Review

Electrostatic

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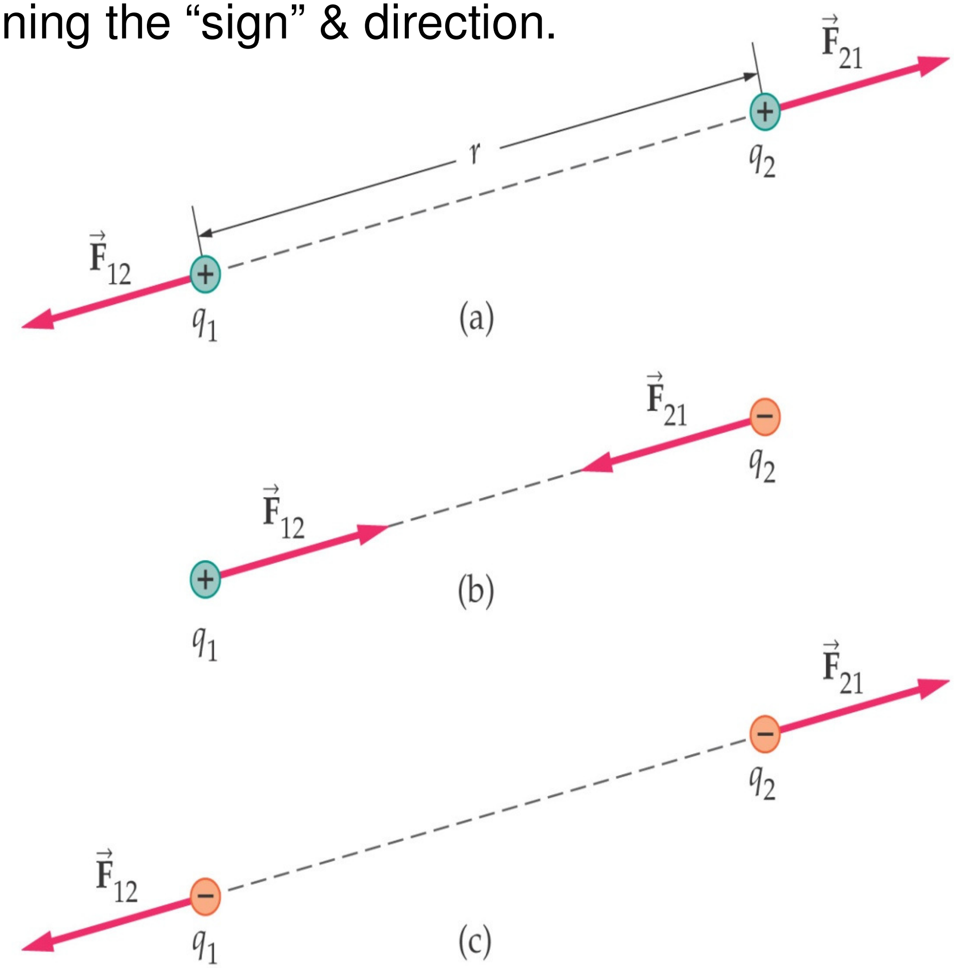
Party Balloons

Coulomb's Law

$$F_e = k\frac{q_1 q_2}{r^2}$$

Coulomb force or electrical force. (vector)
Be careful on determining the "sign" & direction.



$$k = 9 \cdot 10^9 (N \cdot m^2 / C^2)$$

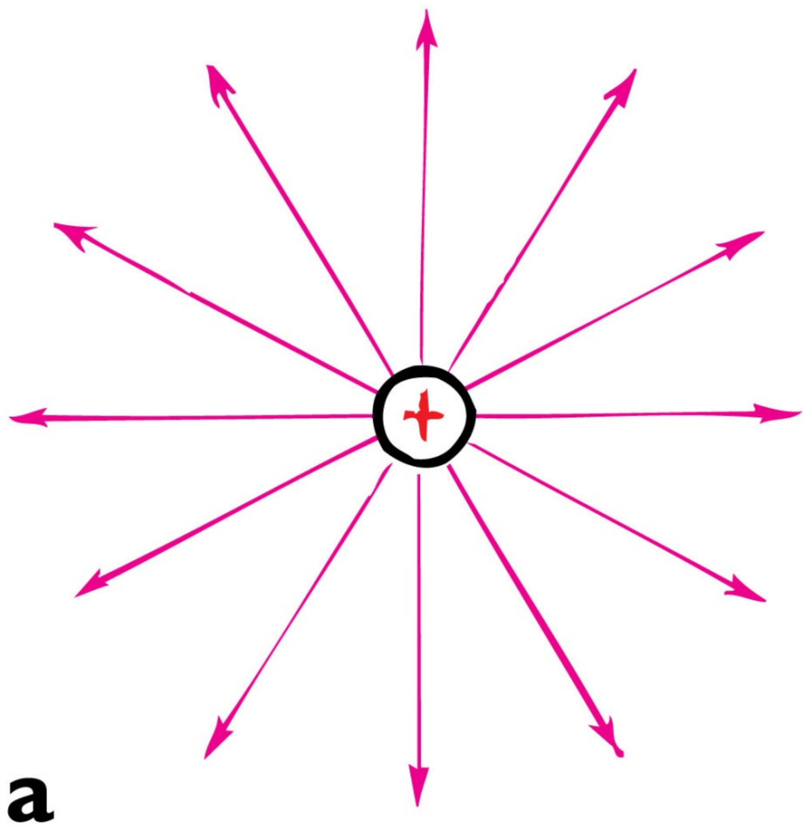
$$k = \frac{1}{4\pi\varepsilon_o}$$

k is the Coulomb's constant,
ε_o is called permittivity of vacuum,
for now it's just a constant.

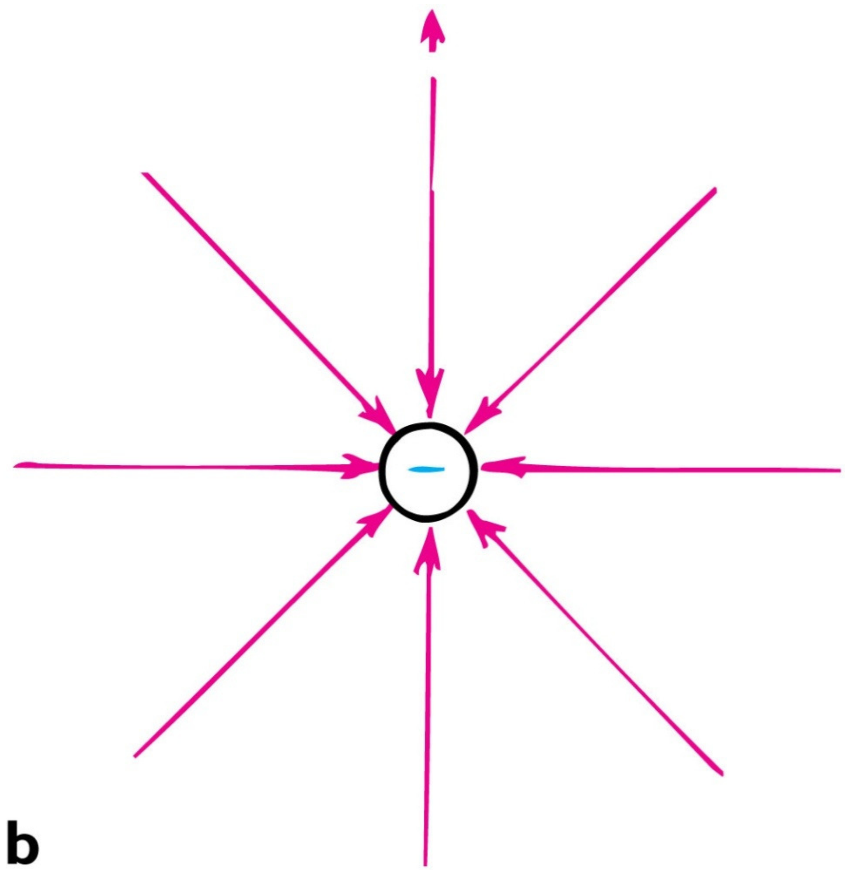
$$\varepsilon_o = \frac{1}{4\pi k} = \frac{10^{-9}}{36\pi} = 8.84 \cdot 10^{-12}\left(\frac{C^2}{N \cdot m^2}\right)$$

Electric Field Lines (F = qE)

Think of the direction of coulomb force on a “+” test charge

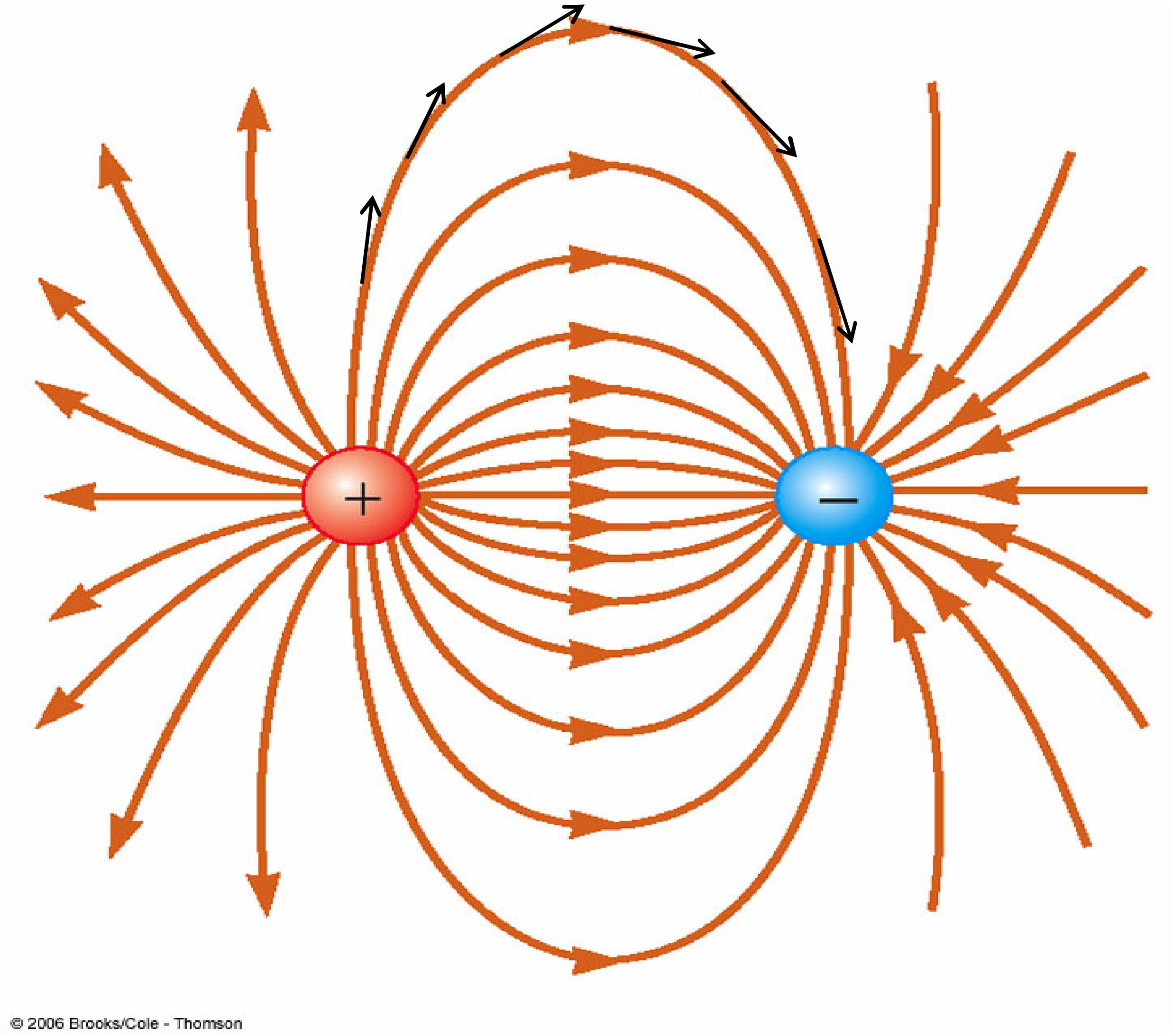


generate from a “+” charge

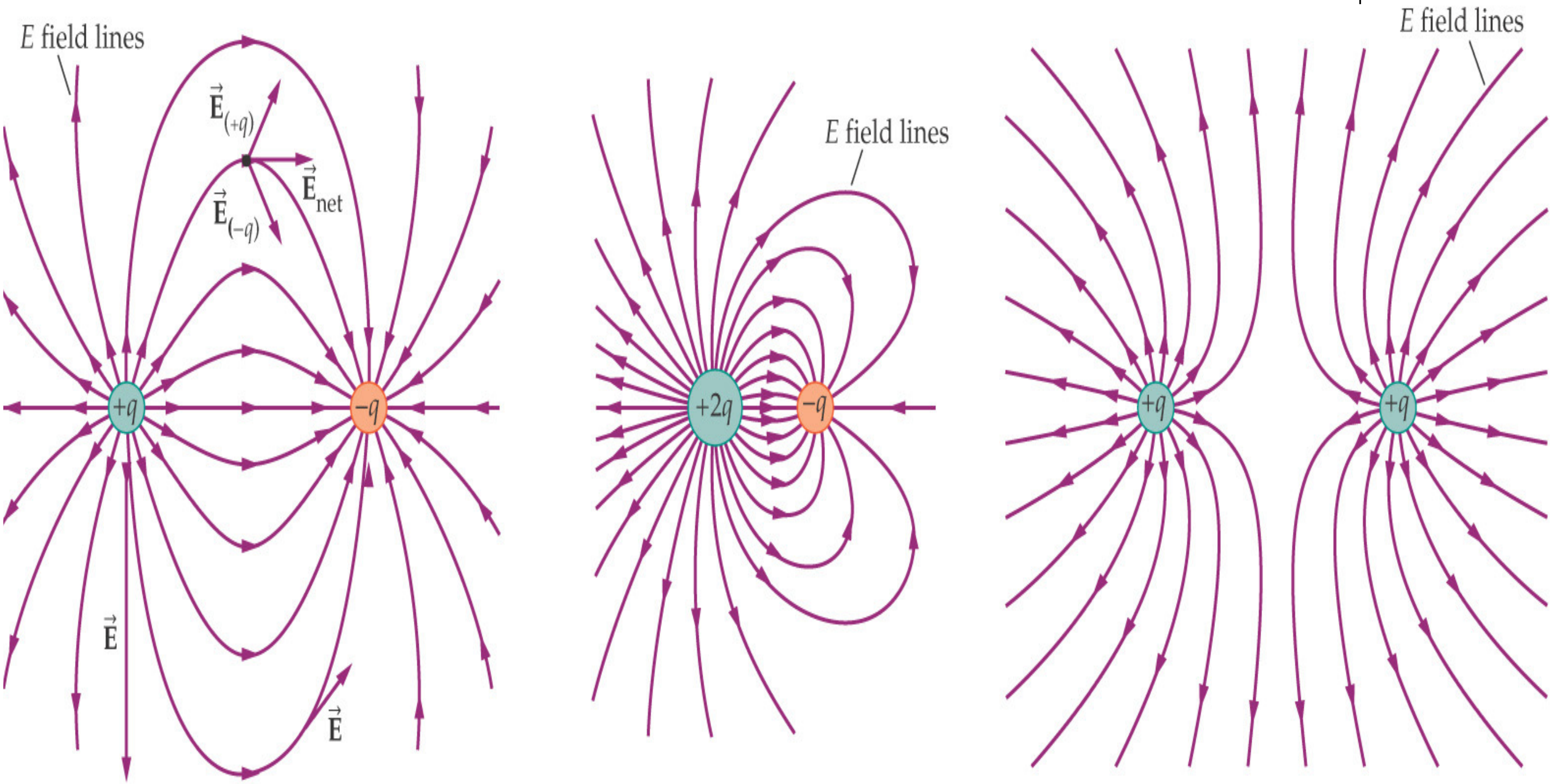


terminate into a “-” charge

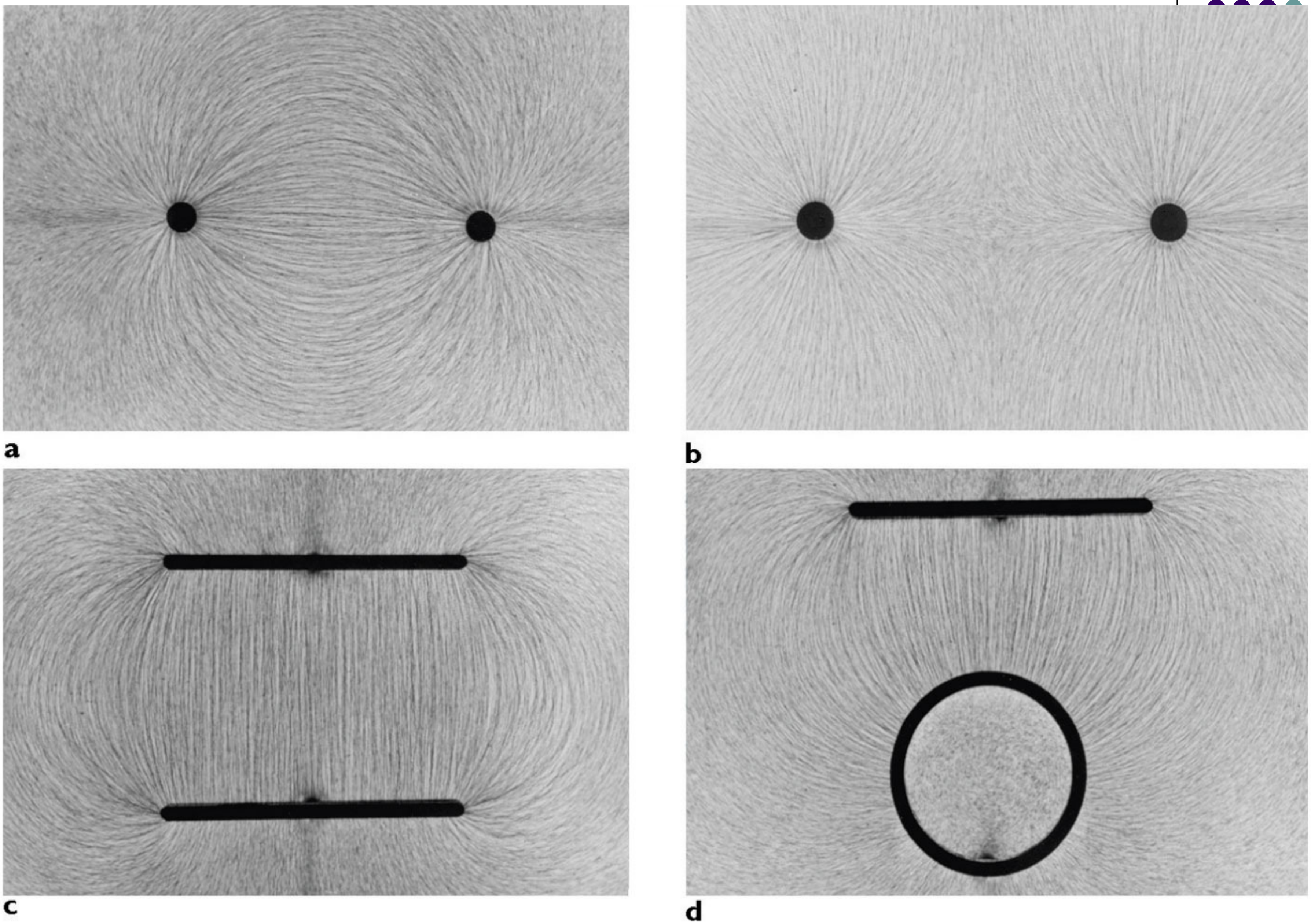
Two opposite charges



Other combinations



(a) (b) (c)


a
b
c
d

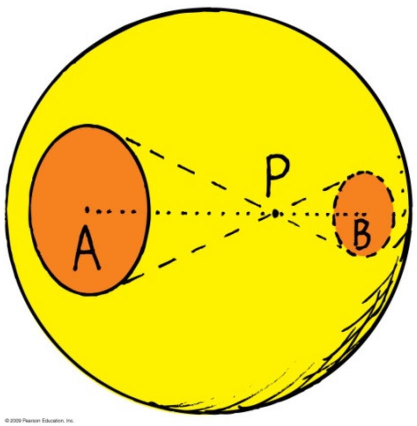
Conductor & E-field: Property 1

- The electric field is zero everywhere inside the conducting material
  - Consider if this were *not* true
    - If there were an electric field inside the conductor, the free charge there would move and there would be a flow of charge
    - If there were a movement of charge, the conductor would not be in equilibrium. (Electrostatic !!)

Property 2

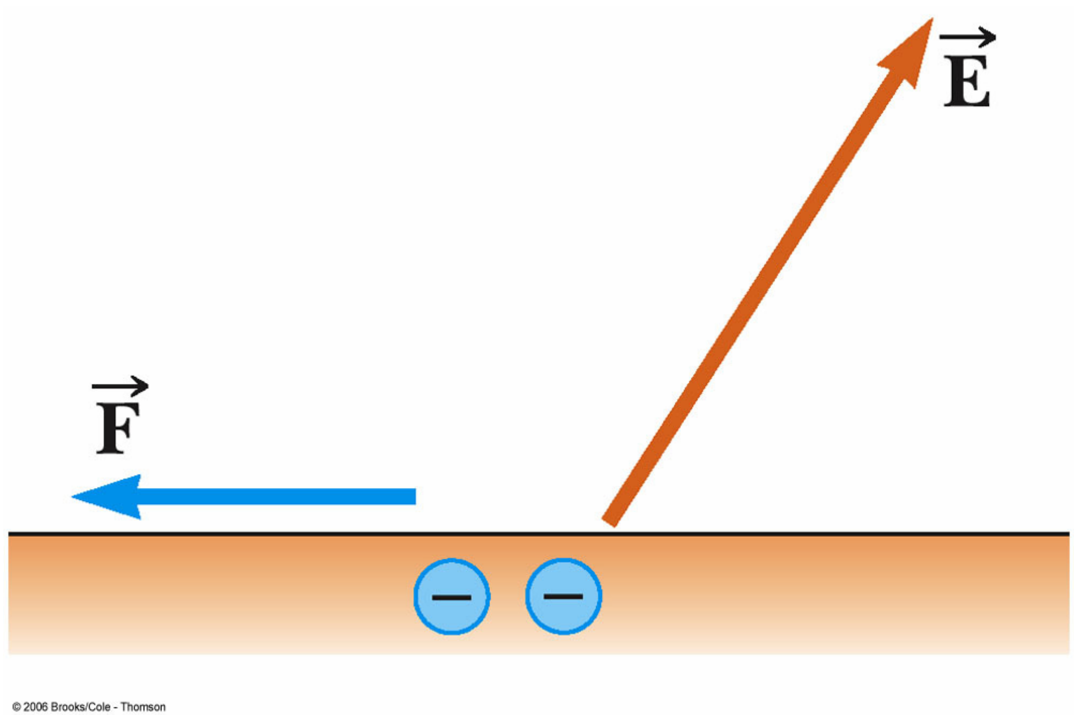
- Any excess charge on an isolated conductor resides entirely on its surface
  - A direct result of the $1/r^2$ repulsion between like charges in Coulomb's Law
  - If some excess of charge could be placed inside the conductor, the repulsive forces would push them as far apart as possible, causing them to migrate to the surface

So E = 0 inside.



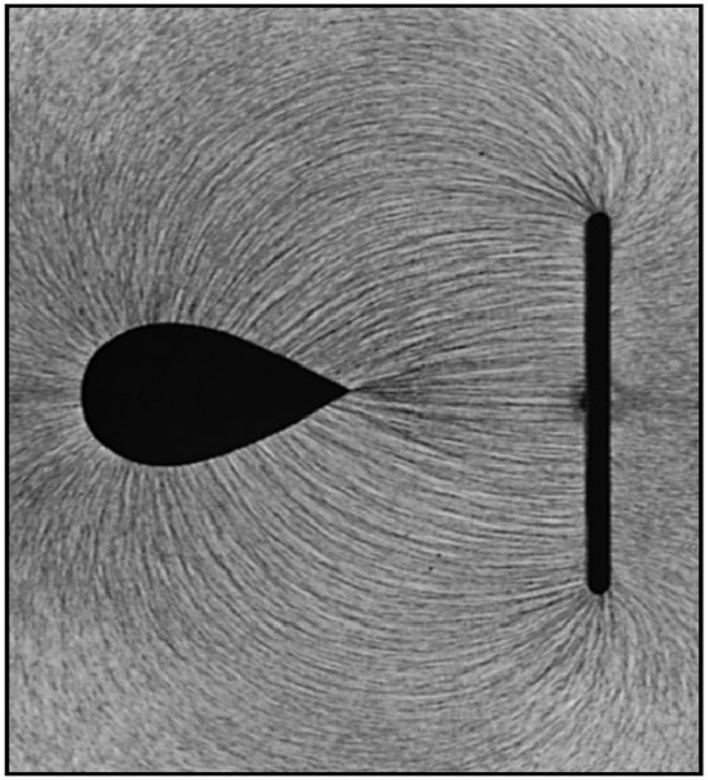
Property 3

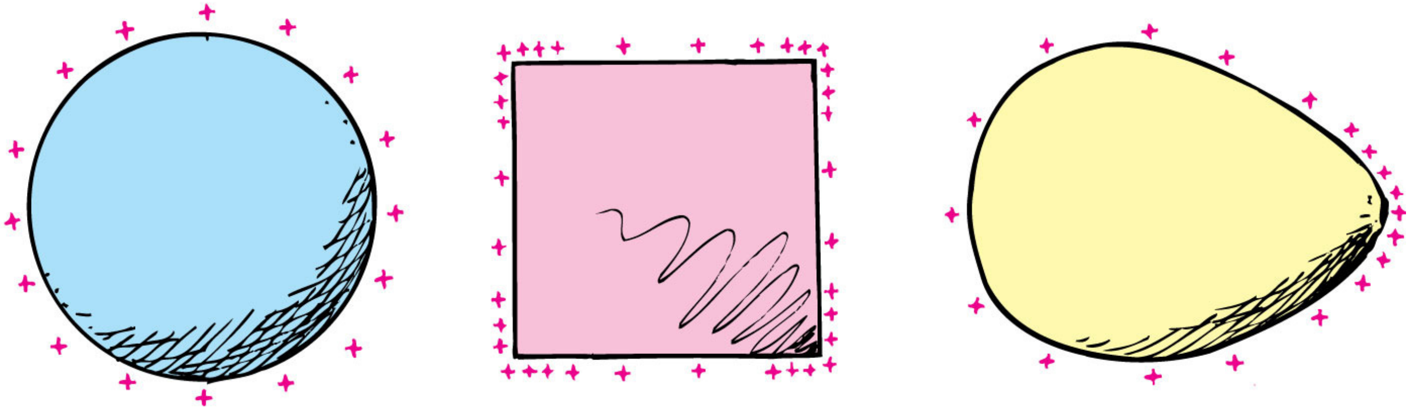
- The electric field just outside a charged conductor is perpendicular to the conductor's surface
  - Consider what would happen it this was not true
  - The component along the surface would cause the charge to move
  - It would not be in equilibrium



Property 4

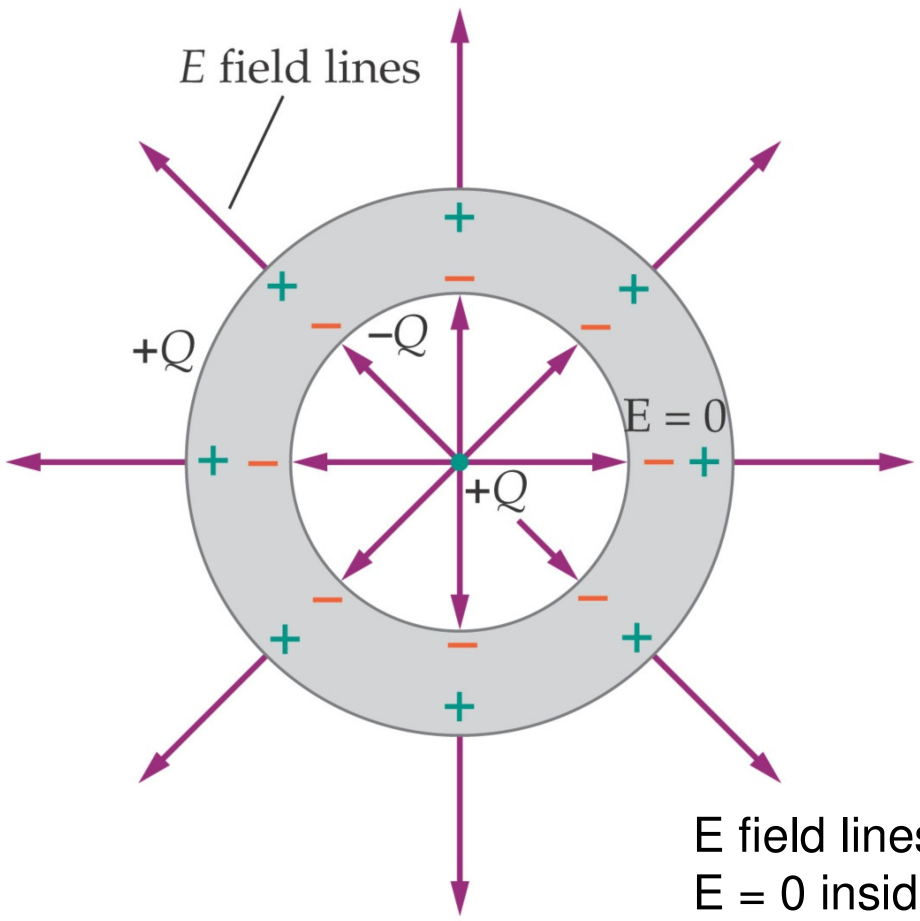
- On an irregularly shaped conductor, the charge accumulates at locations where the radius of curvature of the surface is smallest (that is, at sharp points)





e.g. Metal shielding

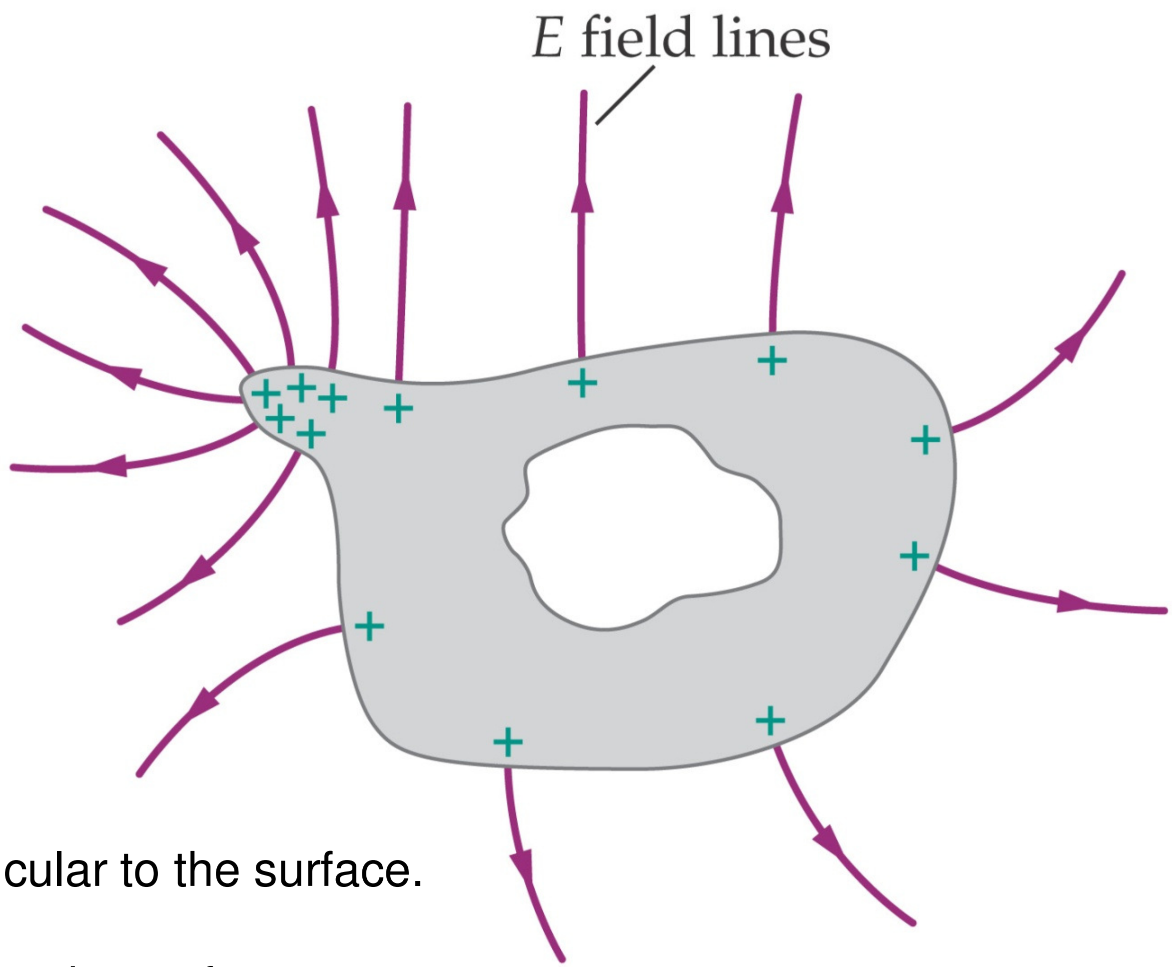
Place a point charge inside a conducting shell.



E field lines are perpendicular to the surface.
E = 0 inside conductor.
Charges are induced on the surface.

E field of a charged conductor

irregular shape…



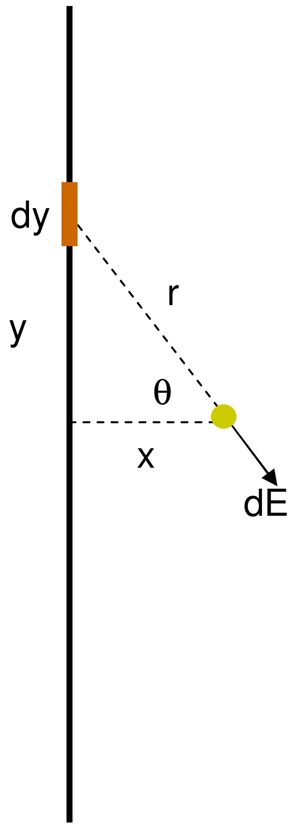
E field lines are perpendicular to the surface.
E = 0 inside conductor.
Charges are cumulated on the surface.
E field is more intense at small radius (higher concentration of charges)

Charge distribution

A non-conducting thin wire of 2 m long, carrying a charge density of +10μC/m. What is the electric field at 10 cm from the center of the wire?

Notation used in most textbook

- ρ = volume charge density
- σ = surface (area) charge density
- λ = linear charge density



$$|dE| = k\frac{|dq|}{r^2} = k\frac{(\lambda dy)}{x^2+y^2}$$

(x is a constant = 10 cm)

$$|dE|_x = |dE|\cos\theta = k\frac{(\lambda dy)}{x^2+y^2}\left(\frac{x}{r}\right) = \frac{k\lambda x(dy)}{(x^2+y^2)^{3/2}}$$

$$E = \int_{-1}^{1}\frac{k\lambda x(dy)}{(x^2+y^2)^{3/2}}$$

$$\int\frac{dy}{r^3} = \int\frac{x\sec^2\theta\, d\theta}{(x\sec\theta)^3} = \frac{1}{x^2}\int\cos\theta\, d\theta = \frac{1}{x^2}\sin\theta = \frac{1}{x^2}\left[\frac{y}{\sqrt{x^2+y^2}}\right]$$

$$\int_{-1}^{1}\frac{dy}{r^3} = \frac{1}{x^2}\left[\frac{y}{\sqrt{x^2+y^2}}\right]_{-1}^{1} = \frac{1}{(0.1)^2}\left[\frac{2}{\sqrt{0.1^2+1}}\right] = 199$$

$$E = k\lambda x\int_{-1}^{1}\frac{dy}{r^3} = (9\cdot 10^9)(10^{-5})(0.1)(199) = 1.8\cdot 10^6$$

N/C, to the right

Gauss

Carl Friedrich Gauss
German mathematician & scientist
1777 - 1855

In cgs, gauss is the unit for magnetic field.

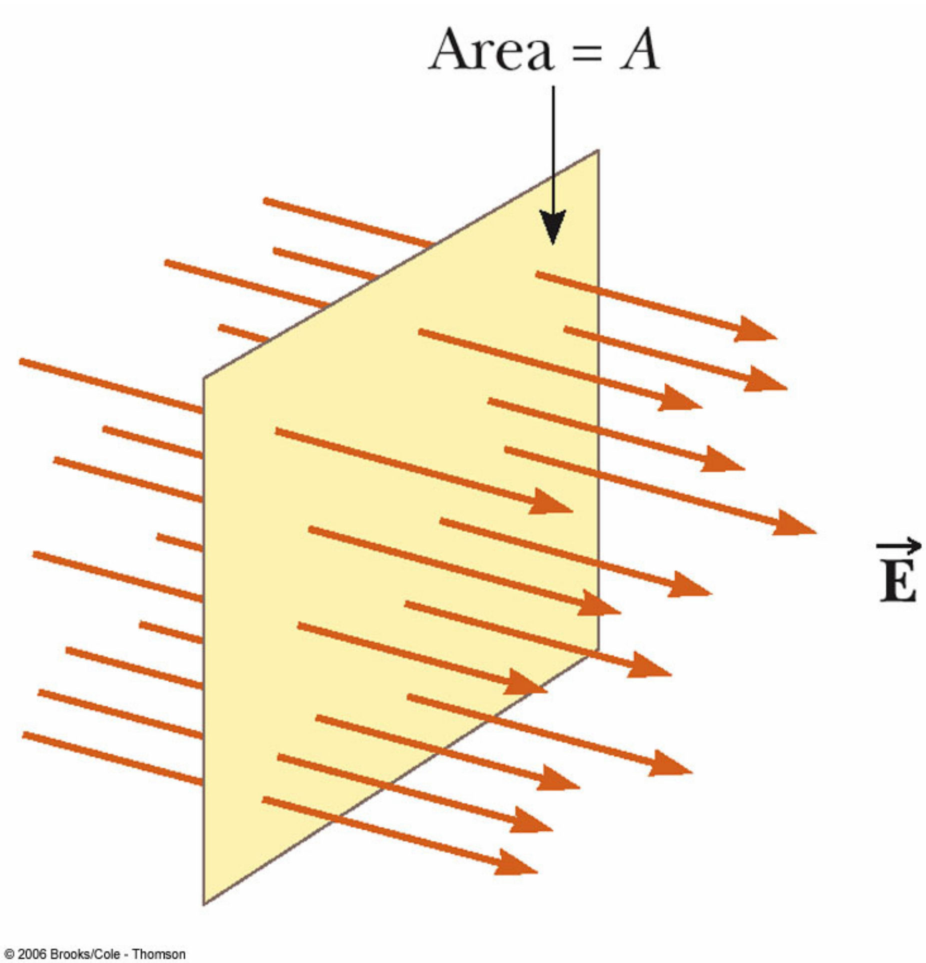
Many contributions in math and geophysics.

Most known for his electrostatic work….
Known as the Gauss's Law.

Electric Flux

- Field lines penetrating an area A perpendicular to the field
- The product of EA is the flux, Φ
- In general:
  - $\Phi_E = E\,A\cos\theta$

$$\Phi_E \equiv \int \vec{E} \cdot d\vec{a}$$



Gauss' Law

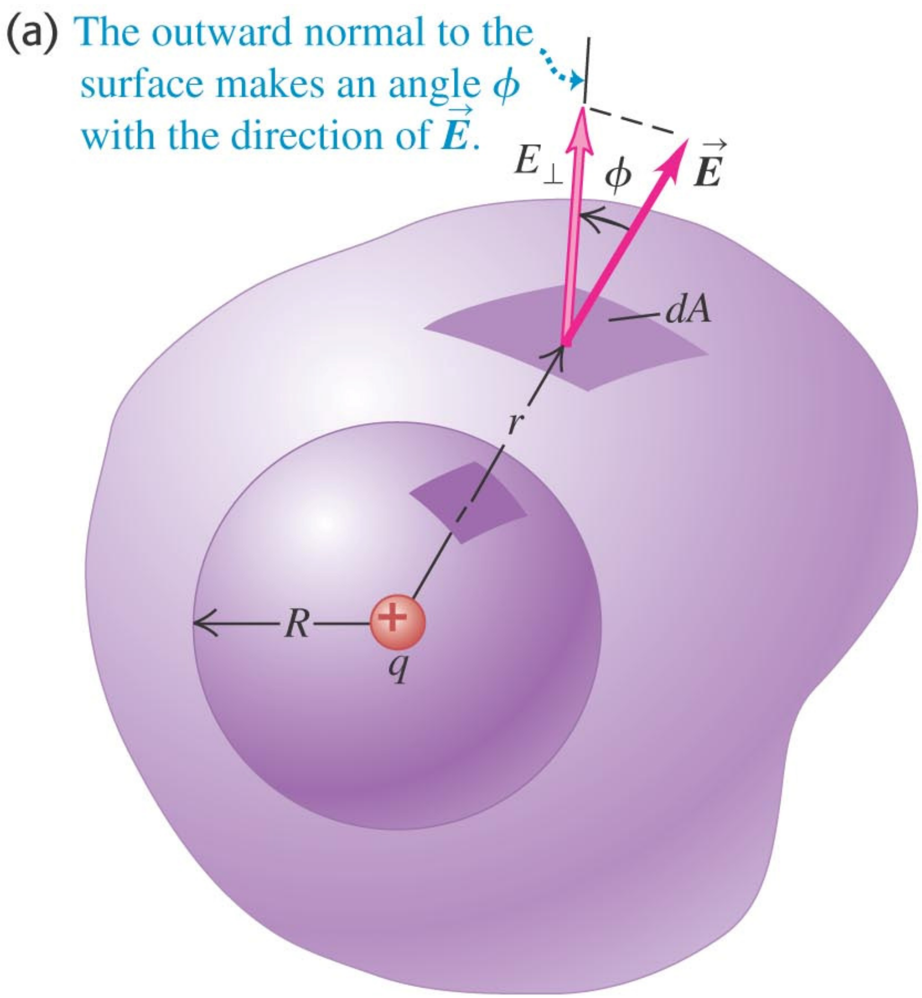
- Gauss' Law states that the electric flux through any closed surface is equal to the net charge Q inside the surface divided by ε_o

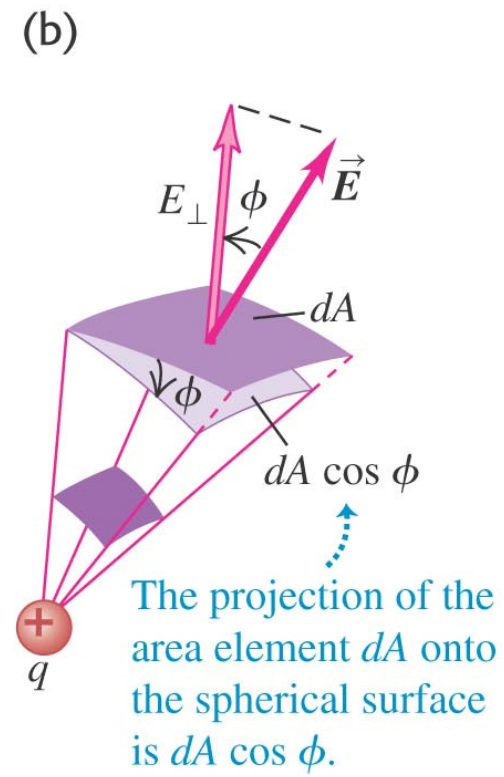
$$\Phi_E = \frac{Q_{inside}}{\varepsilon_o}$$

  - ε_o is the *permittivity of free space* and equals 8.85 x 10^{-12} C^2/Nm^2
  - The area in Φ is an imaginary surface, a Gaussian surface, it does not have to coincide with the surface of a physical object

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{inside}}{\varepsilon_o}$$

Total flux does not depends on the shape chosen





Gaussian Surface

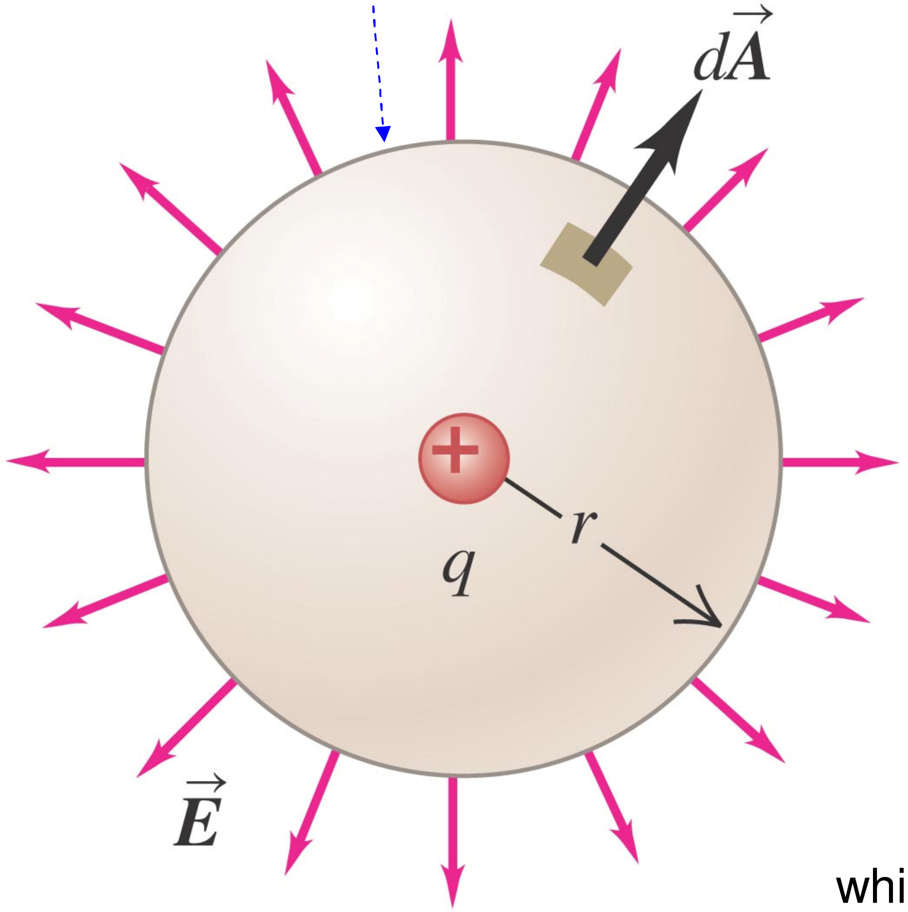
- Have the same symmetry as the charge distribution, such that…
- E field is uniform on the surface
- E field is perpendicular to the surface

$$\Phi_E \equiv \int \vec{E} \cdot d\vec{a} = EA$$

$$EA = \frac{Q_{inside}}{\varepsilon_o}$$

e.g. Point Charge

Gaussian surface is a sphere



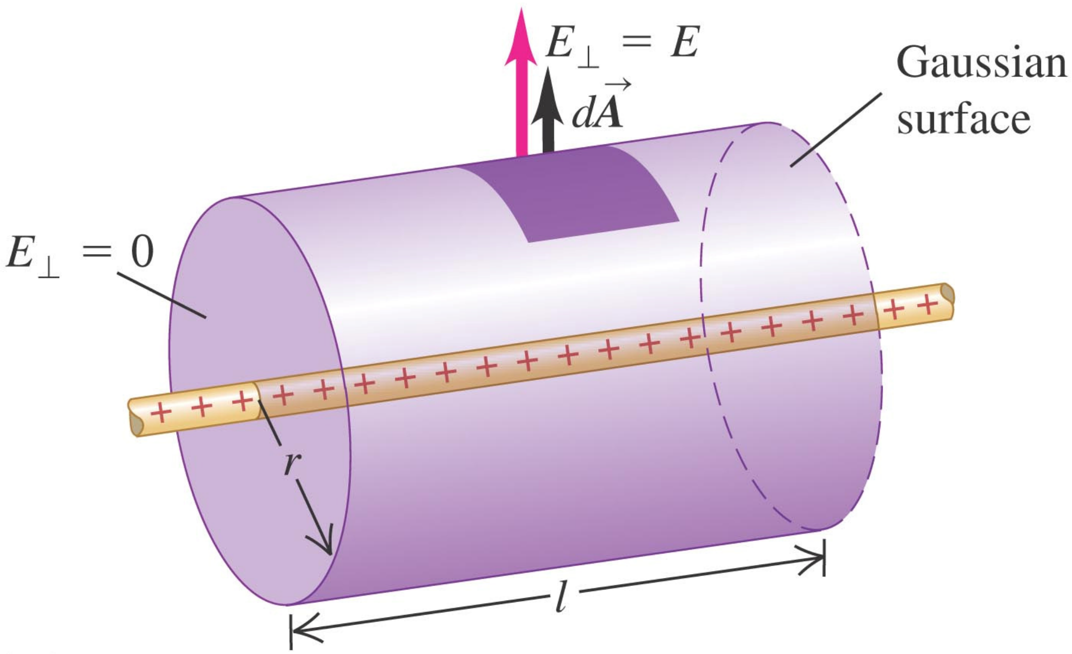
$$EA = \frac{Q_{inside}}{\varepsilon_o}$$

$$E(4\pi r^2) = \frac{q}{\varepsilon_o}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

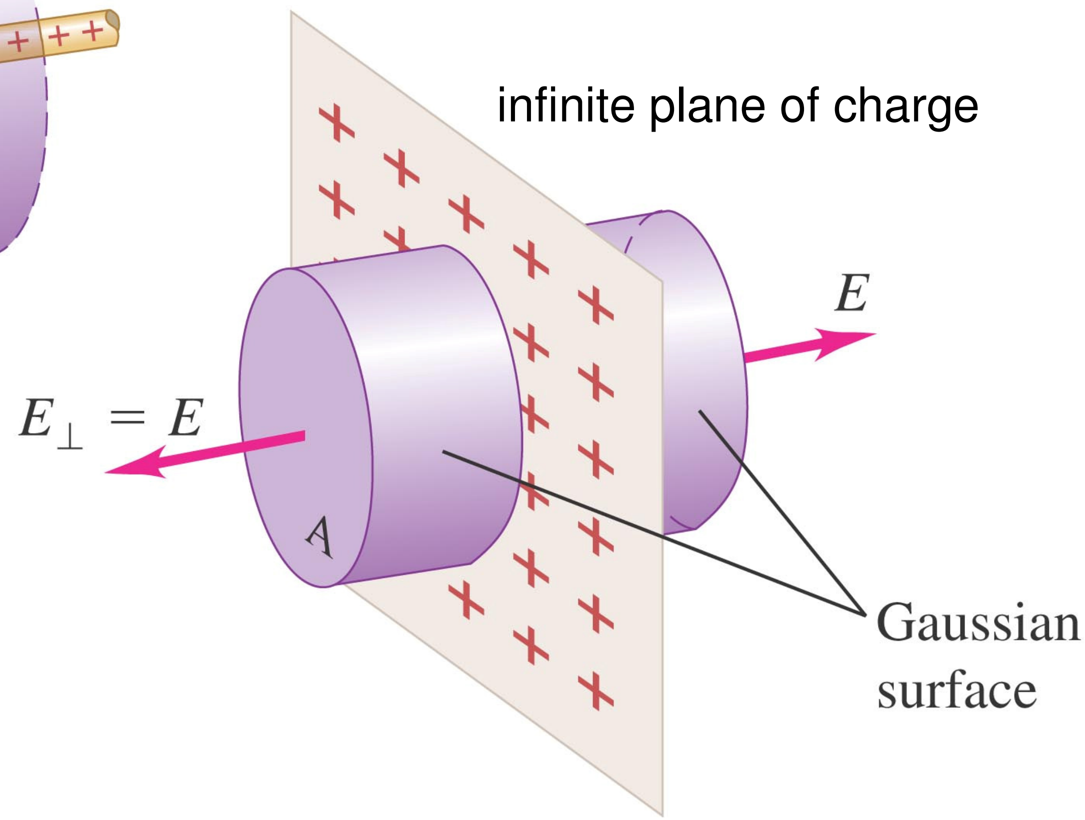
which is the E field for a point charge

E field of a line or plane of charge

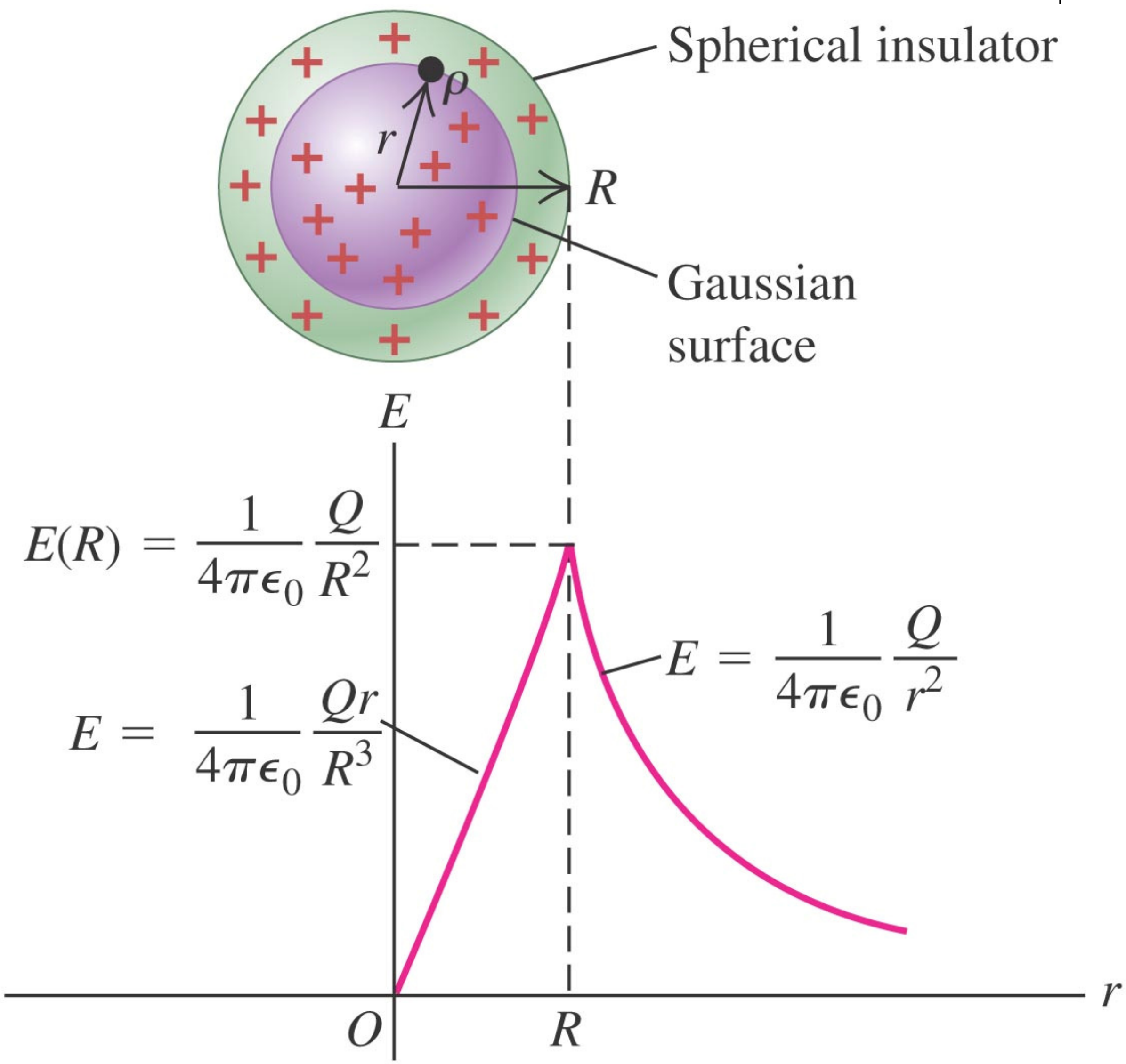


infinite line charge

infinite plane of charge



E field of a uniformly charged sphere



Divergence Theorem

or Gauss's Theorem

$$\int_V \nabla \cdot \vec{E}\, dV = \oint_S \vec{E} \cdot d\vec{a}$$

for any vector E.

Gauss's Law

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{inside}}{\varepsilon_o}$$

$$\int_V \nabla \cdot \vec{E}\, dV = \int_V \frac{\rho\, dV}{\varepsilon_o}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_o}$$

Gauss's Law in differential form

Stoke's Theorem

$$\int_S \nabla \times \vec{E} \cdot d\vec{a} = \oint_L \vec{E} \cdot d\vec{\ell}$$

for any vector E.

In electrostatic, we defined electric potential (voltage): $V = -\int_L \vec{E} \cdot d\vec{\ell}$

and $\oint_L \vec{E} \cdot d\vec{\ell} = 0$ Conservative field, Kirchhoff's rule.

$$\nabla \times \vec{E} = 0$$