**Chapter 3. Manipulator Kinematics**

*Homework #3. Due 9/17/14*

*2.19 (Do not expand your result to arrive at Eq. 2.72. ), 3.1 (Study only), 3.3, 3.16, 3.17*

**Joints, Links, and Joint Offset**

Joint Types – Revolute and Prismatic

Sub categories – Revolute – Twist, Rotational, ReVolving

 Prismatic – Linear, Orthogonal

Symbols - Position, velocity, acceleration

 Revolute –, (), ()

 Prismatic –

**Link parameters: Denavit-Hartenberg Notation**

|  |  |  |  |
| --- | --- | --- | --- |
| αi-1 | a i-1 | di | θi |

*αi-1 =Link twist (0 or ±90º) a i-1=Link Length di=X axis offset θi=Z rotation about X*

*ai-1 = the link distance from to  along*

*α i-1 = the twist angle from to  about*

*di = the offset distance from to along *

*θi = the rotation angle from to about *

*ai-1 = Common perpendicular distance. Zero for coaxial (twist) revolute joints.*

*αi = Normally zero or ±90º between revolute joints. ±90º between revolute and prismatic*

*di = Offset distance. Variable for prismatic. Coaxial distance for revolute.*

*θi = Rotation angle. Zero for prismatic.*

***Assignment of Axes***

 * = for revolute - The axis of rotation either in parallel with or at a right angle to*

 *= for prismatic – In the direction of sliding, pointing away from Joint i-1.*

 *= for revolute – Along Link i-1, pointing toward Joint i+1 (a common perpendicular)*

 *= for prismatic – If ai =0, normal(orthogonal) to a plane formed by and .*

 *= when  and are assigned, simply follow the right hand rule.*

*If  andare not orthogonal to, they will either intersect or cross each other by an offset distance di. If so, orient  such that it is normal to a plane formed by the two Z axes when di=0.*

*Rules: is formed by rotating  about.*

*If and  do not intersect,*

*ai (“the Z distance”) is the perpendicular distance between and .*

*di (“the X distance”) is the perpendicular distance between and .*

*If and intersect,*

*ai=0*

*di is the perpendicular distance between and .*

**Link Transformation**

 (3.6)

 when =0

 when =90.





**Reference Frames**

*Base{B}, Station{S}, Wrist{W}, Tool{T}, Universal{U} or Global{G}*

Use the relationship , , and  to define a position vector with respect to any frame in the chain of frames.

**Link Length (*αi-1*) or Link Offset *(di*)? – A Simple Test**

1. *Starting from the base, successively label straight fixed length link segments by L1, L2, …, .*

*Do the same for variable length segments by D1, D2, …*

1. *If Li (or Di) coincides or is parallel to the axis of rotation or translation of the predecessor joint, Li (or Di) is a link offset* ***(di****). If it is orthogonal (or radial), it is a link length* ***(αi****-1).*

**Between Joint Frame Combinations**

*There are total 16 combinations possible as summarized in the table below. If  andare parallel, connects the two axes if the offset di is zero. If  andare not parallel (orthogonal without the effect of the offset), provides the rotation axis for to transform into. You can imitate the two frame set up with your two hands using two thumbs (Z) and two index fingers (X). The right hand rule determineswhich is normal to the XZ plane.*

Zi, Xi

b

Zi-1, Xi-1

a

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *Joint i-1* | *Joint i* | *Zi-1, Zi* | *Xi-1, Xi* | *ai-1* | *di* |
| R or P | R or P | // or  | // or  | a or b | a or b |

*R – Revolute P – Prismatic // - Parallel  - Orthogonal*

 *di =dist(Zi-1, Zi), measured along Xi-1 ai-1 = dist(Xi-1Xi), measured along Z*

**Six Axes Robot**

Z6

X6

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| i | *αi-1* | *a i-1* | *di* | *θi* |
| 1 | 0 | 0 | L1 | θ1 |
| 2 | -90º | L3 | L2 | θ2 |
| 3 | 180º | L4 | 0 | θ3 |
| 4 | -90º | L6 |  L5 | θ4 |
| 5 | 90º | 0 | L7 | θ5 |
| 6 | -90º | L8 | 0 | θ6 |

X5

L8

Z4

X4

Z5

L7

X3

L6

L44

L5

L3

L2

L1

X2

Z3

Z2

Z1

Z1

X1

X3

Z2

X2

Z3

**SCARA Robot**

L5

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| i | *αi-1* | *a i-1* | *di* | *θi* |
| 1 | 0 | 0 | L1 | θ1 |
| 2 | 0 | L3 | L2+L4 | θ2 |
| 3 | 180º | L5 | d3 | 0 |
| 4 | 0 | 0 | 0 | θ4 |

L3

L5

L4

d3

L2

Z1

X4

X1

Z4

L1

X0

**Cartesian Robot**

X2

X2

Z2

Z1

Z1

D2

L1

D2

L1

L3

L1

X1

L2

L1

L1

L1

D1

L1

D3

Z0

X3

Z3

Z4

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| i | *αi-1* | *a i-1* | *di* | *θi* |
| 1 | 0 | 0 | d1 | 0 |
| 2 | 90º | L1 | d2 | 0 |
| 3 | 90º | L2 | d3 | 0 |
| 4 | 0 | 0 | 0 | θ4 |

**Wrist Joint Equivalence –** *Yaw-Pitch-Roll - Euler Z-Y-Z rotation*

Three Way Socket Joint

(ZYZ Rotation on Moving Frame)

Link Offset

Link Length

d4

Z4

Z6

Z5

X5

X6

X4

**PUMA 560 Joint Configuration**

Joints 4-6

L6

L8

L7

L5

a3

X3

Z3

L3

X3

Z2

L4

L2

Z3

a2

X2

Joints 1-3

d3

Z1

d3=L2-L4

d4=L6+L7

a2=L3

a3=L5

L1

X1

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **i** | ***αi-1*** | ***a i-1*** | ***di*** | ***θi*** |
| 1 | 0 | 0 | 0 | θ1 |
| 2 | -90º | 0 | L1 | θ2 |
| 3 | 0 | L3 | L2-L4 | θ3 |
| 4 | -90º | L5 | L6 | θ4 |
| 5 | 90º | 0 | L7 | θ5 |
| 6 | -90º | 0 | L8 | θ6 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **i** | ***αi-1*** | ***a i-1*** | ***di*** | ***θi*** |
| 1 | 0 | 0 | 0 | θ1 |
| 2 | -90º | 0 | 0 | θ2 |
| 3 | 0 | α2 | d3 | θ3 |
| 4 | -90º | α3 | d4 | θ4 |
| 5 | 90º | 0 | 0 | θ5 |
| 6 | -90º | 0 | 0 | θ6 |