in} = Z_o\left(\frac{Z_L + jZ_o\tan\beta\ell}{Z_o + jZ_L\tan\beta\ell}\right)$$

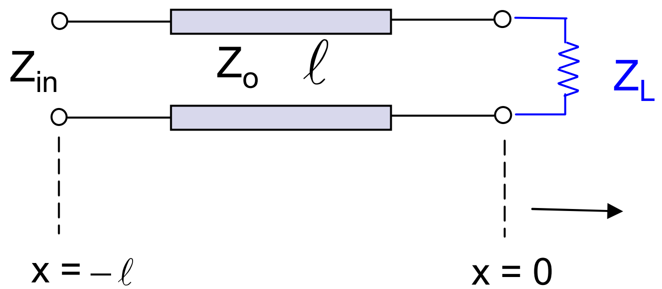
$$\Gamma_{in} = \frac{Z_{in} - Z'_o}{Z_{in} + Z'_o} = \frac{\overline{Z}'_{in} - 1}{\overline{Z}'_{in} + 1}$$

1/3

1/3 (-90^o) only change phase !?!

0.388 (235^o)

Voltage wave in transmission line



$$V(x) = V_o^+ e^{-j\beta x} + V_o^- e^{j\beta x}$$

$$V(x) = V_o^+ e^{-j\beta x}\left(1 + \Gamma_L e^{2j\beta x}\right)$$

$$|V| = |V_o^+|\,|1 + \Gamma_L e^{2j\beta x}|$$

$$\Gamma_L \equiv \rho e^{j\theta}$$

$$|V| = V_o^+ \left|1 + \rho e^{j(\theta + 2\beta x)}\right|$$

$$|V| = V_o^+ \sqrt{(1 + \rho\cos(\theta + 2\beta x))^2 + \rho^2 \sin^2(\theta + 2\beta x)}$$

$$|V| = V_o^+ \sqrt{1 + 2\rho\cos(\theta + 2\beta x) + \rho^2}$$

$$|V| = V_o^+ \sqrt{(1 + \rho)^2 - 2\rho(1 - \cos(\theta + 2\beta x))}$$

$$|V| = V_o^+ \sqrt{(1 + \rho)^2 - 4\rho\sin^2\left(\frac{\theta + 2\beta x}{2}\right)}$$

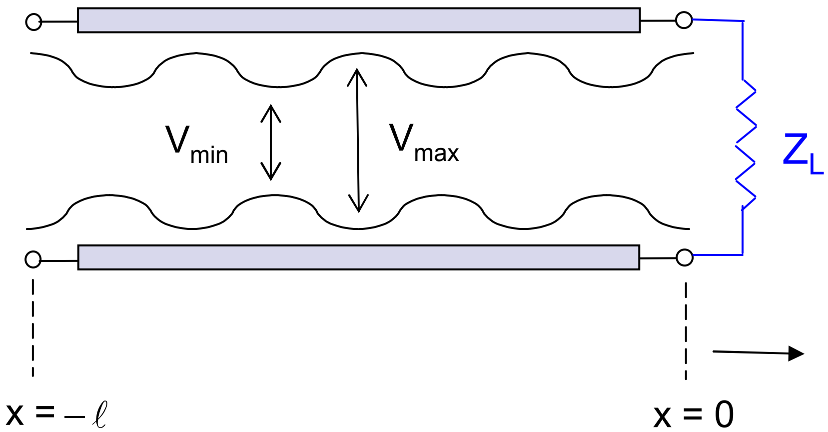
min when sine = 1

$$V_{min} = V_o^+ \sqrt{(1 + \rho)^2 - 4\rho} = V_o^+(1 - \rho)$$

max when sine = 0

$$V_{max} = V_o^+ \sqrt{(1 + \rho)^2} = V_o^+(1 + \rho)$$

Voltage Standing Wave



$$V(x) = V_o^+ e^{-j\beta x} + V_o^- e^{j\beta x}$$

standing wave

If $|V_o^+| = |V_o^-|$, $|\Gamma_L| \equiv \rho = \pm 1$

perfect standing wave with nodes

$$|V| = V_o^+ \sqrt{(1+\rho)^2 - 4\rho \sin^2\left(\frac{\theta + 2\beta x}{2}\right)}$$

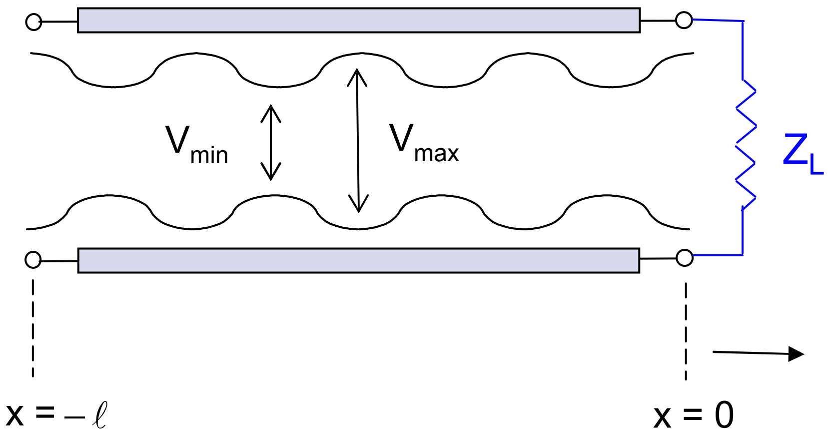
min when $\dfrac{\theta + 2\beta x}{2} = \pm \dfrac{(2n+1)\pi}{2}$

(x < 0) $x = -[\theta \mp (2n+1)\pi]\dfrac{\lambda}{4\pi}$

max when $\dfrac{\theta + 2\beta x}{2} = \pm n\pi$

$x = -[\theta \mp 2n\pi]\dfrac{\lambda}{4\pi}$

VSWR (Voltage Standing Wave Ratio)



$$|V| = V_o^+ \sqrt{(1+\rho)^2 - 4\rho \sin^2\left(\frac{\theta + 2\beta x}{2}\right)}$$

$$V_{min} = V_o^+ \sqrt{(1+\rho)^2 - 4\rho} = V_o^+ (1-\rho)$$

$$V_{max} = V_o^+ \sqrt{(1+\rho)^2} = V_o^+ (1+\rho)$$

$$\text{VSWR} \equiv \frac{V_{max}}{V_{min}} = \frac{1+\rho}{1-\rho} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

perfect match: $\rho = 0$, VSWR = 1.0

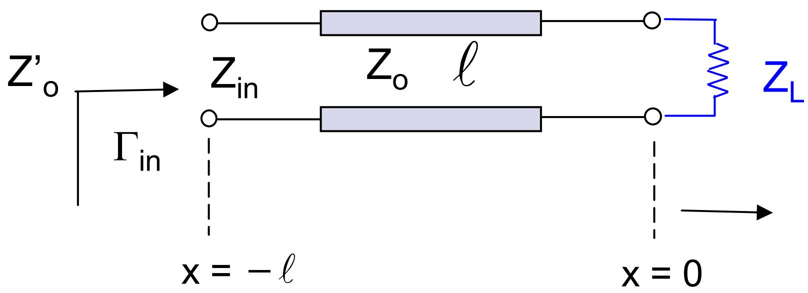
open / short: $\rho = 1$, VSWR $\rightarrow \infty$

It is an indicator on how well the load matches the line.

VSWR is the standing wave pattern INSIDE the line.

Only Γ at the reflected junction that counts

Exercise



$Z_o = 50\ \Omega$
$Z'_o = 75\ \Omega$
$Z_L = 100\Omega$
Length = $\lambda/8$
VSWR = ?

$\Gamma_L = 1/3$
VSWR = 2

Return Loss

$RL \equiv -20 \log \rho$ (dB)

perfect match: $\rho \to 0$, VSWR $\to 1.0$, $RL \to \infty$

open / short: $\rho = 1$, VSWR $\to \infty$, $RL \to 0$ dB

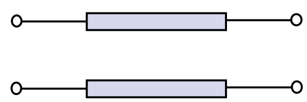
$$VSWR \equiv \frac{1+\rho}{1-\rho} = \frac{1+|\Gamma|}{1-|\Gamma|} \longrightarrow \rho = |\Gamma| = \frac{VSWR-1}{VSWR+1}$$

Typical VSWR = 1.1 to 2
ρ = 0.048 to 0.33
RL = 26 dB to 9.5 dB

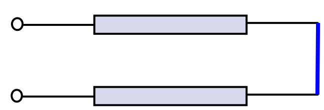
Stub

Transmission line connecting nowhere(?)

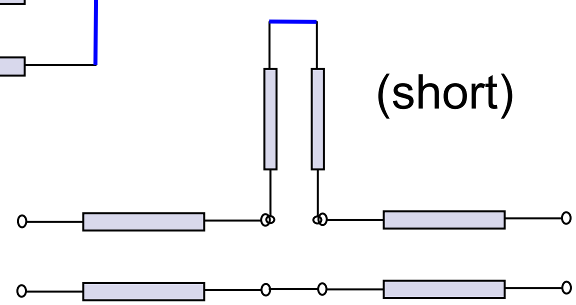
- Open stub



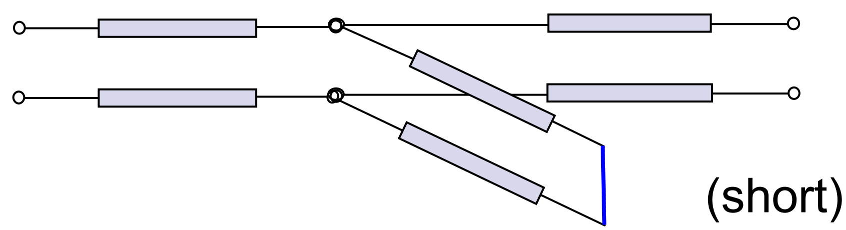
- Short stub



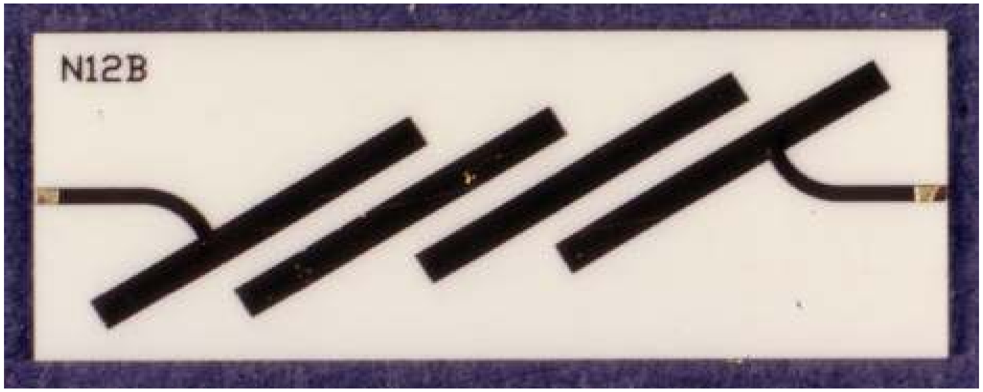
- Series stub

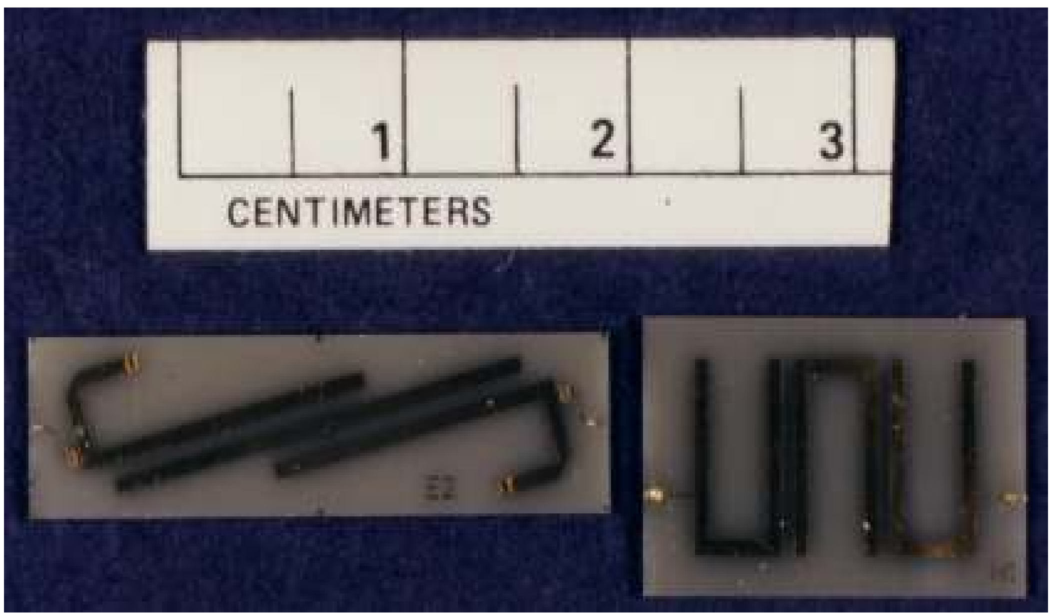


- Shunt stub



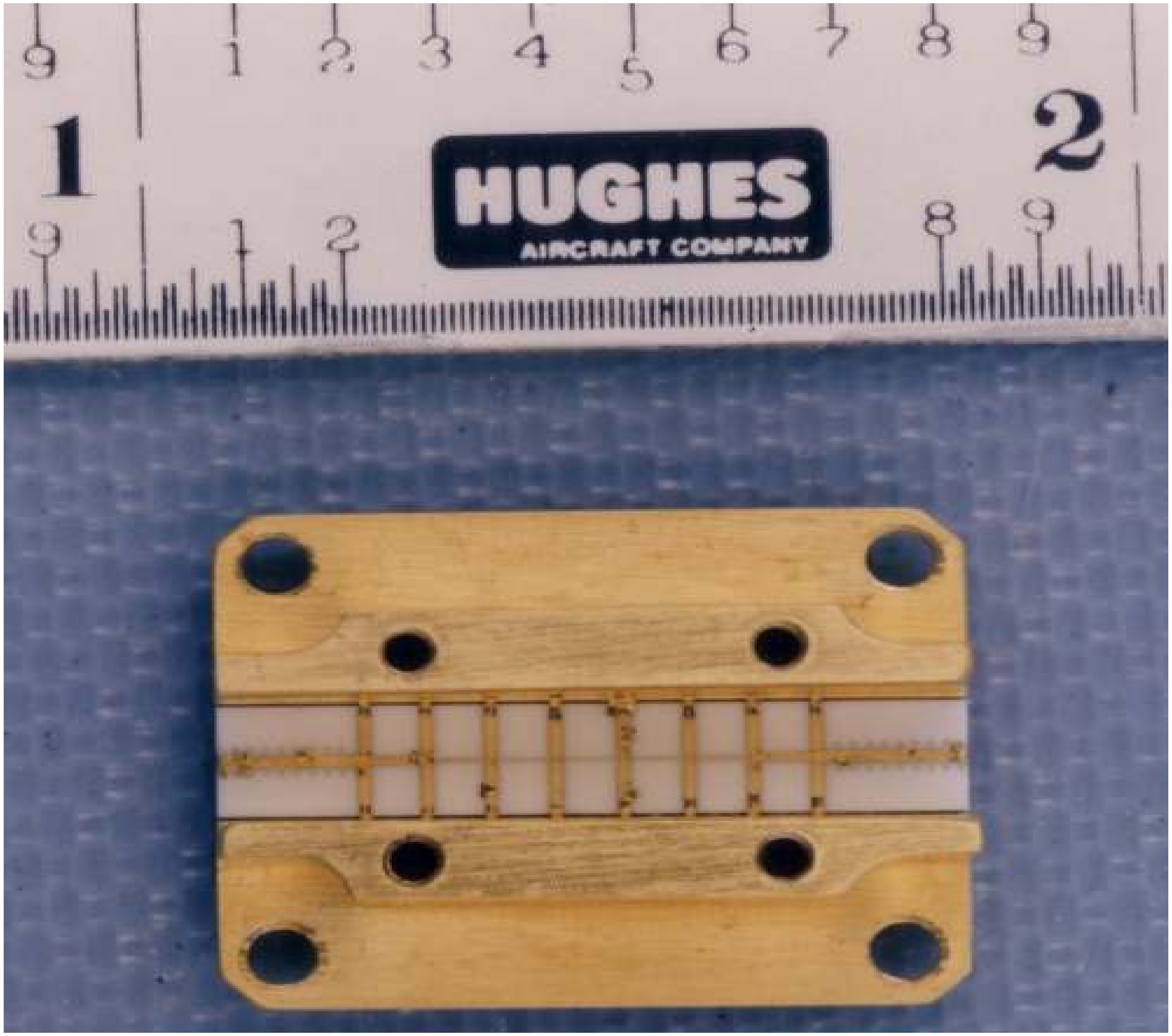
Open Shunt Stub





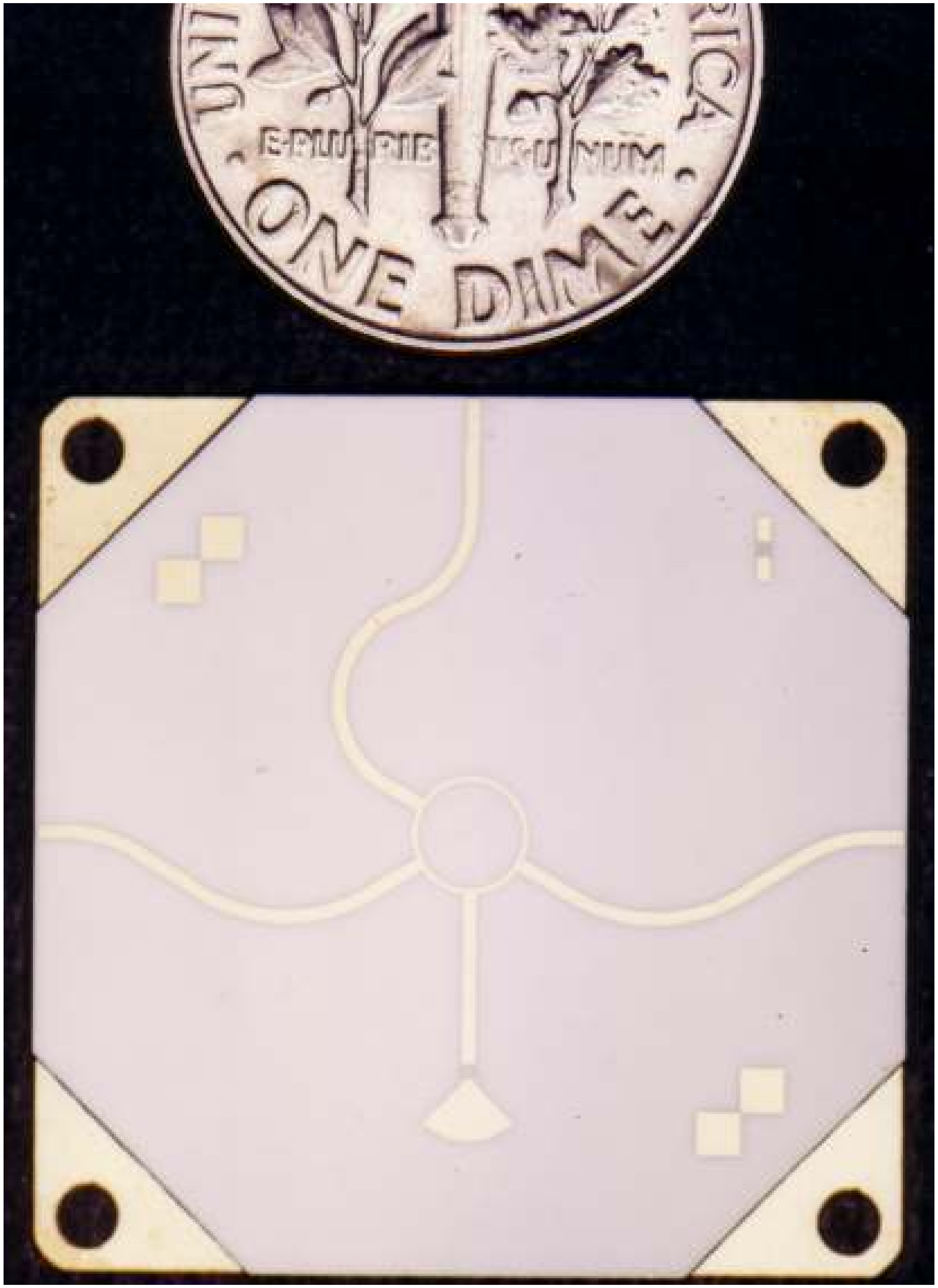
L-Band

Short Shunt Stub



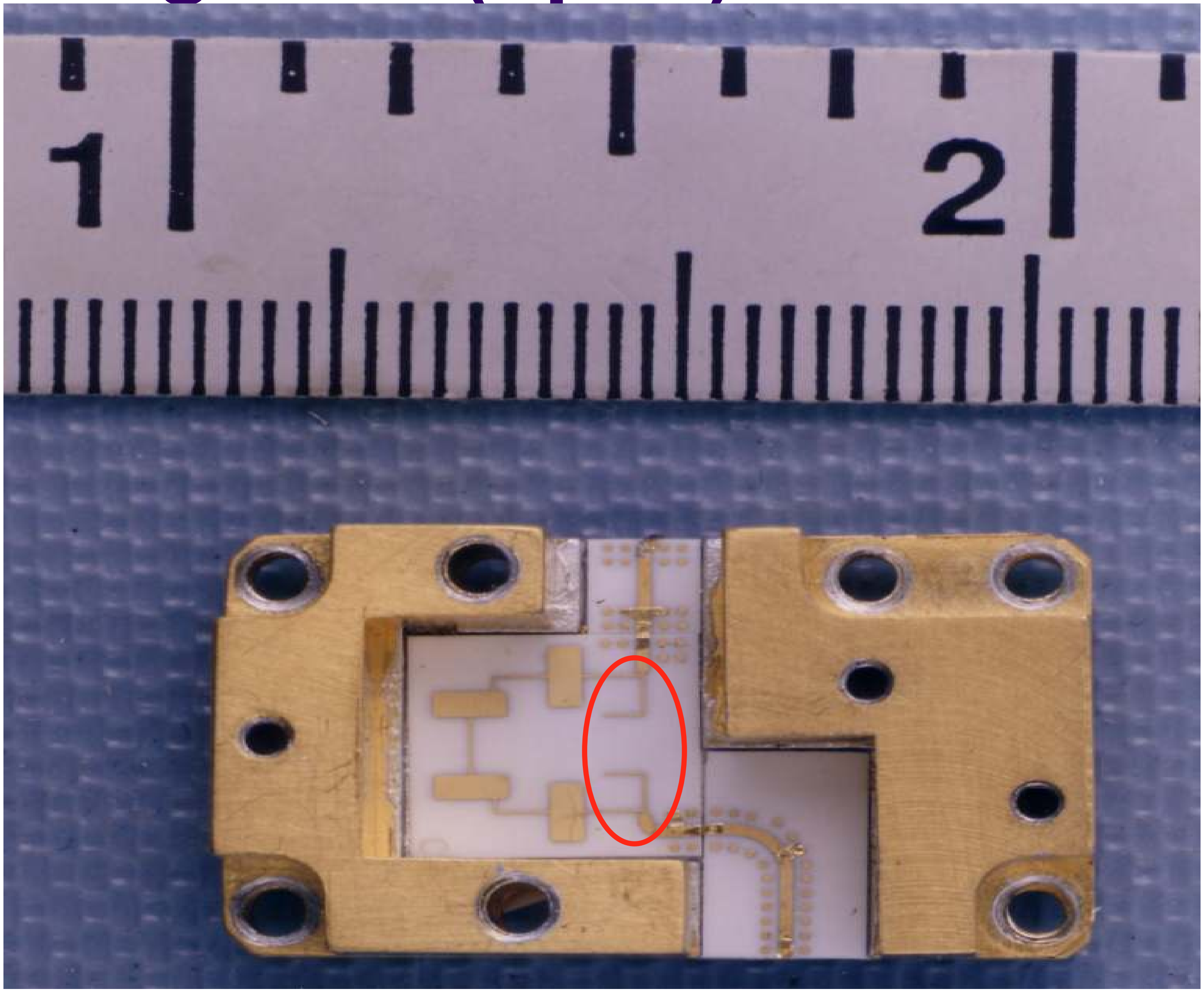
20 GHz
Interdigital
Filter

Radial Stub

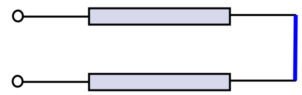


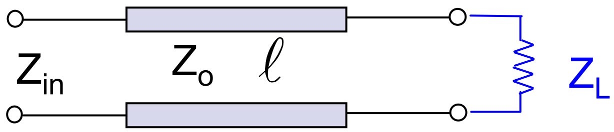
18 GHz
Rat Race

Tuning stub (open)



Short Stub



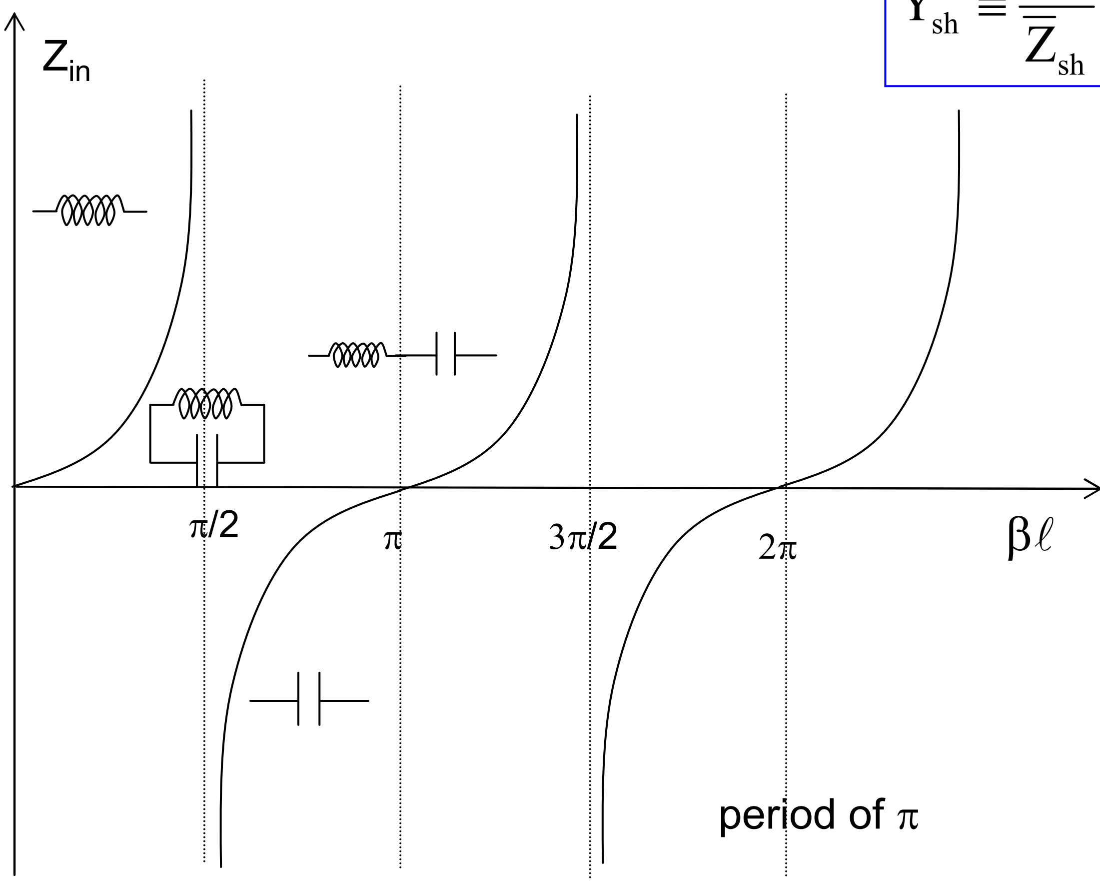


$$\overline{Z}_{in} = \frac{\overline{Z}_L + j\tan\beta\ell}{1 + j\overline{Z}_L \tan\beta\ell}$$

$$(Z_L \rightarrow 0)$$

$$\overline{Z}_{sh} = j\tan\beta\ell$$

$$\overline{Y}_{sh} \equiv \frac{1}{\overline{Z}_{sh}} = -j\cot\beta\ell$$



period of π

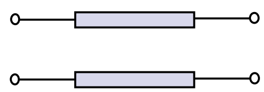
$$Z_{coil} = j\omega L$$

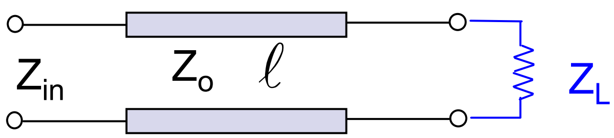
$$Z_{cap} = -j/\omega C$$

$$Z_{sres} = j\omega L\left(1 - \frac{1}{\omega^2 LC}\right)$$

$$Z_{pres} = \frac{1}{j\omega C\left(1 - \frac{1}{\omega^2 LC}\right)}$$

Open Stub



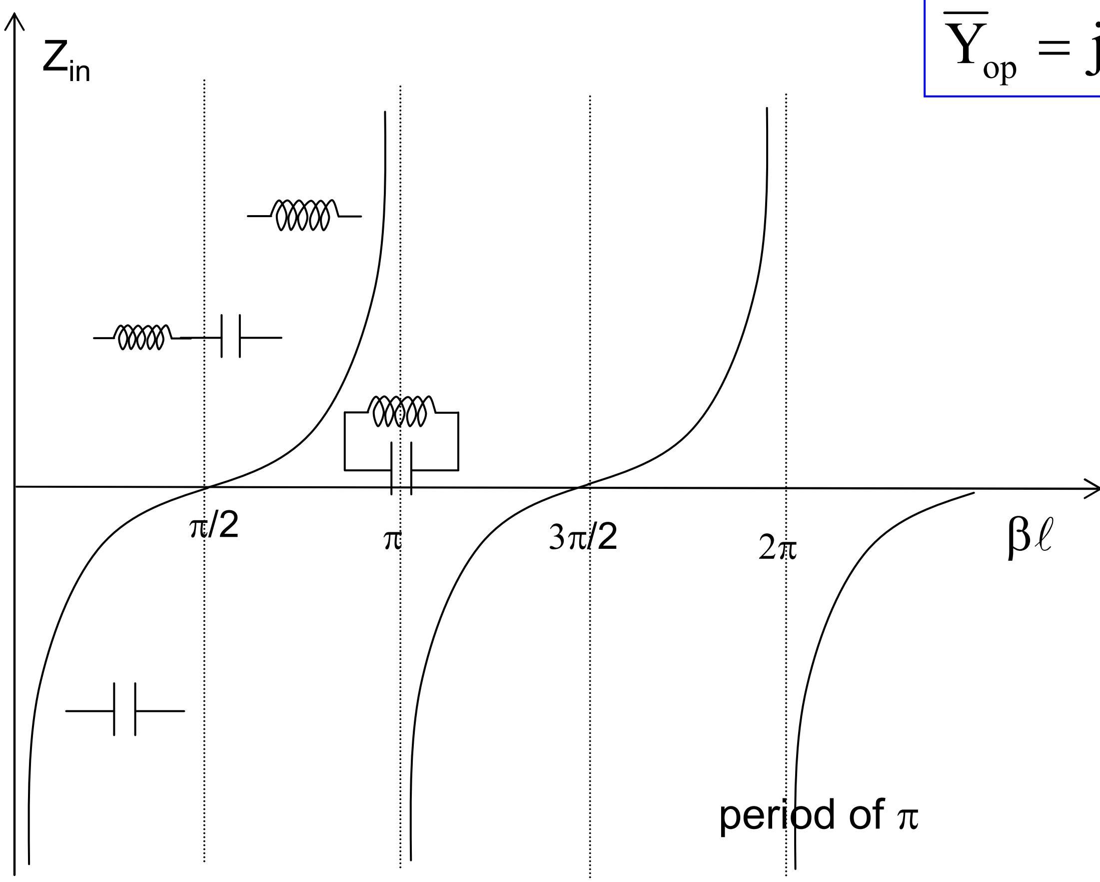


$$\overline{Z}_{in} = \frac{\overline{Z}_L + j\tan\beta\ell}{1 + j\overline{Z}_L \tan\beta\ell}$$

$(Z_L \to \infty)$

$$\overline{Z}_{op} = -j\cot\beta\ell$$

$$\overline{Y}_{op} = j\tan\beta\ell$$



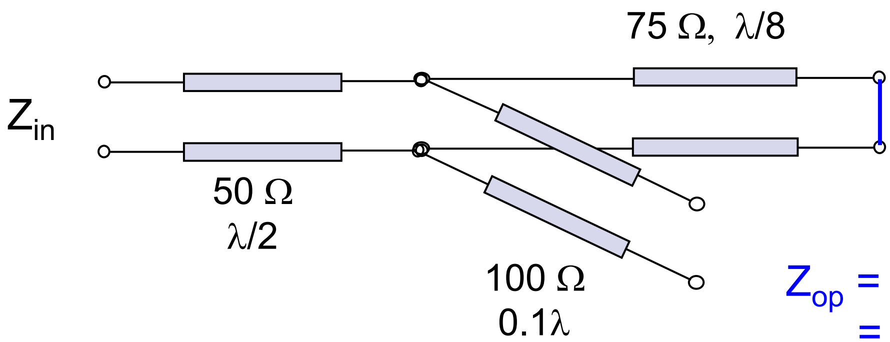
Z_{cap} = -j/ωC

Z_{coil} = jωL

$$Z_{sres} = j\omega L\left(1 - \frac{1}{\omega^2 LC}\right)$$

$$Z_{pres} = \frac{1}{j\omega C\left(1 - \frac{1}{\omega^2 LC}\right)}$$

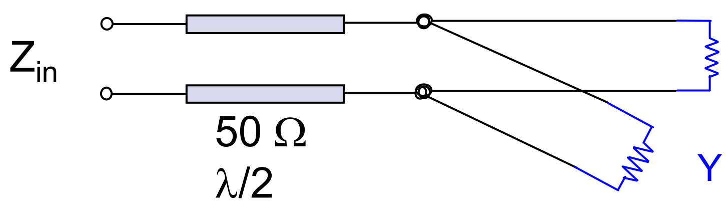
Exercise



Find Z_{in} & Γ_{in}.

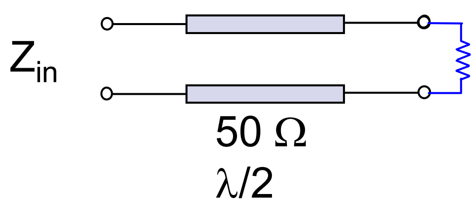
$Z_{sh} = j\, Z_o \tan(\beta l)$
$= j\ 75 \tan(45^o) = j\ 75\ \Omega$

$Z_{op} = -j\, Z_o \cot(\beta l)$
$= -j\ 100 \cot(36^o) = -j\ 138\ \Omega$



$Y = 1/j75 = -j\ 0.0133$

$Y = j/138 = j0.0073$



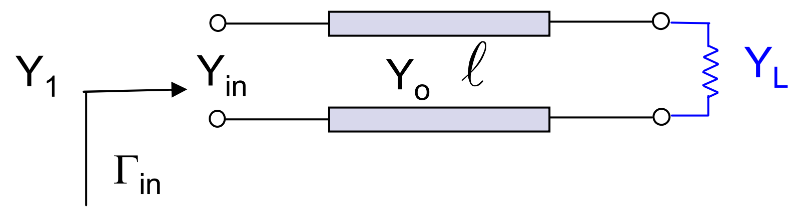
$Z = 1/Y = 1/(-j0.006) = j166\ \Omega$

$$\overline{Z}_{in} = \frac{\overline{Z}_L + j\tan\beta\ell}{1 + j\overline{Z}_L \tan\beta\ell} = \overline{Z}_L$$

$Z_{in} = j\ 166\ \Omega$

$$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = \frac{j166 - 50}{j166 + 50} = 1\angle(107^o - 73^o) = 1\angle 34^o$$

Admittance (Y = 1/Z)



$$\overline{Z}_{in} = \frac{\overline{Z}_L + j\tan\beta\ell}{1 + j\overline{Z}_L \tan\beta\ell}$$

$$\overline{Y}_{in} \equiv \frac{1}{\overline{Z}_{in}} = \frac{1 + j\overline{Z}_L \tan\beta\ell}{\overline{Z}_L + j\tan\beta\ell}$$

$$\overline{Y}_{in} = \frac{1 + j(1/\overline{Y}_L)\tan\beta\ell}{1/\overline{Y}_L + j\tan\beta\ell}$$

$$\overline{Y}_{in} = \frac{\overline{Y}_L + j\tan\beta\ell}{1 + j\overline{Y}_L \tan\beta\ell}$$

$$Y_{in} = Y_o\left(\frac{Y_L + jY_o\tan\beta\ell}{Y_o + jY_L\tan\beta\ell}\right)$$

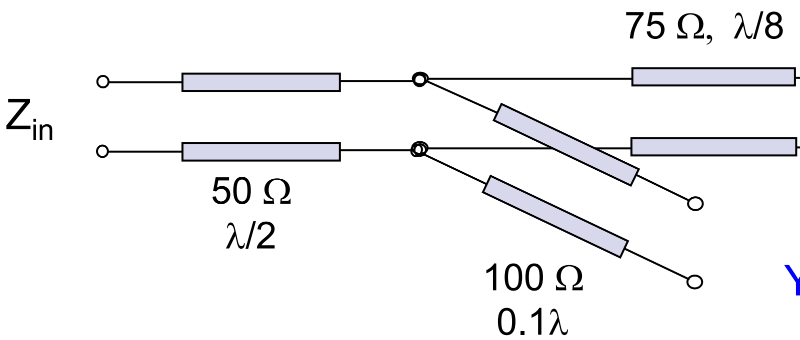
$$\Gamma_{in} = \frac{Z_{in} - Z_1}{Z_{in} + Z_1}$$

$$\Gamma_{in} = \frac{1/Y_{in} - 1/Y_1}{1/Y_{in} + 1/Y_1}$$

$$\Gamma_{in} = \frac{Y_1 - Y_{in}}{Y_1 + Y_{in}} = \frac{1 - \overline{Y}_{in}}{1 + \overline{Y}_{in}}$$

useful for shunt circuits

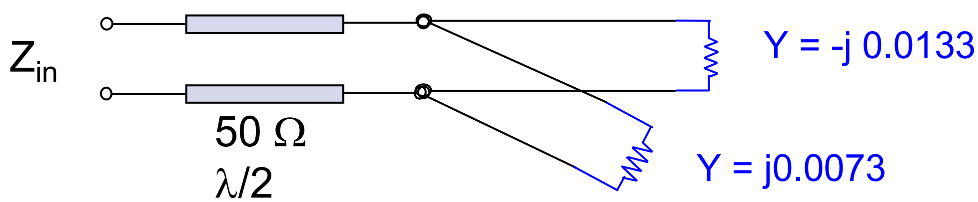
Earlier exercise

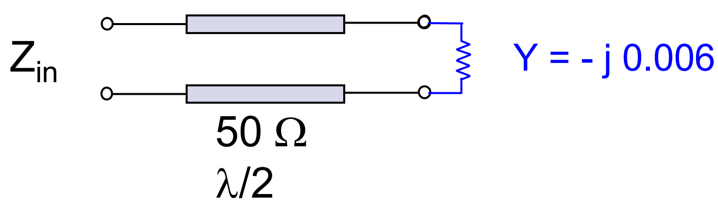


Find Z_{in} & Γ_{in}.

Y_{sh} = - j $Y_o$cot(βl)
= - j (1/75) cot(45°) = - j 0.0133

Y_{op} = j $Y_o$tan(βl)
= j 0.01 tan(36°) = j 0.0073 Ω



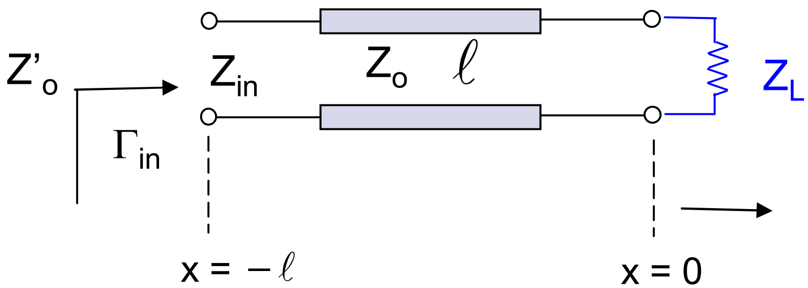


$$\overline{Y}_{in} = \frac{\overline{Y}_L + j\tan\beta\ell}{1 + j\overline{Y}_L \tan\beta\ell} = \overline{Y}_L$$

Y_L = - j 0.006
Z_{in} = j 166 Ω

$$\Gamma_{in} = \frac{Y_o - Y_{in}}{Y_o + Y_{in}} = \frac{1/50 + j0.006}{1/50 - j0.006} = 1\angle(17^o + 17^o) = 1\angle 34^o$$

Earlier Exercise – power consideration



Z_o = 50 Ω
Z'_o = 50 Ω
Z_L = 100Ω
Length = λ/8
Γ_L = 1/3 Γ_{in} = 1/3 (-90°)
only change phase

$$\overline{Z}_{in} = \frac{\overline{Z}_L + j\tan\beta\ell}{1 + j\overline{Z}_L \tan\beta\ell}$$

$$Z_{in} = Z_o\left(\frac{Z_L + jZ_o\tan\beta\ell}{Z_o + jZ_L\tan\beta\ell}\right)$$

$$\Gamma_{in} = \frac{Z_{in} - Z'_o}{Z_{in} + Z'_o} = \frac{\overline{Z}'_{in} - 1}{\overline{Z}'_{in} + 1}$$

Power reflected = ? $\left|\frac{V^-}{V^+}\right|^2 = |\Gamma|^2 = \left|\frac{1}{3}\right|^2 = 11\%$

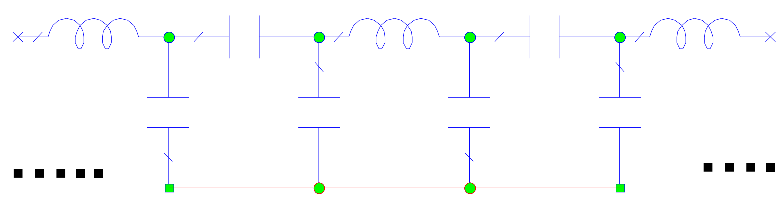
Power delivered = ? $1 - |\Gamma|^2 = 89\%$

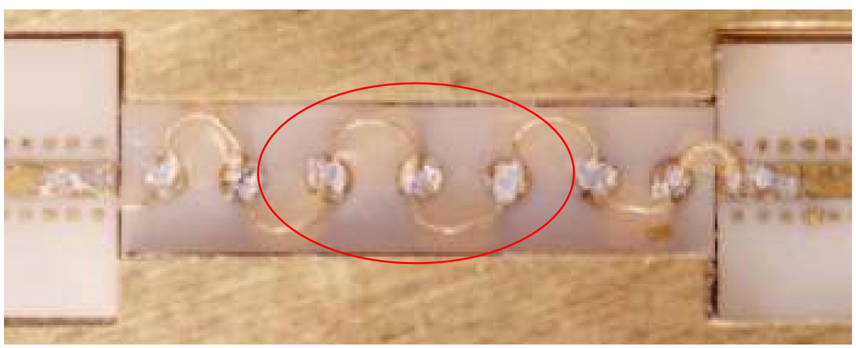
Don't double count reflection…. Γ_L & Γ_{in}

Return Loss (RL) = - 20 log|ρ| = + 9.5 dB

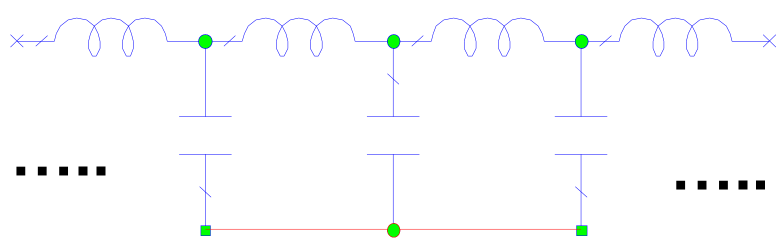
High f circuit elements

1 GHz lumped element
Band pass filter



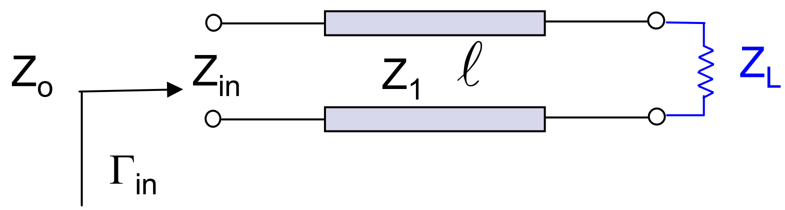


12 GHz lumped element
Low pass filter
much smaller



A small loop of thin wire is an inductor !!

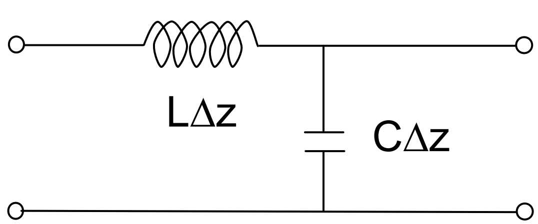
High-Z Line as inductor



$$Z_{in} = Z_1 \left(\frac{Z_L + jZ_1 \tan \beta \ell}{Z_1 + jZ_L \tan \beta \ell} \right)$$

$Z_1 >> Z_L$
line length < $\lambda/4$ ($\pi/2$)
$Z_L \sim Z_o$ (order of magnitude)

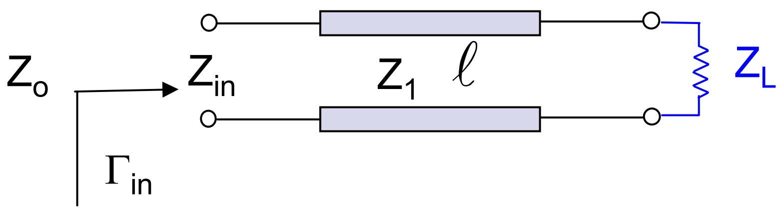
$$Z_{in} = Z_1 \left(\frac{a\angle + \Psi}{b\angle + \varphi} \right) = |Z_{in}| \angle + \theta$$



small C, large L, series inductor

Z_{in} has a positive phase
→ inductor-like !!!

Low-Z Line as capacitor



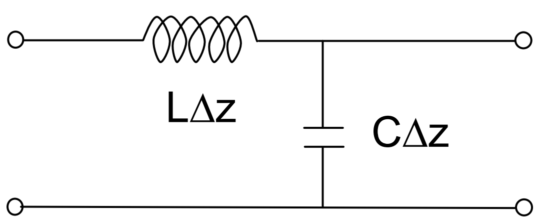
$$Z_{in} = Z_1\left(\frac{Z_L + jZ_1 \tan\beta\ell}{Z_1 + jZ_L \tan\beta\ell}\right)$$

$Z_1 << Z_L$
line length << $\lambda/4$ ($\pi/2$)
$Z_L \sim Z_o$

$$Z_{in} = Z_1\left(\frac{a\angle+\varphi}{b\angle+\Psi}\right) = |Z_{in}|\angle-\theta$$

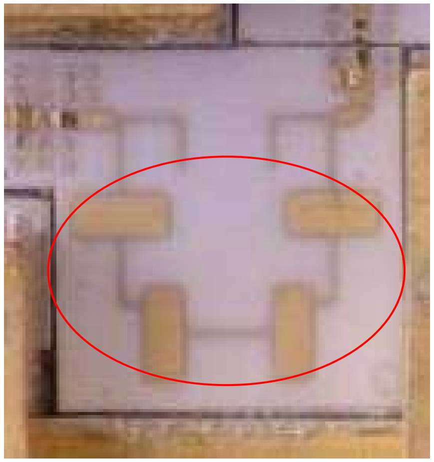
$$Y_{in} = |Y_{in}|\angle+\theta$$

Y_{in} has a positive phase
→ capacitor-like !!!

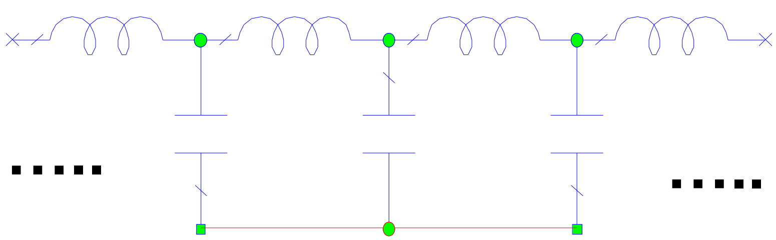


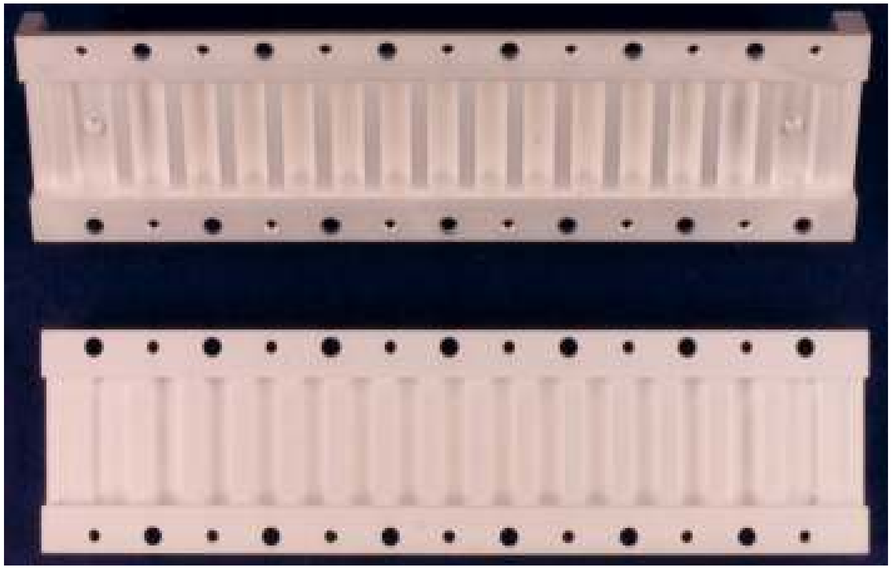
small L, large C, shunt capacitor

Low pass filter



5 GHz low pass filter

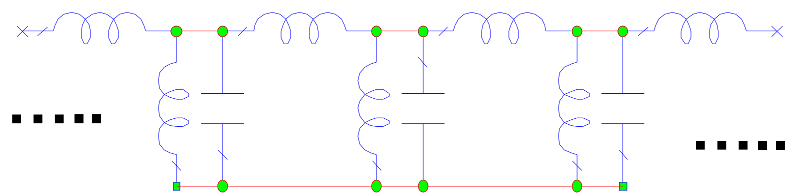


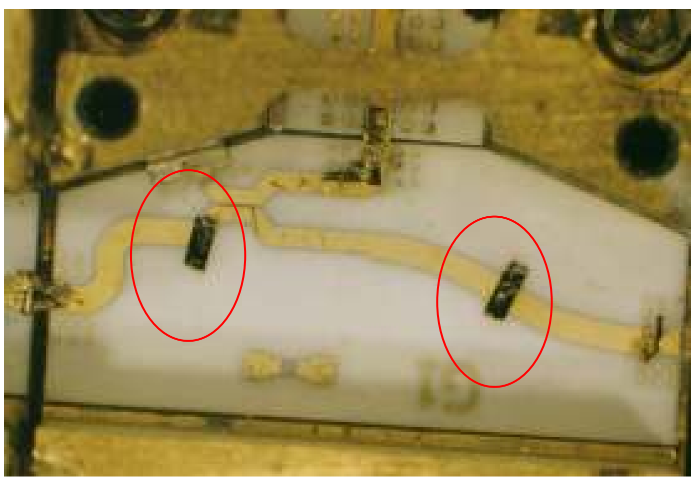


14 GHz low pass filter
high-low impedance lines
waveguide
high power
low loss

High-Low-Z lines

20 GHz band pass filter
high Z lines → inductors
Short shunt stubs $\lambda/4$ resonators





13 GHz coupler
Tuning with stubs (shunt open)
Think of them as shunt capacitors
→ low Z lines

Homework

1. A 100 Ω tranmission line has an effective dielectric constant of 1.65. Find the shortest open-circuited length of this line that appears at its input as a capacitor of 5 pF at 2.5 GHz. Repeat for an inductance of 5 nH.

2. A radio transmitter is connected to an antenna having an impedance 80 + j40 Ω with a 50 Ω coaxial cable. If the 50 Ω transmitter can deliver 30 W when connected to a 50 Ω load, how much power is delivered to the antenna?

3. A 75 Ω coaxial transmission line has a length of 2 cm and is terminated with a load impedance of 37.5 + j75 Ω. If the dielectric constant of the line is 2.56 and the frequency is 3 GHz, find the input impedance to the line, the reflection coefficient a the load, the reflection coefficient at the input, and the SWR on the line.

4. The VSWR on a lossless 300 Ω transmission line terminated in an unknown load impedance is 2.0, and the nearest voltage minimum is at a distance 0.3λ from the load. Determine (a) Γ_L, (b) Z_L.

5. Calculate VSWR, ρ, and return loss values to complete the entries in the following table.

VSWR	ρ	RL (dB)
1.00	0.00	∞
1.01		
	0.01	
1.05		32
		30
1.10		
1.20		
	0.10	
1.50		
		10
2.00		
2.50		

6. Measurements on a 0.6 m lossless coaxial cable at 100 kHz show a capacitance of 54 pF when the cable is open-circuited, and an inductance of 0.30 μH when it is short-circuited. (a) Determine Z_o and ε_r of the medium.