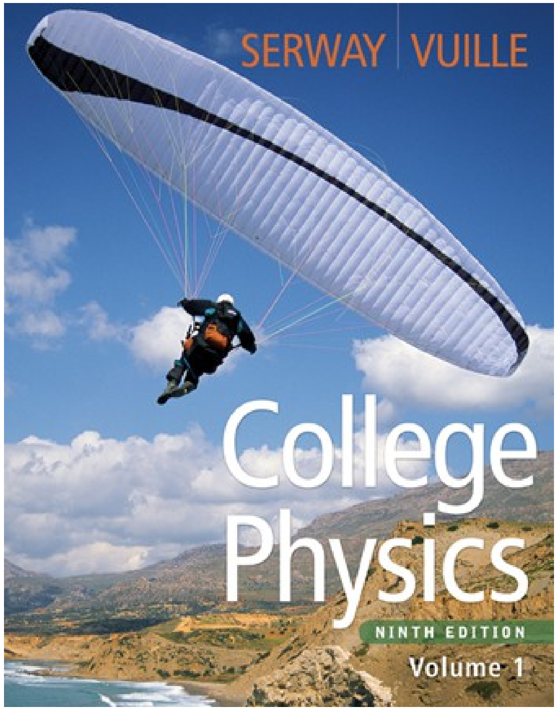

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CENGAGE Learning

Chapter 20

Induced Voltages and Inductance

Quiz 8: 1. Which of the following actions would double the magnitude of the magnetic force per unit length between two parallel current-carrying wires? Choose all correct answers. (a) Double one of the currents. (b) Double the distance between them. (c) Reduce the distance between them by half. (d) Double both currents.

2. • True or false:

(*a*) The magnetic moment of a bar magnet points from its north pole to its south pole.

(*b*) Inside the material of a bar magnet, the magnetic field due to the bar magnet points from the magnet's south pole toward its north pole.

(*c*) If a current loop simultaneously has its current doubled and its area cut in half, then the magnitude of its magnetic moment remains the same.

(*d*) The maximum torque on a current loop placed in a magnetic field occurs when the plane of the loop is perpendicular to the direction of the magnetic field.

Connections Between Electricity and Magnetism

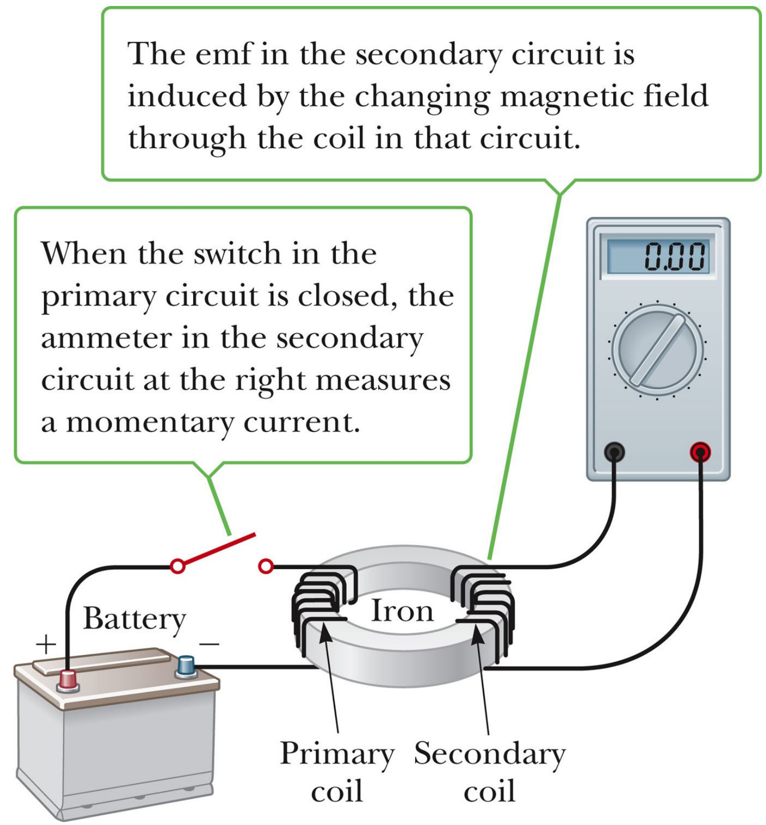
- 1819
  - Hans Christian Oersted discovered an electric current exerts a force on a magnetic compass.
  - First evidence of a link between electricity and magnetism
- 1831
  - Faraday and Henry showed a changing magnetic field could induce an electric current in a circuit.
  - Led to Faraday's Law

Michael Faraday

- 1791 – 1867
- Great experimental scientist
- Invented electric motor, generator and transformers
- Discovered electromagnetic induction
- Discovered laws of electrolysis

Faraday's Experiment – Set Up

- A current can be produced by a changing magnetic field.
  - First shown in an experiment by Michael Faraday
- A primary coil is connected to a battery.
- A secondary coil is connected to an ammeter.



Faraday's Experiment

- There is no battery in the secondary circuit.
- When the switch is closed, the ammeter reads a current and then returns to zero.
- When the switch is opened, the ammeter reads a current in the opposite direction and then returns to zero.
- When there is a steady current in the primary circuit, the ammeter reads zero.

Faraday's Conclusions

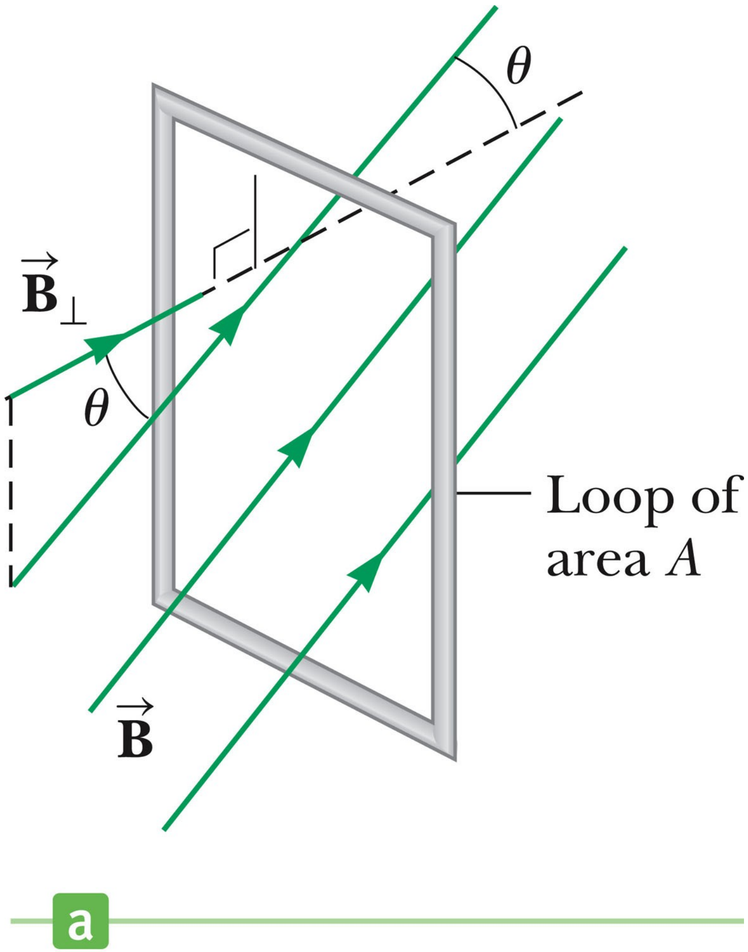
- An electrical current is produced by a *changing* magnetic field.
- The secondary circuit acts as if a source of emf were connected to it for a short time.
- It is customary to say that *an induced emf is produced in the secondary circuit by the changing magnetic field.*

Magnetic Flux

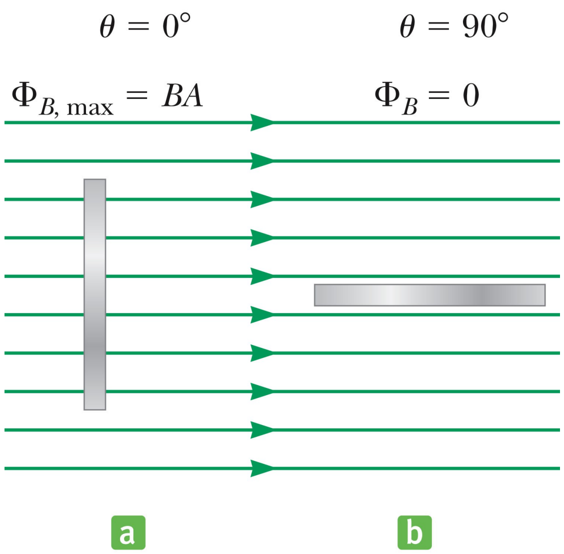
- The emf is actually induced by a change in the quantity called the *magnetic flux* rather than simply by a change in the magnetic field.
- Magnetic flux is defined in a manner similar to that of electrical flux.
- Magnetic flux is proportional to both the strength of the magnetic field passing through the plane of a loop of wire and the area of the loop.

Magnetic Flux, 2

- You are given a loop of wire.
- The wire is in a uniform magnetic field.
- The loop has an area A.
- The flux is defined as
  - $\Phi_B = B_{\geq} A = B A \cos \theta$
  - θ is the angle between B and the normal to the plan
- SI unit: weber (Wb)
  - $Wb = T \cdot m^2$



Magnetic Flux, 3



- When the field is perpendicular to the plane of the loop, as in a, $\theta = 0$ and $\Phi_B = \Phi_{B,\,max} = BA$
- When the field is parallel to the plane of the loop, as in b, $\theta = 90°$ and $\Phi_B = 0$
  - The flux can be negative, for example if $\theta = 180°$

Magnetic Flux, Final

- The flux can be visualized with respect to magnetic field lines.
  - **The value of the magnetic flux is proportional to the total number of lines passing through the loop.**
- When the area is perpendicular to the lines, the maximum number of lines pass through the area and the flux is a maximum.
- When the area is parallel to the lines, no lines pass through the area and the flux is 0.

A conducting circular loop of radius 0.250 m is placed in the *xy*-plane in a uniform magnetic field of 0.360 T that points in the positive *z*-direction, the same direction as the normal to the plane. **(a)** Calculate the magnetic flux through the loop. **(b)** Suppose the loop is rotated clockwise around the *x*-axis, so the normal direction now points at a 45.0° angle with respect to the *z*-axis. Recalculate the magnetic flux through the loop. **(c)** What is the change in flux due to the rotation of the loop?

$$A = \pi r^2 = \pi (0.250 m)^2 = 0.196 m^2$$

$$\phi_B = \vec{A} \cdot \vec{B} = AB\cos\theta = (0.196 m^2)(0.360 T)\cos 0^\circ$$

$$= 0.0706 T m^2 = 0.0706 Wb$$

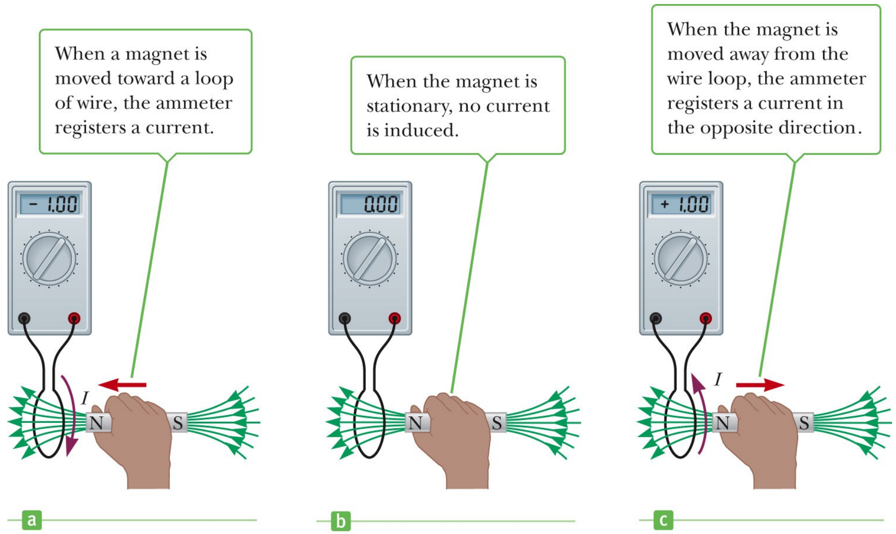
$$\phi_B = \vec{A} \cdot \vec{B} = (0.196 m^2)(0.360 T)\cos 45^\circ$$

$$= 0.0499 Wb$$

$$\Delta\phi_B = (0.0499 - 0.0706) Wb$$

$$= -0.0207 Wb.$$

Electromagnetic Induction – An Experiment



- When a magnet moves toward a loop of wire, the ammeter shows the presence of a current (a).
- When the magnet is held stationary, there is no current (b).
- When the magnet moves away from the loop, the ammeter shows a current in the opposite direction (c).
- If the loop is moved instead of the magnet, a current is also detected.

Electromagnetic Induction – Results of the Experiment

- A current is set up in the circuit as long as there is *relative motion* between the magnet and the loop.
  - The same experimental results are found whether the loop moves or the magnet moves.
- The current is called an *induced current* because is it produced by an induced emf.

Faraday's Law and Electromagnetic Induction

- The instantaneous emf induced in a circuit equals the negative time rate of change of magnetic flux through the circuit.
- If a circuit contains N tightly wound loops and the flux changes by $\Delta\Phi_B$ during a time interval Δt, the average emf induced is given by *Faraday's Law:*
- $$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

Faraday's Law and Lenz' Law

- The change in the flux, $\Delta\Phi_B$, can be produced by a change in B, A or θ
  - Since $\Phi_B = B\ A \cos\theta$
- The negative sign in Faraday's Law is included to indicate the polarity of the induced emf, which is found by *Lenz' Law.*
  - The current caused by the induced emf travels in the direction that creates a magnetic field with flux opposing the change in the original flux through the circuit.

A coil with 25 turns of wire is wrapped on a frame with a square cross section 1.80 cm on a side. Each turn has the same area, equal to that of the frame, and the total resistance of the coil is 0.350 V. An applied uniform magnetic field is perpendicular to the plane of the coil, as in Figure 20.7. **(a)** If the field changes uniformly from 0.00 T to 0.500 T in 0.800 s, what is the induced emf in the coil while the field is changing? Find **(b)** the magnitude and **(c)** the direction of the induced current in the coil while the field is changing.

$$A = L^2 = (0.018\,m)^2 = 3.24 \times 10^{-4}\,m^2$$

$$\phi_{Bf} = BA\cos\theta = (0.500T)(3.24 \times 10^{-4}\,m^2)\cos 0^\circ$$

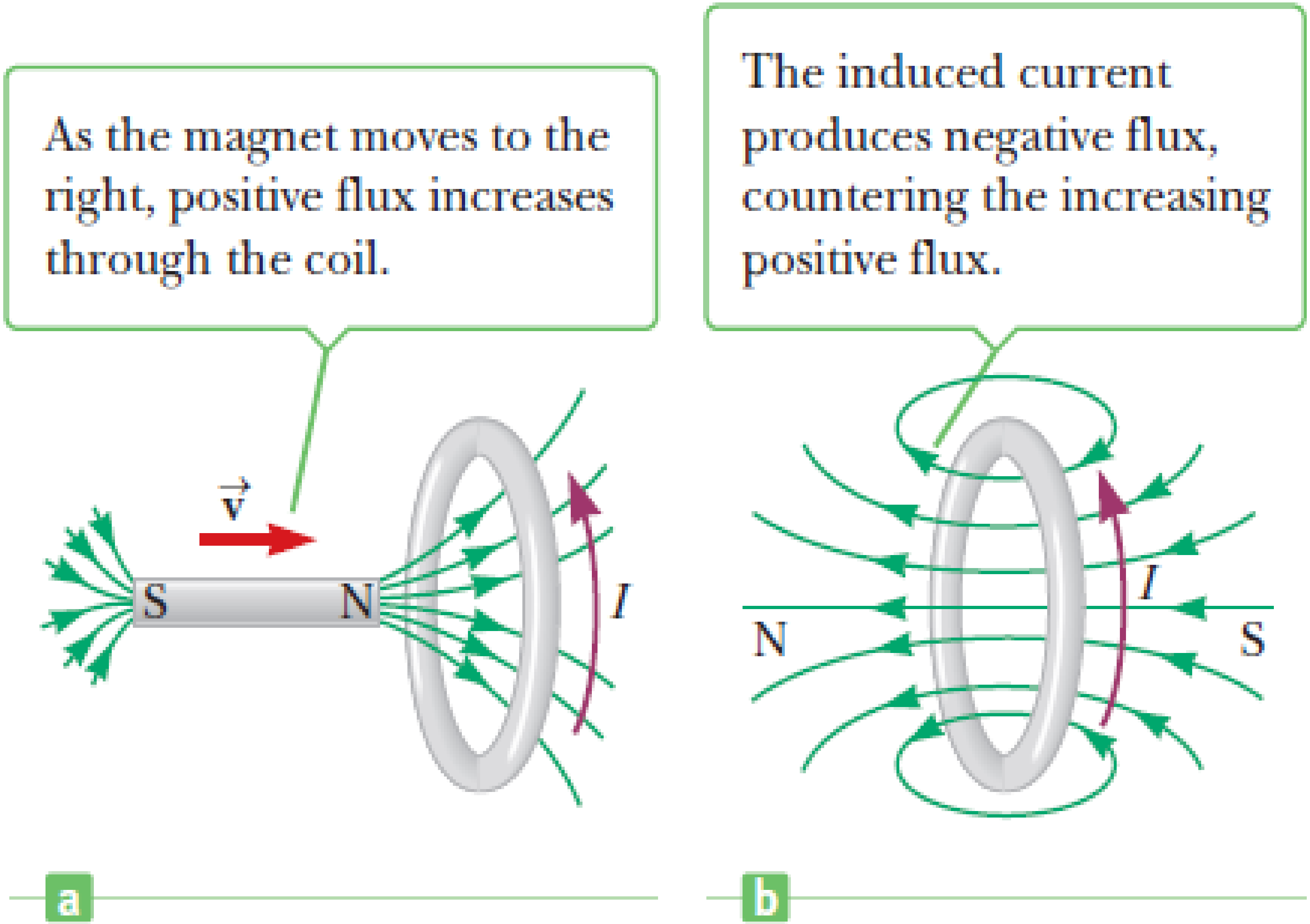
$$= 1.62 \times 10^{-4}\,Wb$$

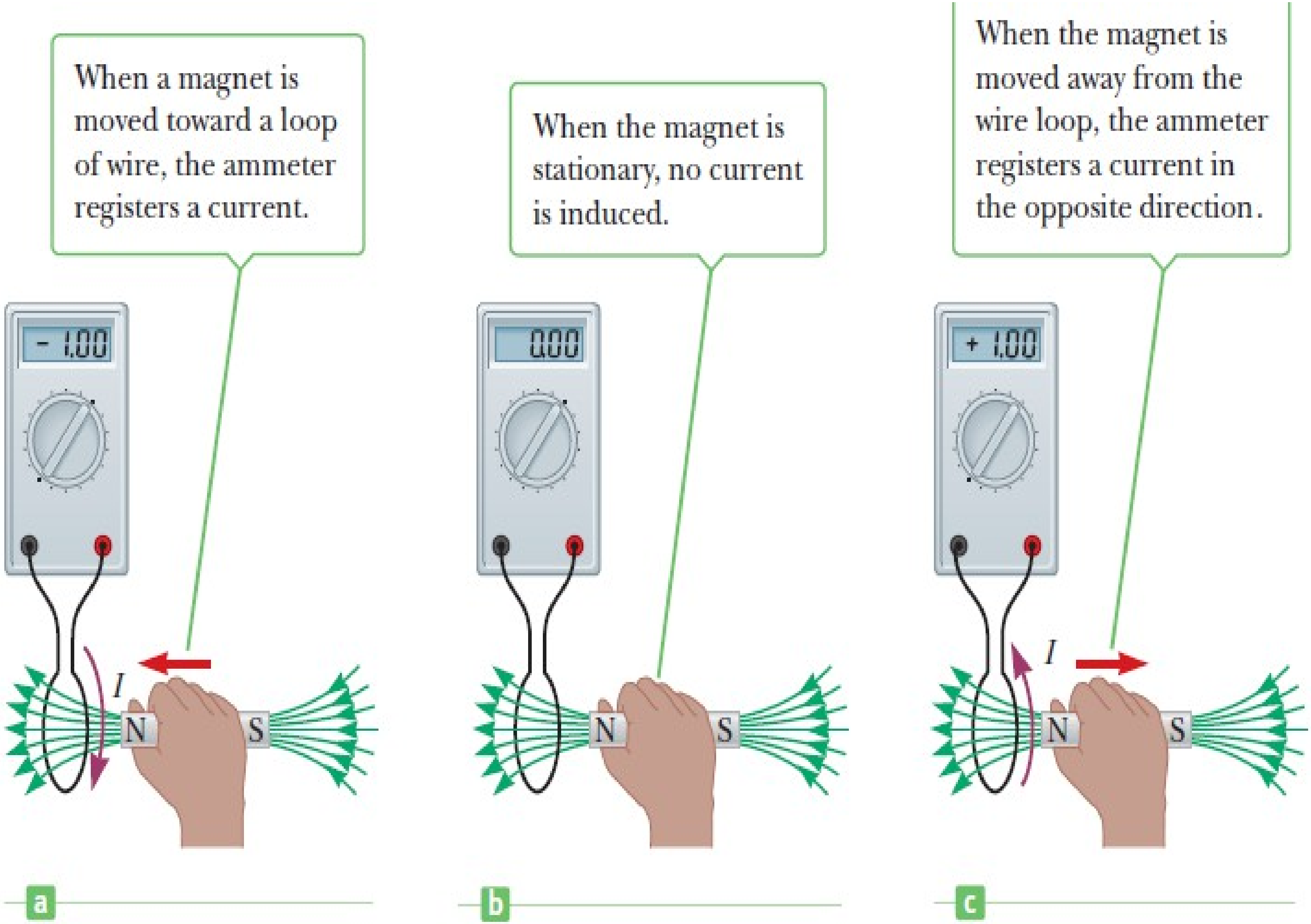
$$\phi_{Bi} = 0 \ (at\ t = 0)$$

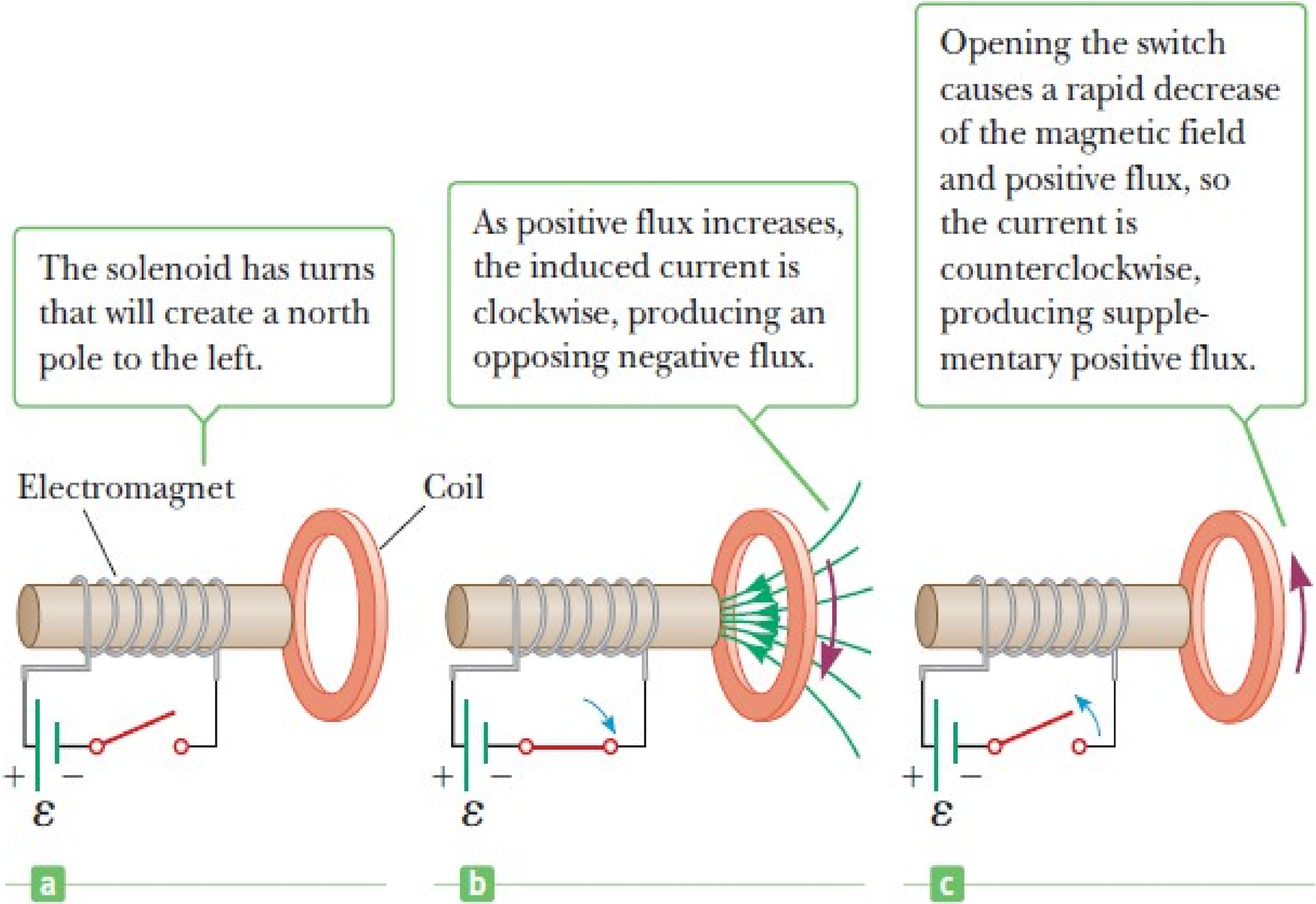
$$\mathcal{E} = -N\frac{\Delta\phi_B}{\Delta t} = -(25\,turns)\frac{(1.62 \times 10^{-4}\,Wb)}{0.800\,s}$$

$$= -5.06 \times 10^{-5}\,V.$$

$$I = \Delta V/R = 5.06 \times 10^{-5}\,V / 0.350\,\Omega = 1.45 \times 10^{-2}\,A$$

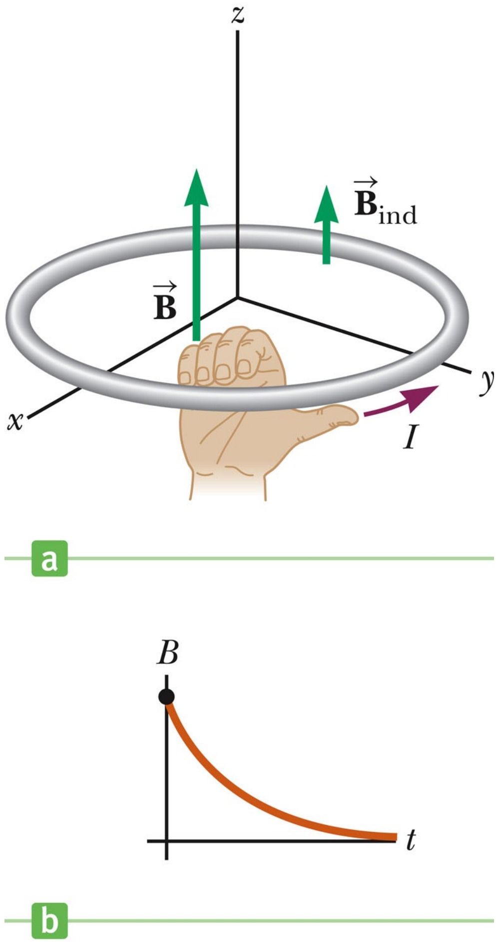

As the magnet moves to the right, positive flux increases through the coil.
The induced current produces negative flux, countering the increasing positive flux.
v
S
N
I
N
S
I
a
b


When a magnet is moved toward a loop of wire, the ammeter registers a current.
- 1.00
I
N
S
a
When the magnet is stationary, no current is induced.
0.00
N
S
b
When the magnet is moved away from the wire loop, the ammeter registers a current in the opposite direction.
+ 1.00
I
N
S
c


The solenoid has turns that will create a north pole to the left.
As positive flux increases, the induced current is clockwise, producing an opposing negative flux.
Opening the switch causes a rapid decrease of the magnetic field and positive flux, so the current is counterclockwise, producing supplementary positive flux.
Electromagnet
Coil
+
−
ℰ
a
b
c

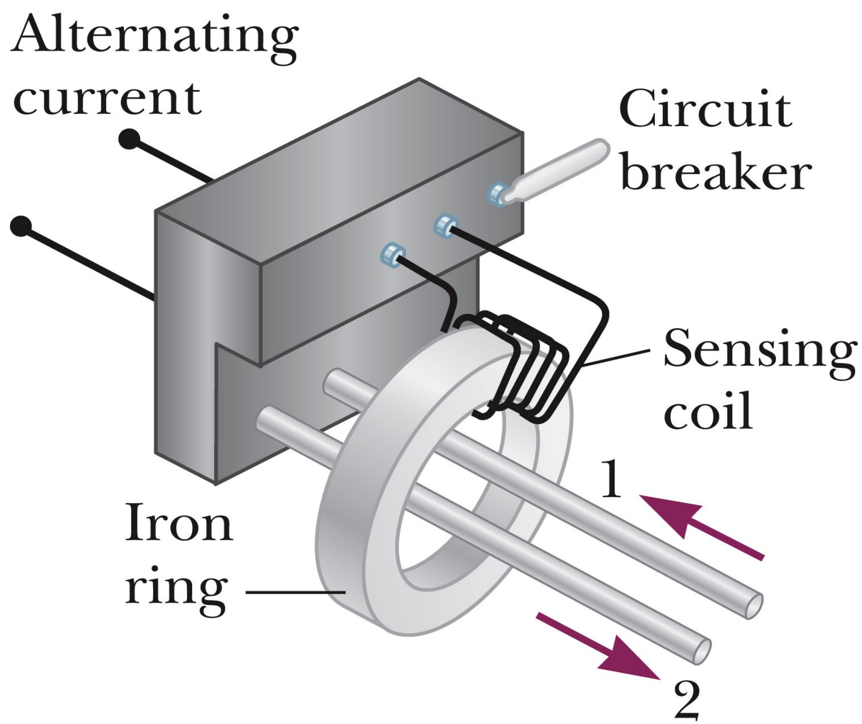
Lenz' Law – Example

- The magnetic field, $\vec{\mathbf{B}}$ becomes smaller with time.
  - This reduces the flux.
- The induced current will produce an induced field, $\vec{\mathbf{B}}_{ind}$, in the same direction as the original field.



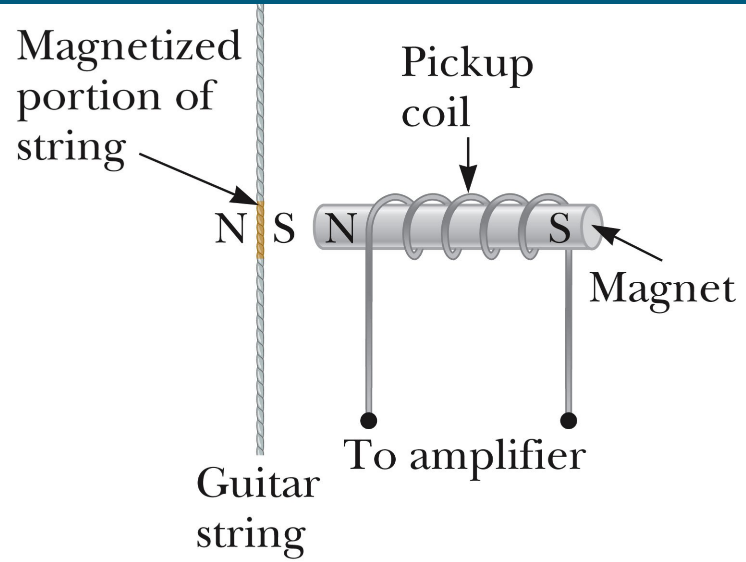
Applications of Faraday's Law – Ground Fault Interrupters

- The ground fault interrupter (GFI) is a safety device that protects against electrical shock.
  - Wire 1 leads from the wall outlet to the appliance.
  - Wire 2 leads from the appliance back to the wall outlet.
  - The iron ring confines the magnetic field, which is generally 0.
  - If a leakage occurs, the field is no longer 0 and the induced voltage triggers a circuit breaker shutting off the current.



Applications of Faraday's Law – Electric Guitar

- A vibrating string induces an emf in a coil.
- A permanent magnet inside the coil magnetizes a portion of the string nearest the coil.
- As the string vibrates at some frequency, its magnetized segment produces a changing flux through the pickup coil.
- The changing flux produces an induced emf that is fed to an amplifier.



a

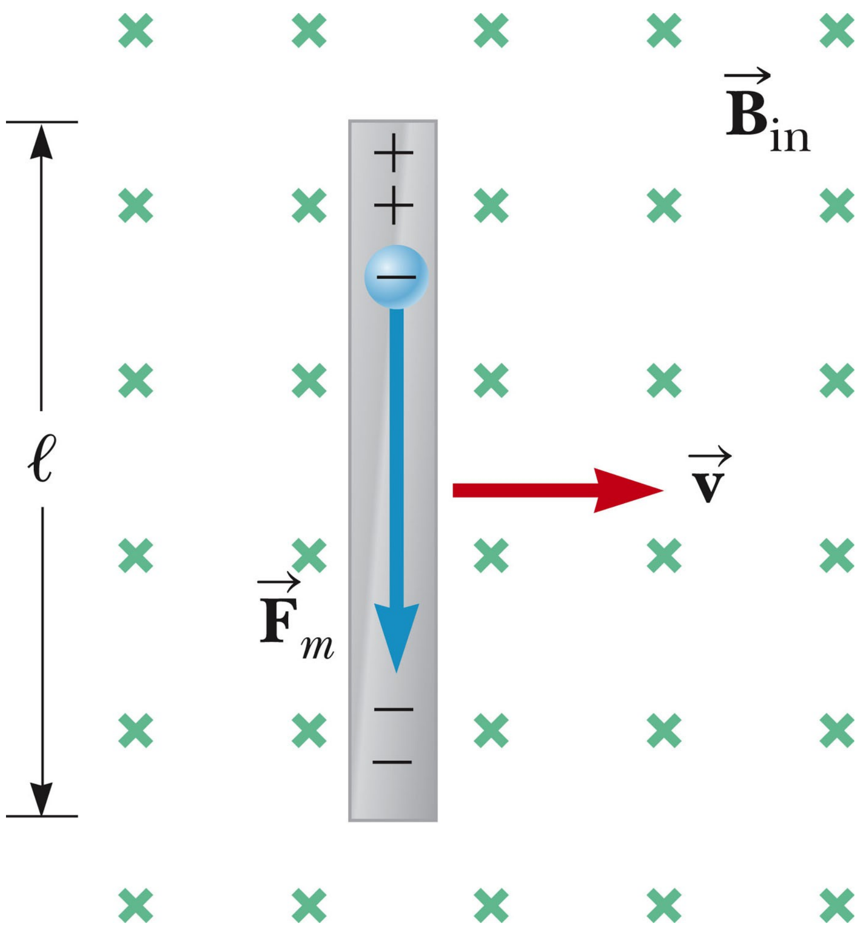
b

Applications of Faraday's Law – Apnea Monitor

- The coil of wire attached to the chest carries an alternating current.
- An induced emf produced by the varying field passes through a pick up coil.
- When breathing stops, the pattern of induced voltages stabilizes and external monitors sound an alert.

Application of Faraday's Law – Motional emf

- A straight conductor of length ℓ moves perpendicularly with constant velocity through a uniform field.
- The electrons in the conductor experience a magnetic force.
  - $F = q\,v\,B$
- The electrons tend to move to the lower end of the conductor.



Motional emf

- As the negative charges accumulate at the base, a net positive charge exists at the upper end of the conductor.
- As a result of this charge separation, an electric field is produced in the conductor.
- Charges build up at the ends of the conductor until the downward magnetic force is balanced by the upward electric force.
- There is a potential difference between the upper and lower ends of the conductor.

Motional emf, Cont.

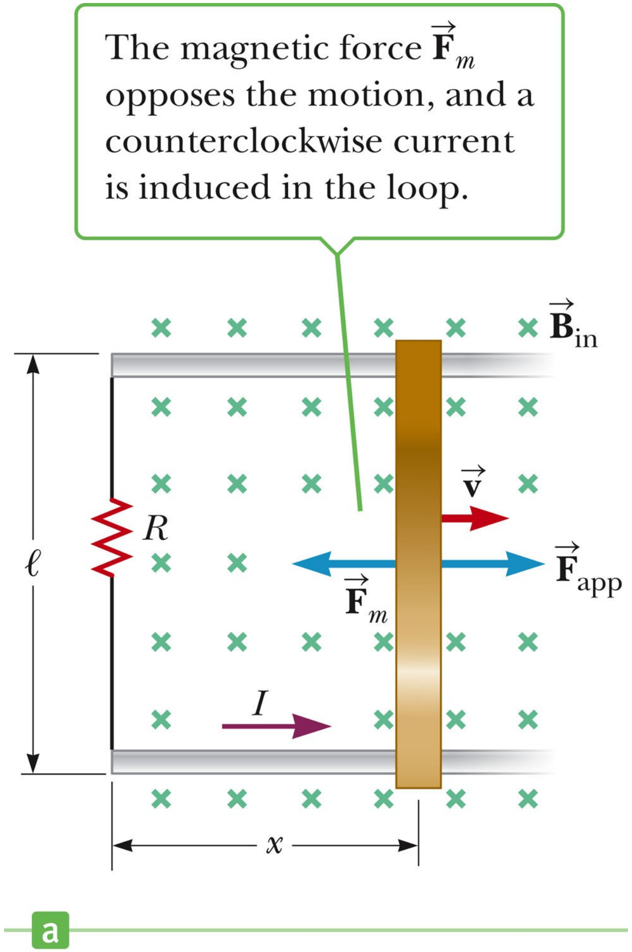
- The potential difference between the ends of the conductor can be found by
  - $\Delta V = E \ell = B \ell v$
  - The upper end is at a higher potential than the lower end
- A potential difference is maintained across the conductor as long as there is motion through the field.
  - If the motion is reversed, the polarity of the potential difference is also reversed.

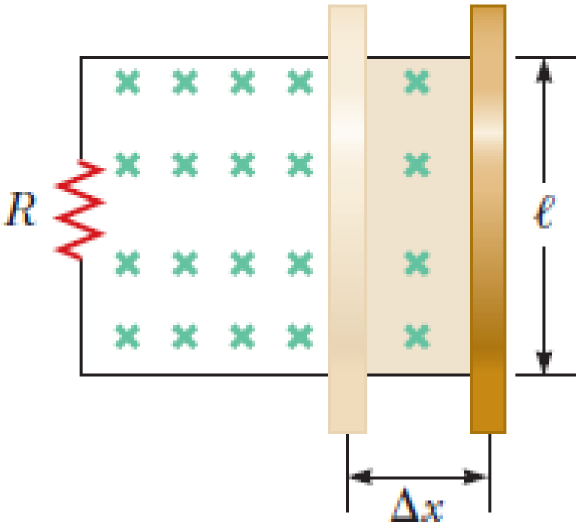
the condition for equilibrium requires that

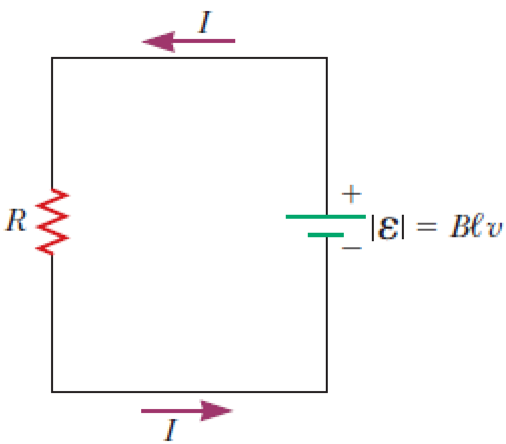
$$qE = qvB \qquad \text{or} \qquad E = vB$$

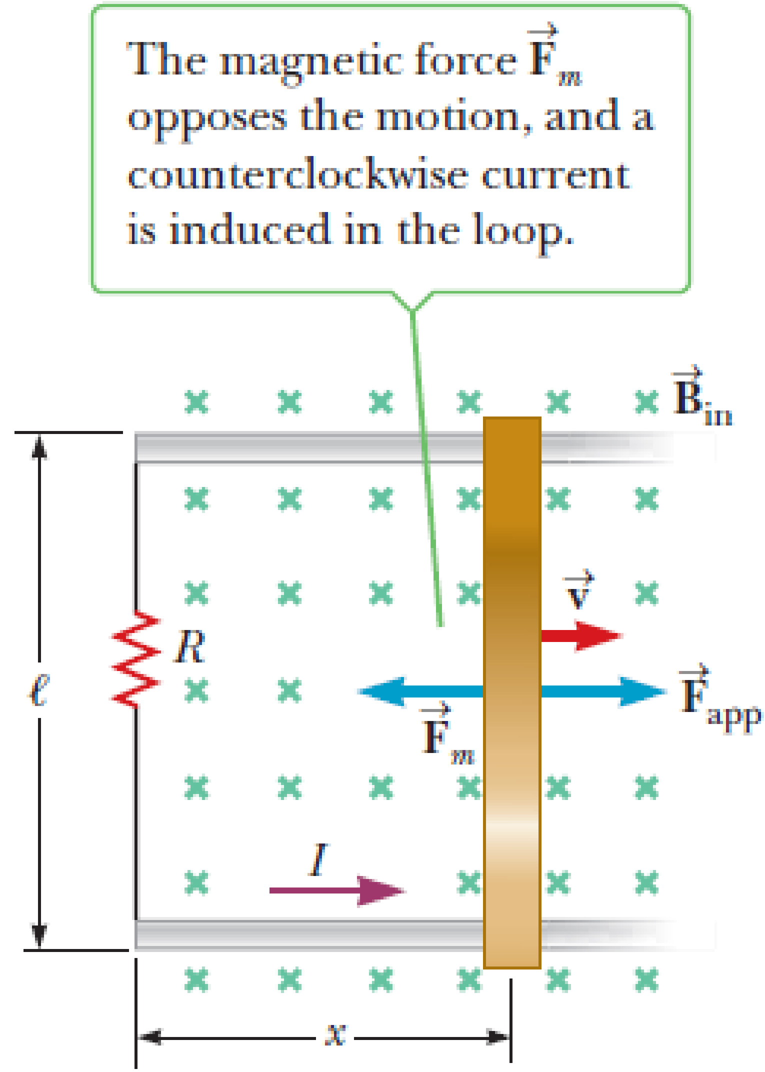
Motional emf in a Circuit

- Assume the moving bar has zero resistance.
- As the bar is pulled to the right with a given velocity under the influence of an applied force, the free charges experience a magnetic force along the length of the bar.
- This force sets up an induced current because the charges are free to move in the closed path.









$$\Delta\Phi_B = BA = B\ell\,\Delta x$$

$$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = B\ell\frac{\Delta x}{\Delta t} = B\ell v$$

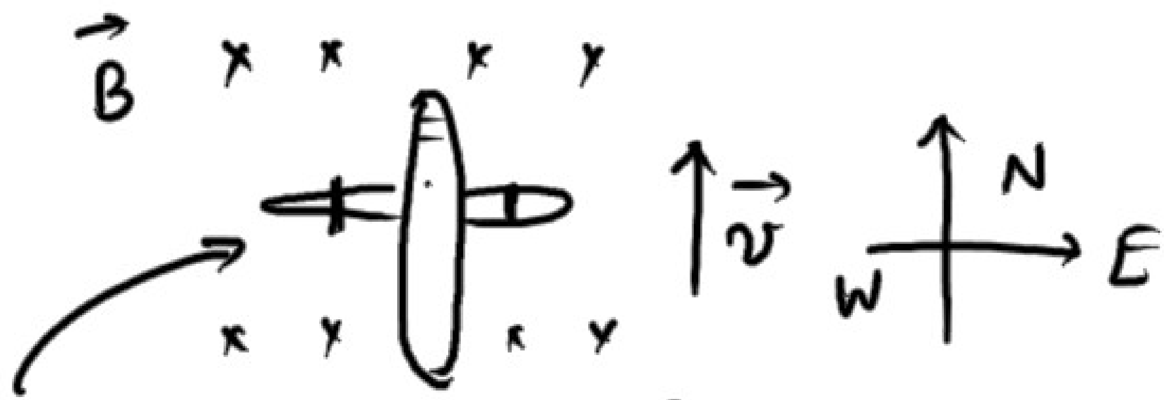
$$I = \frac{|\mathcal{E}|}{R} = \frac{B\ell v}{R}$$

An airplane with a wingspan of 30.0 m flies due north at a location where the downward component of Earth's magnetic field is 0.600×10^{-4} T. There is also a component pointing due north that has a magnitude of 0.470×10^{-4} T.

(a) Find the difference in potential between the wingtips when the speed of the plane is 2.50×10^{2} m/s. **(b)** Which wingtip is positive?

$$\mathcal{E} = Blv = (0.600 \times 10^{-4}\,T)(30.0\,m) \times (2.50 \times 10^{2}\,m/s)$$

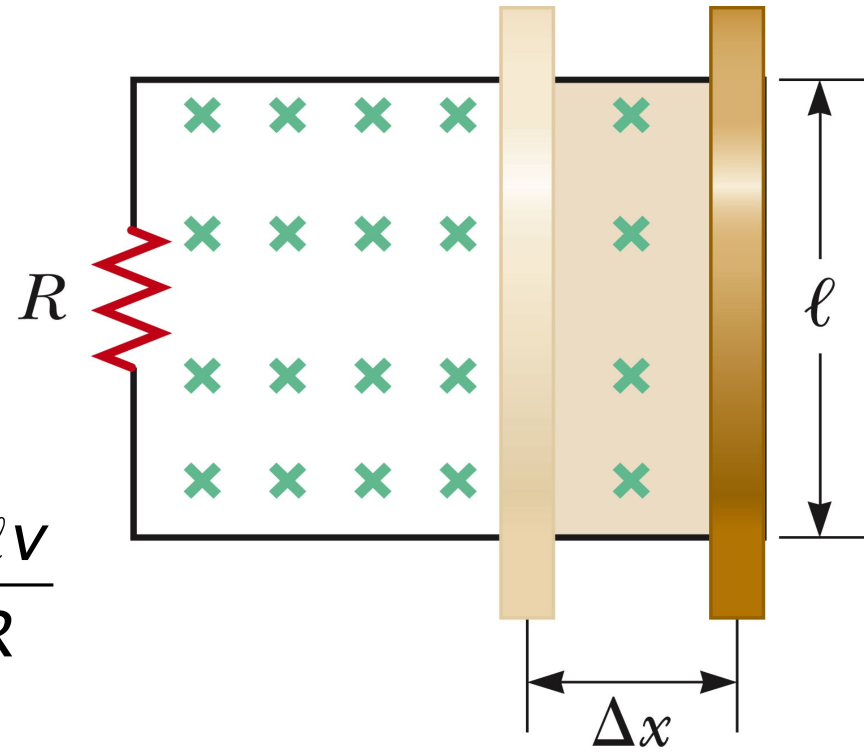
$$= 0.450\,V!$$



West wing is positive

Motional emf in a Circuit, Cont.

- The changing magnetic flux through the loop and the corresponding induced emf in the bar result from the *change in area* of the loop.
- The induced, motional, emf acts like a battery in the circuit.
- $$\left|\varepsilon\right| = B\ell v \text{ and } I = \frac{B\ell v}{R}$$

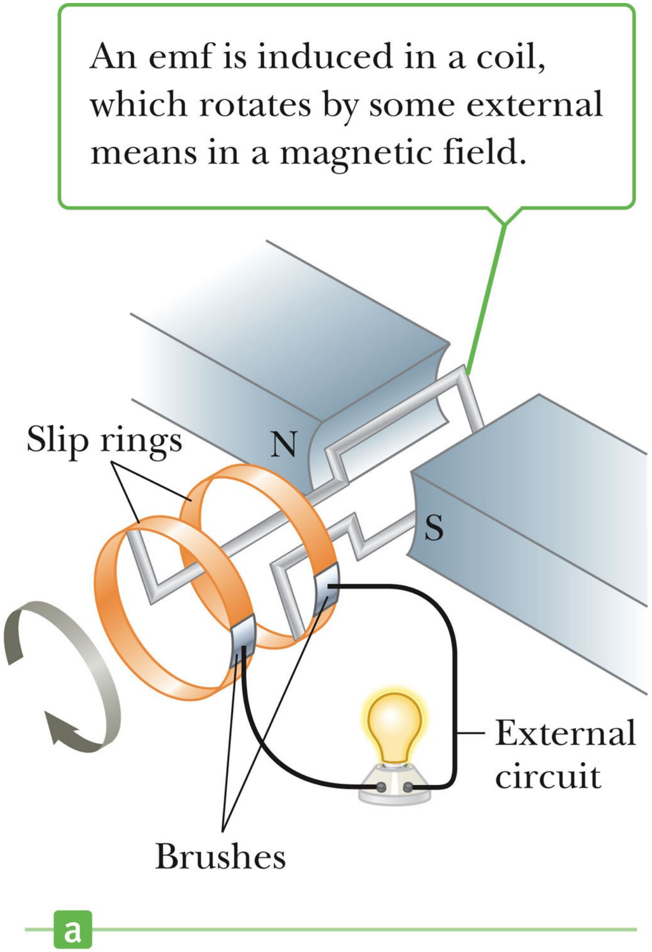


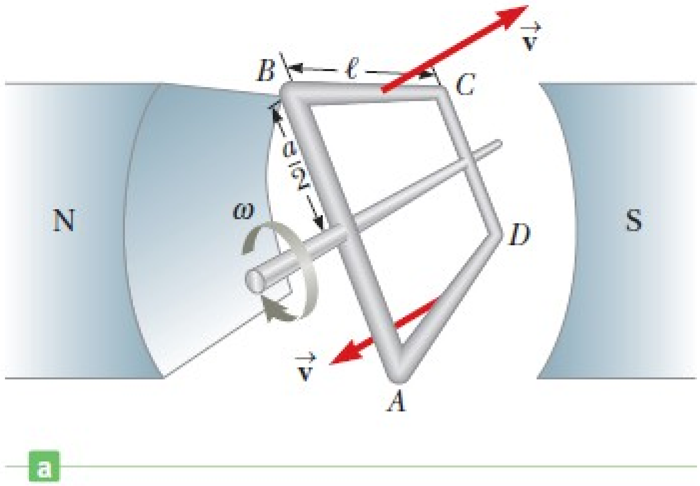
Generators

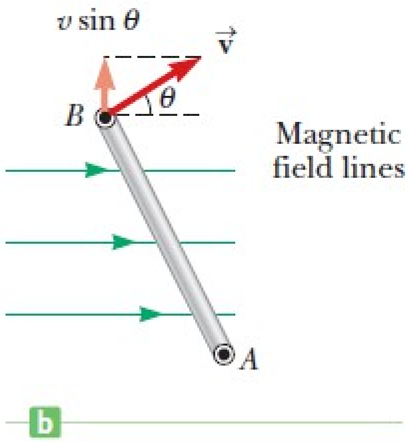
- Alternating Current (AC) generator
  - Converts mechanical energy to electrical energy
  - Consists of a wire loop rotated by some external means
  - There are a variety of sources that can supply the energy to rotate the loop.
- These may include falling water, heat by burning coal to produce steam

AC Generators, Cont.

- Basic operation of the generator
  - As the loop rotates, the magnetic flux through it changes with time.
  - This induces an emf and a current in the external circuit.
  - The ends of the loop are connected to slip rings that rotate with the loop.
  - Connections to the external circuit are made by stationary brushes in contact with the slip rings.







$$\mathcal{E} = 2B\ell v_{\perp} = 2B\ell v \sin\theta$$

$$\mathcal{E} = 2B\ell\left(\frac{a}{2}\right)\omega \sin\omega t = B\ell a\omega \sin\omega t$$

$$\mathcal{E} = NBA\omega \sin\omega t$$

AC Generators, Final

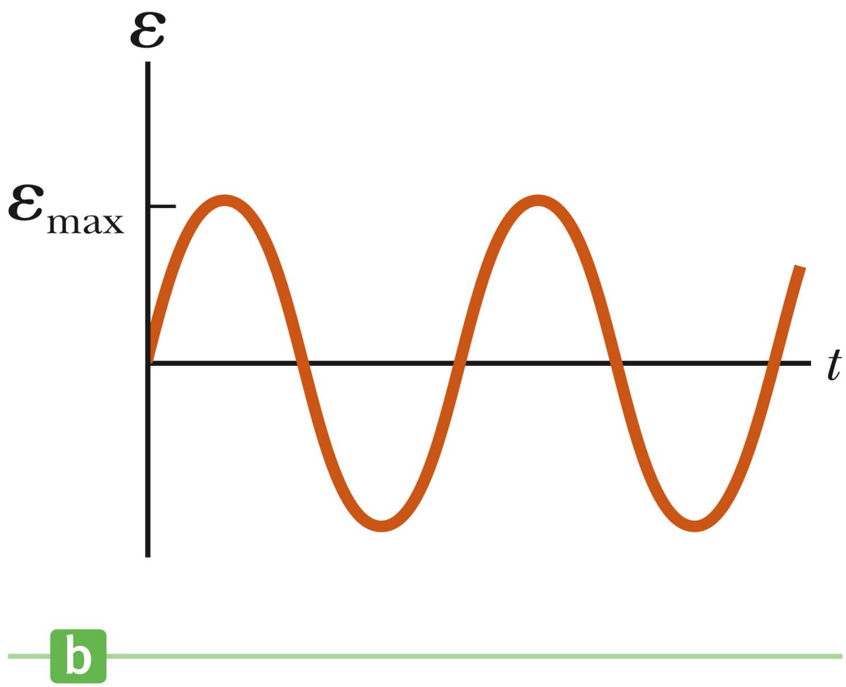
- The emf generated by the rotating loop can be found by

$$\varepsilon = 2 B \ell v_{\geq} = 2 B \ell \sin \theta$$

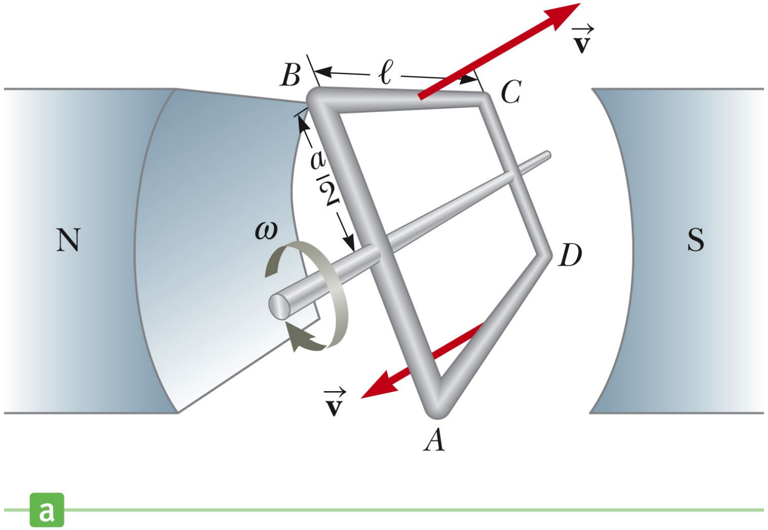
- If the loop rotates with a constant angular speed, ω, and N turns

$$\varepsilon = N B A \omega \sin \omega t$$

- $\varepsilon = \varepsilon_{max}$ when loop is parallel to the field
- $\varepsilon = 0$ when the loop is perpendicular to the field



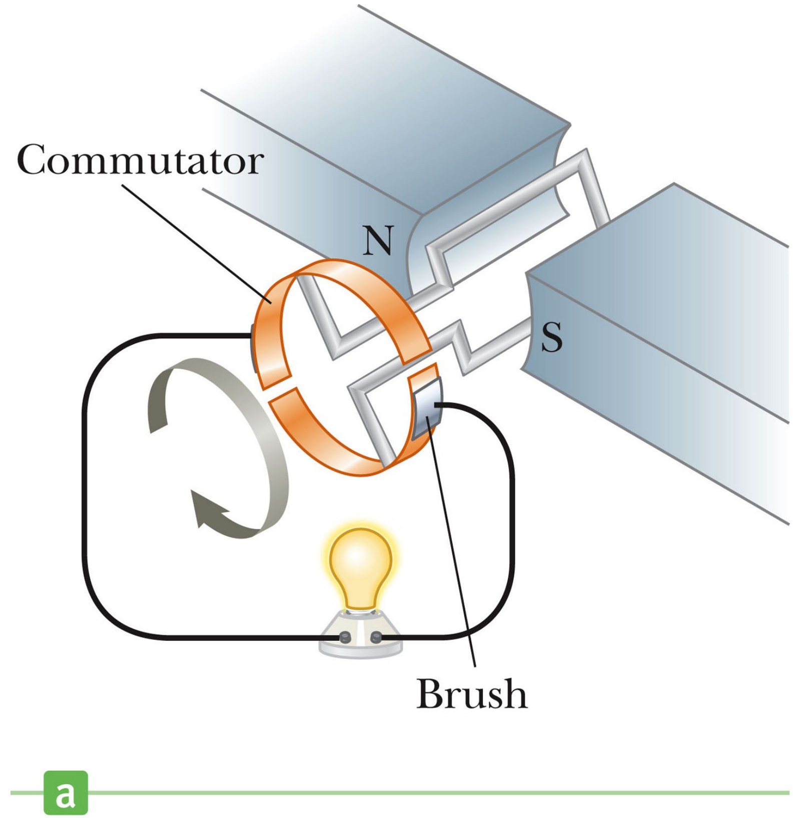
AC Generators – Detail of Rotating Loop



- The magnetic force on the charges in the wires AB and CD is perpendicular to the length of the wires.
- An emf is generated in wires BC and AD.
- The emf produced in each of these wires is $\varepsilon = B \ell v_{\geq} = B \ell \sin \theta$
-

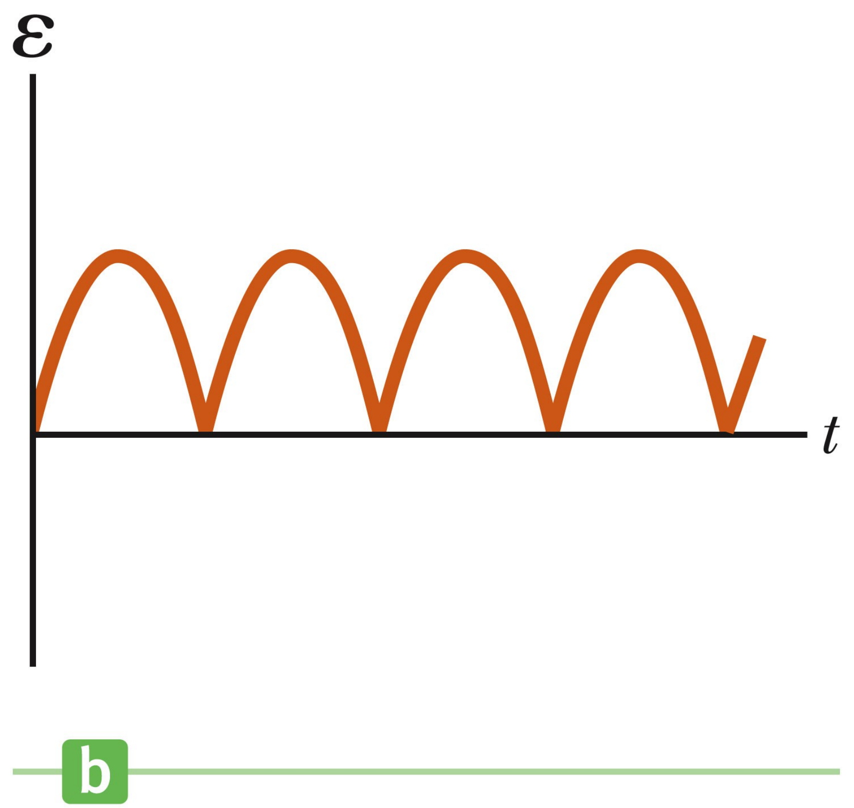
DC Generators

- Components are essentially the same as that of an ac generator
- The major difference is the contacts to the rotating loop are made by a split ring, or commutator



DC Generators, Cont.

- The output voltage always has the same polarity.
- The current is a pulsing current.
- To produce a steady current, many loops and commutators around the axis of rotation are used.
  - The multiple outputs are superimposed and the output is almost free of fluctuations.

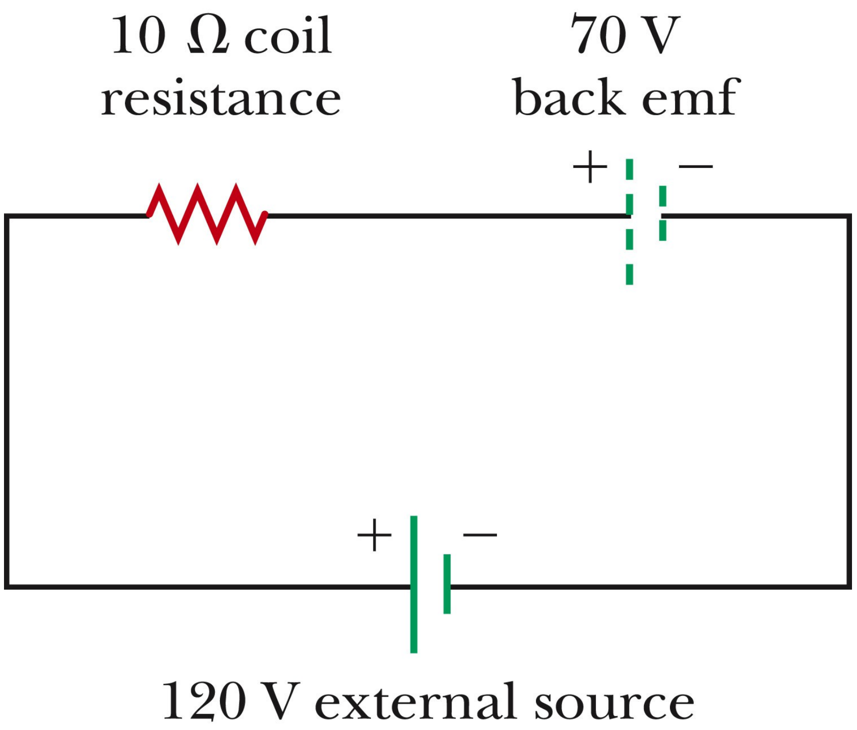


Motors

- Motors are devices that convert electrical energy into mechanical energy.
  - A motor is a generator run in reverse.
- A motor can perform useful mechanical work when a shaft connected to its rotating coil is attached to some external device.

Motors and Back emf

- The phrase *back emf* is used for an emf that tends to reduce the applied current.
- When a motor is turned on, there is no back emf initially.
- The current is very large because it is limited only by the resistance of the coil.



Motors and Back emf, Cont.

- As the coil begins to rotate, the induced back emf opposes the applied voltage.
- The current in the coil is reduced.
- The power requirements for starting a motor and for running it under heavy loads are greater than those for running the motor under average loads.

Self-inductance

- *Self-inductance* occurs when the changing flux through a circuit arises from the circuit itself.
  – As the current increases, the magnetic flux through a loop due to this current also increases.
  – The increasing flux induces an emf that opposes the change in magnetic flux.
  – As the magnitude of the current increases, the rate of increase lessens and the induced emf decreases.
  – This decreasing emf results in a gradual increase of the current.

Self-inductance, Cont.

- The self-induced emf must be proportional to the time rate of change of the current.
- $$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

– L is a proportionality constant called the **inductance** of the device.

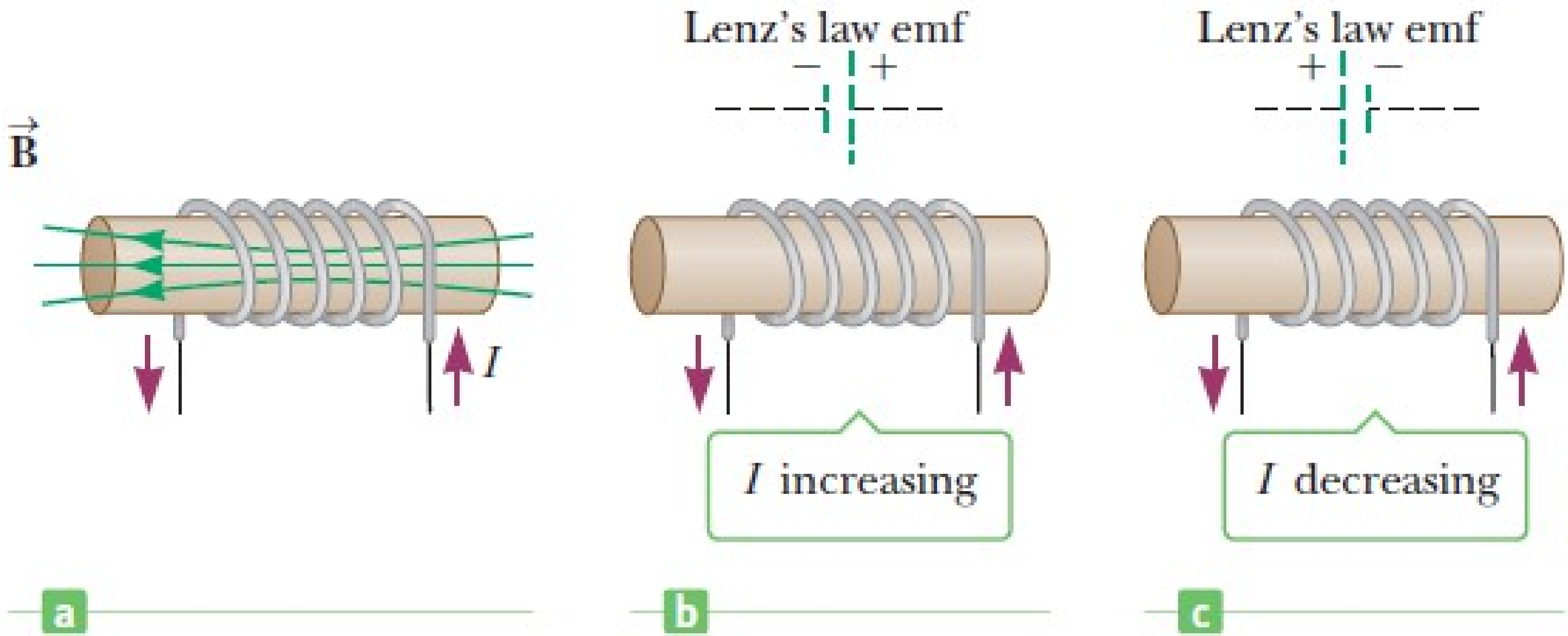
– The negative sign indicates that a changing current induces an emf in opposition to that change.

Self-inductance, Final

- The inductance of a coil depends on geometric factors.
- The SI unit of self-inductance is the *Henry*
  - 1 H = 1 (V · s) / A
- You can determine an expression for L
- $$L = N\frac{\Delta\Phi_B}{\Delta I} = \frac{N\Phi_B}{I}$$

Joseph Henry

- 1797 – 1878
- First director of the Smithsonian
- First president of the Academy of Natural Science
- First to produce an electric current with a magnetic field
- Improved the design of the electro-magnet and constructed a motor
- Discovered self-inductance



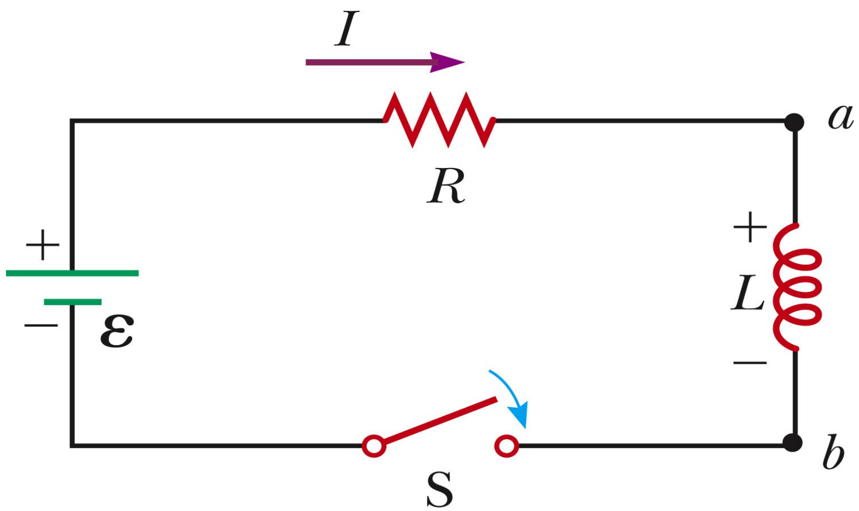
B
I
Lenz's law emf
− +
I increasing
Lenz's law emf
+ −
I decreasing
a
b
c

Inductor in a Circuit

- Inductance can be interpreted as a measure of opposition to the rate of change in the current.
  - Remember resistance R is a measure of opposition to the current.
- As a circuit is completed, the current begins to increase, but the inductor produces an emf that opposes the increasing current.
  - Therefore, the current doesn't change from 0 to its maximum instantaneously.

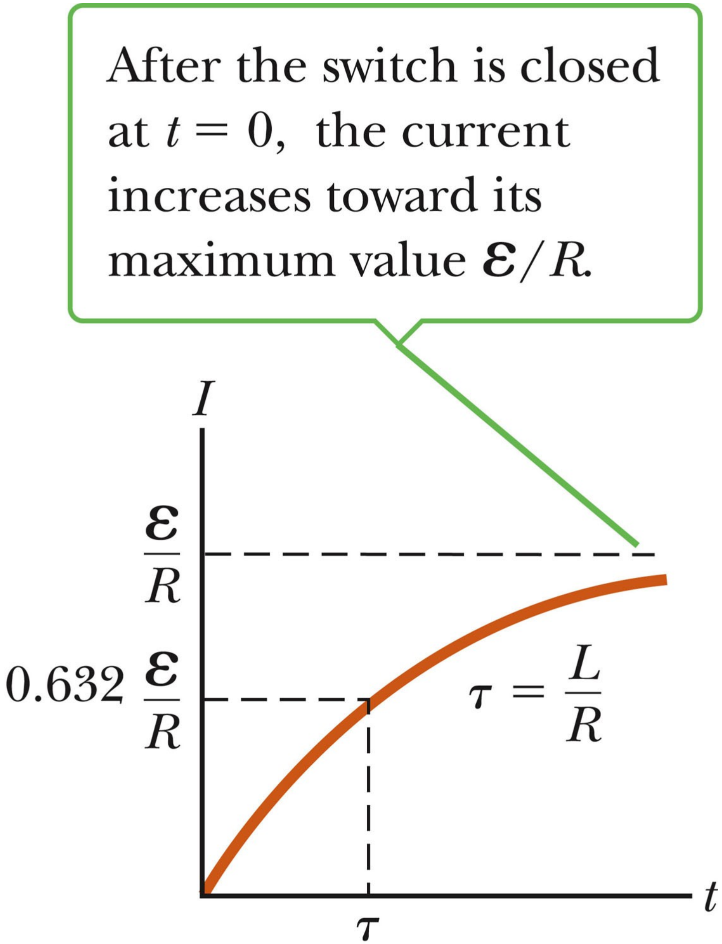
RL Circuit

- When the current reaches its maximum, the rate of change and the back emf are zero.
- The time constant, ≥ , for an RL circuit is the time required for the current in the circuit to reach 63.2% of its final value.



RL Circuit, Graph

- The current increases toward the maximum value of ε/R

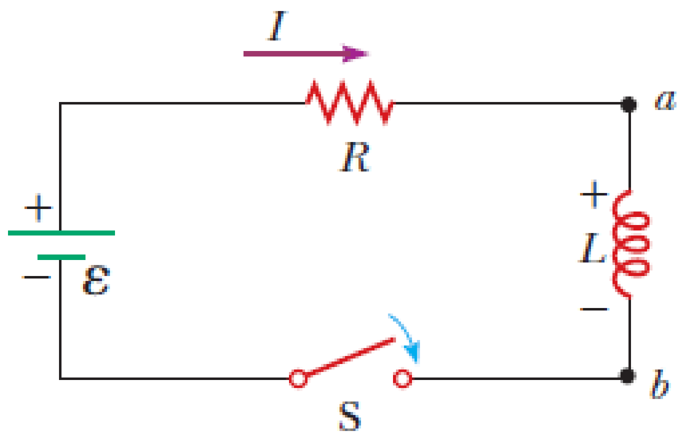


RL Circuit, Cont.

- The time constant depends on R and L.
-
$$\tau = \frac{L}{R}$$
-
- The current at any time can be found by

$$I = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$$

PROBLEM A 12.6-V battery is in a circuit with a 30.0-mH inductor and a 0.150-Ω resistor, as in Active Figure 20.26. The switch is closed at $t = 0$. (a) Find the time constant of the circuit. (b) Find the current after one time constant has elapsed. (c) Find the voltage drops across the resistor when $t = 0$ and $t =$ one time constant. (d) What's the rate of change of the current after one time constant?



$$\tau = \frac{L}{R} = \frac{30.0 \times 10^{-3}\,\text{H}}{0.150\,\Omega} = 0.200\,\text{s}$$

$$I_{\text{max}} = \frac{\mathcal{E}}{R} = \frac{12.6\,\text{V}}{0.150\,\Omega} = 84.0\,\text{A}$$

$$I_{1\tau} = (0.632)I_{\text{max}} = (0.632)(84.0\,\text{A}) = 53.1\,\text{A}$$

$$\Delta V_R = IR$$

$$\Delta V_R\,(t = 0\text{ s}) = (0\text{ A})(0.150\ \Omega) = \boxed{0}$$

$$\Delta V_R\,(t = 0.200\text{ s}) = (53.1\text{ A})(0.150\ \Omega) = \boxed{7.97\text{ V}}$$

$$\mathcal{E} + \Delta V_R + \Delta V_L = 0$$

$$\Delta V_L = -\mathcal{E} - \Delta V_R = -12.6\text{ V} - (-7.97\text{ V}) = -4.6\text{ V}$$

Energy Stored in a Magnetic Field

- The emf induced by an inductor prevents a battery from establishing an instantaneous current in a circuit.
- The battery has to do work to produce a current.
  - This work can be thought of as energy stored by the inductor in its magnetic field.
  - $PE_L = ½ L I^2$

In general, determining the inductance of a given current element can be challenging. Finding an expression for the inductance of a common solenoid, however, is straightforward. Let the solenoid have N turns and length ℓ. Assume ℓ is large compared with the radius and the core of the solenoid is air. We take the interior magnetic field to be uniform and given by Equation 19.16,

$$B = \mu_0 n I = \mu_0 \frac{N}{\ell} I$$

where $n = N/\ell$ is the number of turns per unit length. The magnetic flux through each turn is therefore

$$\Phi_B = BA = \mu_0 \frac{N}{\ell} AI$$

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 A}{\ell} \quad [20.11a]$$

This equation shows that L depends on the geometric factors ℓ and A and on μ_0 and is proportional to the square of the number of turns. Because $N = n\ell$, we can also express the result in the form

$$L = \mu_0 \frac{(n\ell)^2}{\ell} A = \mu_0 n^2 A\ell = \mu_0 n^2 V \quad [20.11b]$$

where $V = A\ell$ is the volume of the solenoid.

PROBLEM (a) Calculate the inductance of a solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is $4.00 \times 10^{-4}\ m^2$. (b) Calculate the self-induced emf in the solenoid described in part (a) if the current in the solenoid decreases at the rate of 50.0 A/s.

$$L = \frac{\mu_0 N^2 A}{\ell}$$

$$= (4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}) \frac{(300)^2(4.00 \times 10^{-4}\ \mathrm{m^2})}{25.0 \times 10^{-2}\ \mathrm{m}}$$

$$= 1.81 \times 10^{-4}\ \mathrm{T \cdot m^2/A} = \boxed{0.181\ \mathrm{mH}}$$

$$\mathcal{E} = -L\frac{\Delta I}{\Delta t} = -(1.81 \times 10^{-4}\ \mathrm{H})(-50.0\ \mathrm{A/s})$$

$$= \boxed{9.05\ \mathrm{mV}}$$