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$$a) \nabla \cdot D = \rho_v = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle xy^2z^3, 0, 0 \rangle = y^2z^3 \frac{C}{m}$$

$$b) Q = \int_V \rho_v dv = \int_0^2 \int_0^2 \int_0^2 y^2 z^3 dx dy dz = \frac{64}{3} C$$

$$c) \oint_S \vec{D} \cdot d\vec{s} = Q$$

$$\oint_{\text{left face}} \vec{D} \cdot d\vec{s} = \int \int xy^2z^3 \hat{x} \cdot -\hat{y} dx dz = 0$$

$$\oint_{\text{right face}} \vec{D} \cdot d\vec{s} = \int \int xy^2z^3 \hat{x} \cdot \hat{y} dx dz = 0$$

$$\oint_{\text{bottom face}} \vec{D} \cdot d\vec{s} = \int \int xy^2z^3 \hat{x} \cdot -\hat{z} dx dz = 0$$

$$\oint_{\text{top face}} \vec{D} \cdot d\vec{s} = \int \int xy^2z^3 \hat{x} \cdot \hat{z} dx dz = 0$$

$$\oint_{\text{front face}} \vec{D} \cdot d\vec{s} = \int_0^2 \int_0^2 xy^2z^3 \hat{x} \cdot \hat{x} dx dz = \frac{64}{3} C$$

$$\oint_{\text{back face}} \vec{D} \cdot d\vec{s} = \int_0^2 \int_0^2 xy^2z^3 \hat{x} \cdot -\hat{x} dx dz = 0$$