METR 130: Lecture 3

- Atmospheric Surface Layer (SL)
- Neutral Stratification (Log-law wind profile)
- Stable/Unstable Stratification (Monin-Obukhov Similarity Theory)

Spring Semester 2011

March 1, 2011

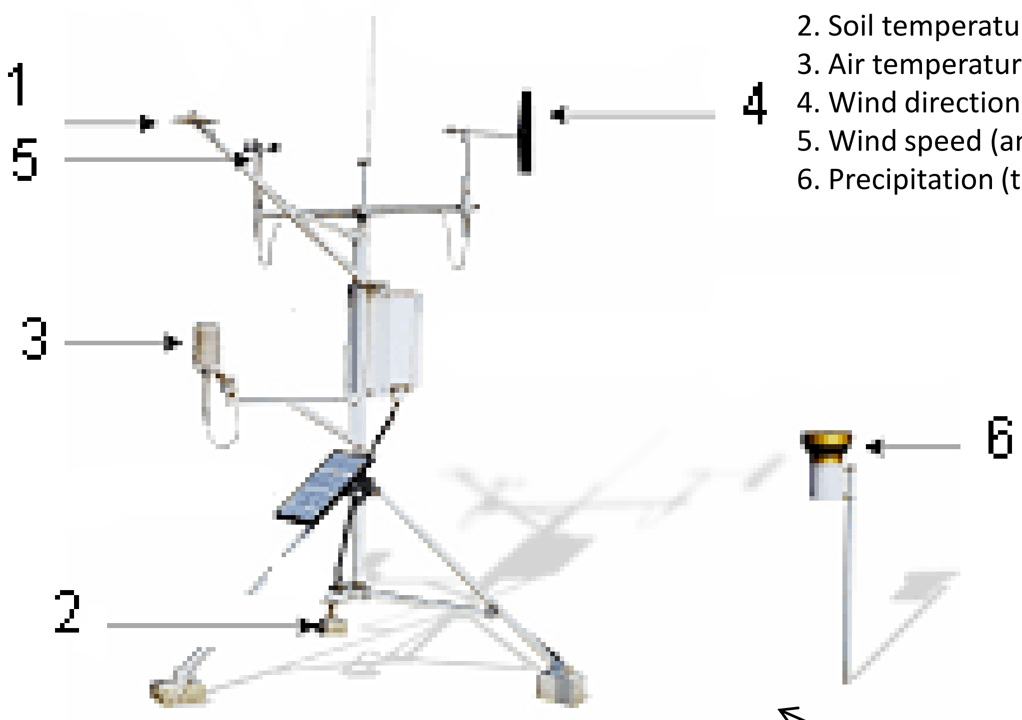
Reading

- **Arya, Chapter 10 (“log-law”, neutral SL)**
- **Arya, Chapter 11 (MO theory, stable/unstable SL)**
- **Review …**
  - Arya, Chapters 5 through 7 (review material covered so far)
  - Arya, Chapter 8.1 (review turbulence & Reynolds number)

Before we begin

(Diagnosing surface fluxes from routine observations)

Example: Observational platform for California DWR “CIMIS” Network …



1. Total solar radiation (pyranometer)
2. Soil temperature (thermistor)
3. Air temperature/relative humidity (HMP35)
4. Wind direction (wind vane)
5. Wind speed (anemometer)
6. Precipitation (tipping-bucket rain gauge)

Can compute (“diagnose”) surface fluxes from these routine near-surface meteorological observations coupled with certain assumptions (e.g. on C_H, $R_{L\downarrow}$). See Lecture 2 and Exam #1 HW calculation for details on this procedure.

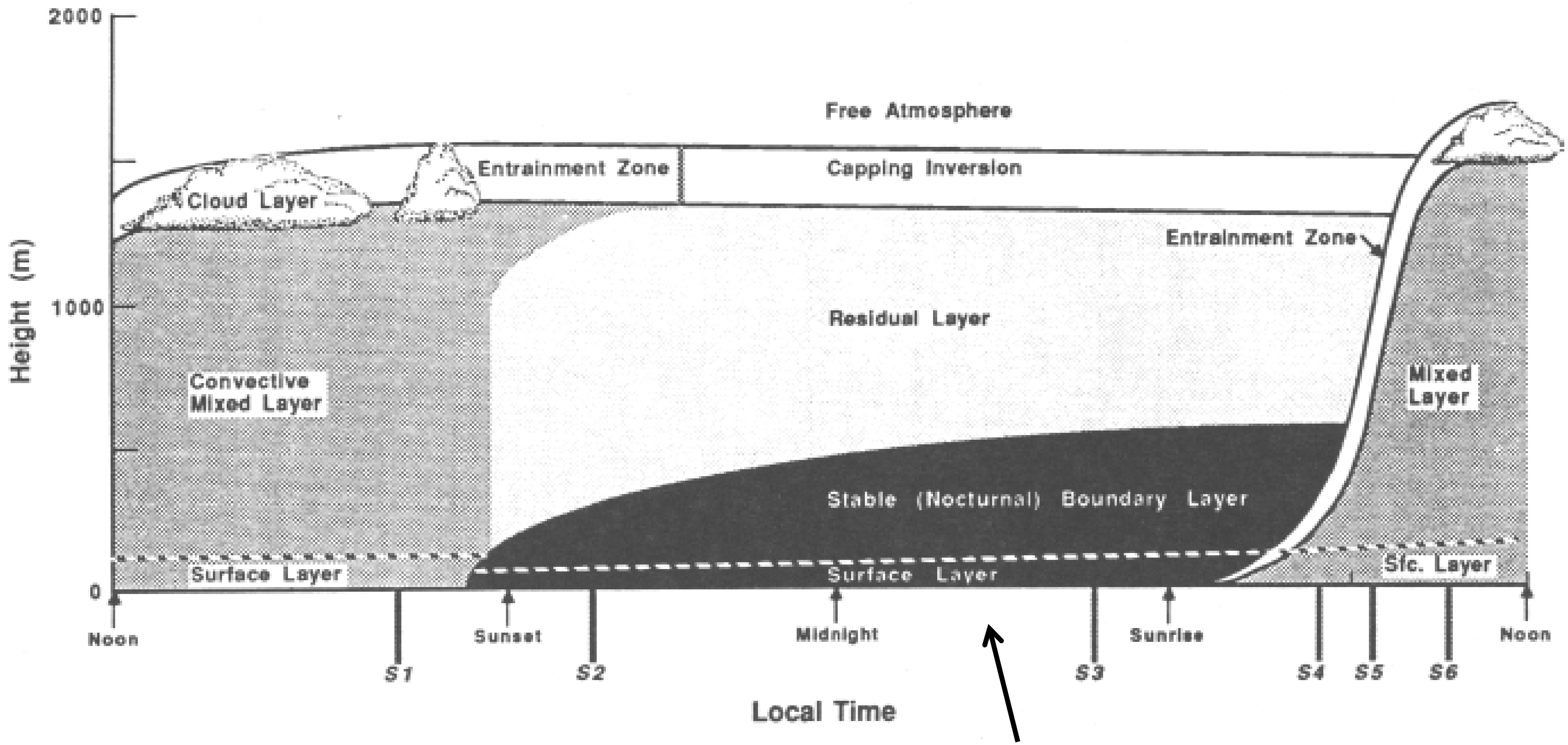
http://wwwcimis.water.ca.gov/cimis/welcome.jsp

Why study "surface layer" theory?

1. To understand often used expressions used for turbulent exchange coefficients (C_M, C_H, C_Q, etc …) when diagnosing fluxes, boundary layer heights, and other things from measured data.
2. When predictions of near-surface met variables are needed (e.g. in models …). Expressions resulting from surface layer theory are used in models to make predictions of near-surface mean profiles as well as surface fluxes.

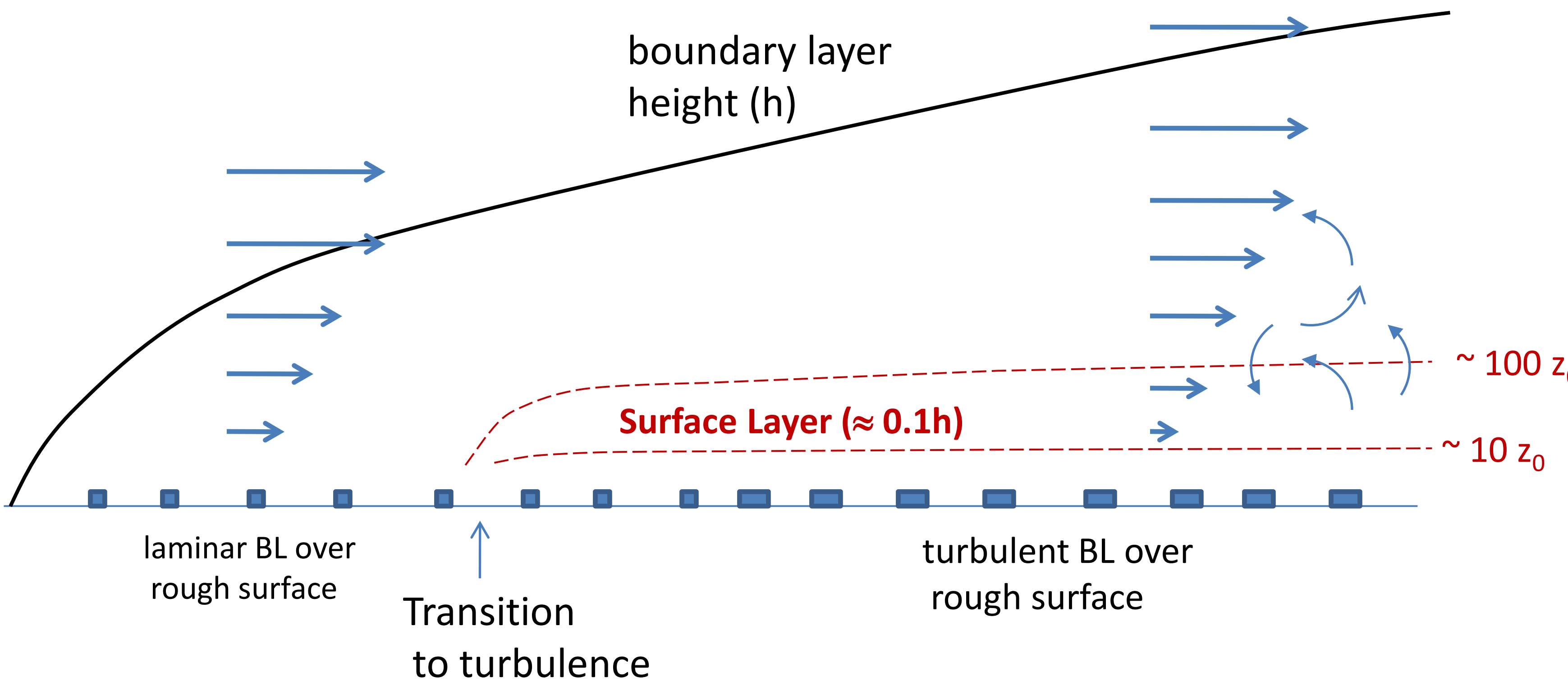
Surface Layer: Definitions

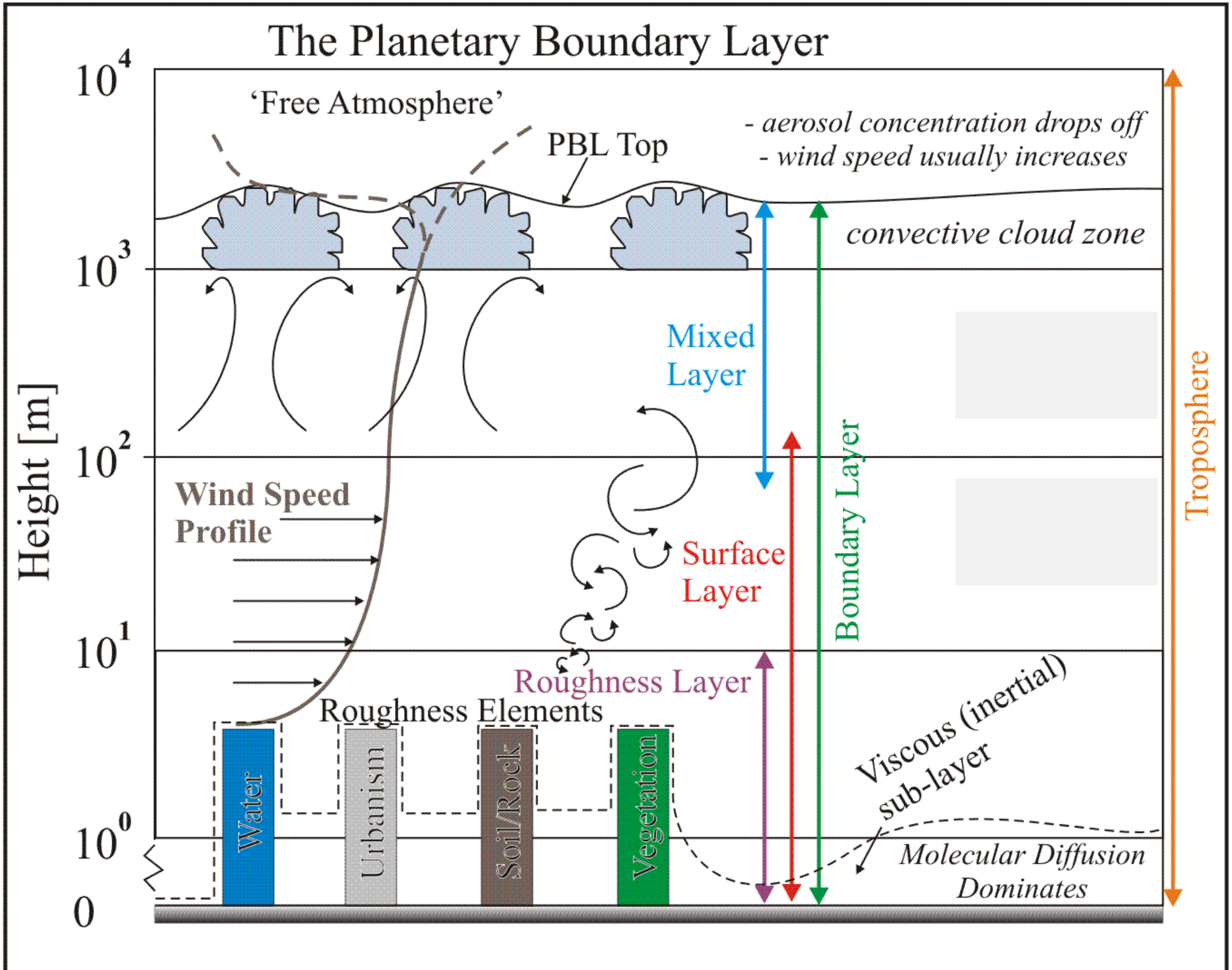
- Let H = surface layer depth, h = boundary layer depth
- H $\approx$ lowest 10% of boundary layer
  - H $\approx$ 0.1h
- H $\approx$ 10 – 100 times roughness length (z_0)
  - $10z_0 < H < 100z_0$
  - $z < 10z_0$ is "roughness sublayer"
  - $z > 100z_0$ is "outer layer"
- In models, surface layer is effectively the lowest model grid layer adjacent to surface (typically lowest grid layer spacing is around 10 meters).



Surface Layer

Surface Layer is an inherent part of a turbulent boundary layer …





The Planetary Boundary Layer
Height [m]
10^4
10^3
10^2
10^1
10^0
0
'Free Atmosphere'
PBL Top
- aerosol concentration drops off
- wind speed usually increases
convective cloud zone
Mixed Layer
Surface Layer
Boundary Layer
Troposphere
Wind Speed Profile
Roughness Layer
Roughness Elements
Water
Urbanism
Soil/Rock
Vegetation
Viscous (inertial) sub-layer
Molecular Diffusion Dominates

Neutral SL Theory
(Log-Law Wind Speed Profile)

Derivation: "Log-Law" Wind Speed Profile (Neutral SL)

Dimensional analysis …

Let friction velocity be velocity scale, i.e. $u_* = (\tau_0/\rho_a)^{1/2}$

Let height above surface (z) be length scale

Therefore …

$$\frac{\mathrm{d}\overline{\mathrm{U}}}{\mathrm{dz}} = \frac{u_*}{kz}$$ where k = 0.4 is the "von karman" constant

Integrate with respect to z …

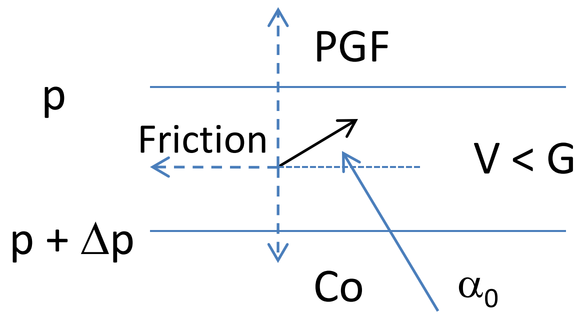
$$\overline{\mathrm{U}} = (u_* / k) \ln(z) + \text{constant}$$

Define constant of integration in terms of height where extrapolated wind profile equals zero. Define this height as the "momentum roughness length", $z_{0,m}$ This leads to 'constant' = -(u_*/k)ln($z_{0,m}$), and therefore to the "log-law" wind speed profile …

$$\overline{\mathrm{U}} = (u_* / k) \ln(z / z_{0,m})$$

What about wind direction?

Near the surface



Wind at angle α_0 to isobars (“cross isobaric flow angle”).

- Can be shown that α_0 does not change with height in surface layer
- Value of α_0 for neutral conditions a function of surface Rossby Number, Ro = G/fz_0
- See handout for Assignment 1 for reminder …
- α_0 also depends on stability, will be covered later …

Modified Form for Flow over Forests and Buildings

$$\overline{U}(z) = \frac{u_*}{k} \ln\left(\frac{z - D}{z_{0,m}}\right)$$

- D defined as "zero-plane displacement height".
- Some "rule-of-thumb" relationships …
  - $z_0 = 0.1 \times$ mean canopy (or building) height ('mch' or 'mbh')
  - $D = 0.7 \times$ mean canopy (or building) height

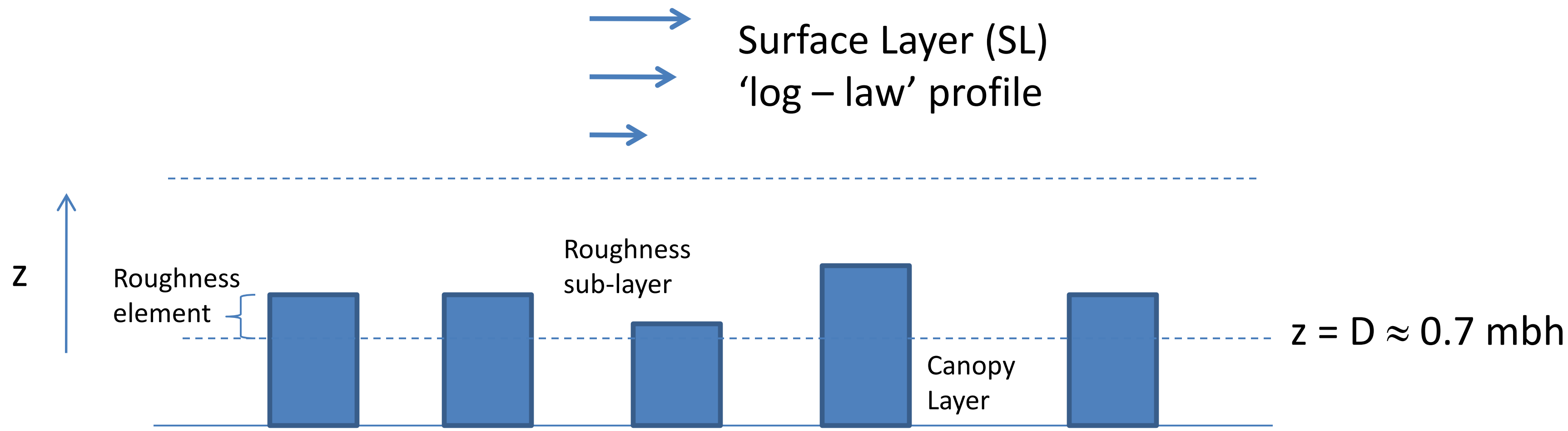


figure not drawn to scale

Calculating wind speed at one height given wind speed at another …

(applying log-law)

$$\frac{\overline{U}(z_1)}{\overline{U}(z_2)} = \frac{\ln(z_1/z_{0,m})}{\ln(z_2/z_{0,m})}$$

$$\overline{U}(z_1) = \cancel{(u_* / k)} \ln(z_1 / z_{0,m})$$

$$\overline{U}(z_2) = \cancel{(u_* / k)} \ln(z_2 / z_{0,m})$$

or with zero-plane displacement

$$\frac{\overline{U}(z_1)}{\overline{U}(z_2)} = \frac{\ln[(z_1 - D)/z_{0,m}]}{\ln[(z_2 - D)/z_{0,m}]}$$

Surface Drag Coefficient

(Momentum)

Momentum Flux = $\tau_0 = -\rho_a u_*^{\,2}$

From log-law and given mean wind speed U_a at height z_a …

$$\mathrm{u}_* = \frac{\mathrm{kU_a}}{\ln\left(z_a / z_{0,m}\right)}$$

Substituting for u_*

$$\tau_0 = -\rho_a u_*^2 = -\rho_a \frac{\mathrm{k}^2}{\ln^2\left(z_a / z_{0,m}\right)} U_a^2 = -\rho_a C_m U_a^2$$

where …

$$C_M = \frac{k^2}{\ln^2\left(z_a / z_{0,m}\right)}$$

Profiles for Passive Scalars (Neutral SL)

A “passive scalar” is a scalar variable that is purely transported around with the wind, and does not affect the dynamics of the wind field or the thermodynamics of the temperature field. Examples include: concentration of an air pollution species in many cases, humidity (provided that air is not saturated and that virtual temperature effects are small), temperature (provided very close to neutral stability and radiation effects are neglected).

Dimensional analysis …

Let passive scalar = A, and $F_{A,0}$ be the surface flux of A

Let surface layer scale for this quantity then equal $A_* = -F_{A,0}/(\rho_a u_*)$

Let height above surface, z, be length scale

Therefore …

$$\frac{\mathrm{dA}}{\mathrm{dz}} = \frac{A_*}{kz}$$ where k = 0.4 can be assumed (based on most recent data)

Will finish derivations for profile and exchange coefficients (e.g. C_Q) in Assignment #2 …

Passive Scalar Profiles in Neutral Surface Layer (2)

From Assignment 2 (Problem 1), can be shown …

$$\overline{A}(z) = A_0 + (A_* / k)\ln(z / z_{0,A})$$

where $z_{0,A}$ is the "roughness length" for A, determined by extrapolating the log profile to the point where A = A_0, the surface value of A. In general, $z_{0A} \neq z_{0m}$.
A common relationship is $z_{0a} = 0.1 z_{0m}$.

Surface Drag Coefficient

(Passive Scalar)

Scalar Flux = F_A = $-\rho_a u_* A_*$

Substituting into the above …

- ➢ u_* and A_* from log-law relationships
- ➢ mean wind speed (U_r) and scalar (A_r) at reference height z_r within SL
- ➢ Surface value of scalar A_0

leads to …

$$F_A = -\rho_a u_* A_* = -\rho_a \left[\frac{k}{\ln\left(z_r / z_{0,m}\right)}\right]\left[\frac{k}{\ln(z_r / z_{0,A})}\right] U_r (A_r - A_0) = -\rho_a C_A U_r (A_r - A_0)$$

where …

$$C_A = \frac{k^2}{\ln\left(z_r / z_{0,m}\right)\ln(z_r / z_{0,A})}$$

Data confirming log-law
(Figures 10.4 and 10.9; Arya)

Tables & Figures for Roughness Length & Displacement Height values
(Figures 10.5, 10.6, and 10.8, Arya)

Stable/Unstable Surface Layer
(Monin-Obukhov Similarity Theory)

Some Background on Turbulence

- **Mechanical Turbulence**
  - Caused by shear instability (i.e. instabilities in wind shear)
  - Friction velocity, u_*, appropriate SL scale to characterize this.
  - Possible in all static stabilities (neutral, stable or unstable).

- **Buoyant Turbulence**
  - Caused by positive buoyancy (buoyant instability)
  - Associated with unstable air ($\partial\theta/\partial z < 0$)
  - More generally, when turbulent heat flux > 0

- **Buoyant Suppression of Turbulence**
  - Caused by negative buoyancy
  - Associated with stable air ($\partial\theta/\partial z > 0$)
  - More generally, when turbulent heat flux < 0

Derivation

(Buoyant Turbulence and Heat Flux)

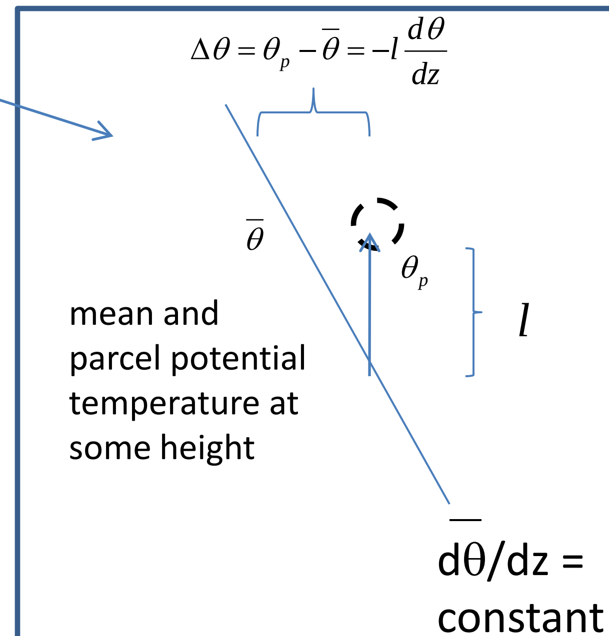
Summary of derivation done on white-board …

$$\frac{dw}{dt} = -\frac{g}{\bar{\theta}}(\bar{\theta} - \theta_p) \longrightarrow \frac{dw}{dt} = -\frac{g}{\bar{\theta}} l \frac{d\bar{\theta}}{dz} \longrightarrow \frac{d(\overline{w^2}/2)}{dt} = -\frac{g}{\bar{\theta}} \overline{wl} \frac{d\bar{\theta}}{dz}$$

"parcel theory"

letting $\theta_p = \bar{\theta} - l \frac{d\bar{\theta}}{dz}$

$$\frac{d(\overline{w^2}/2)}{dt} = -\frac{g}{\bar{\theta}} K_H \frac{d\bar{\theta}}{dz} \qquad \frac{d(\overline{w^2}/2)}{dt} = \frac{g}{\bar{\theta}}\left(\frac{H_S}{\rho_a c_p}\right) = B$$



Key Points

- Heat flux affects vertical turbulence (dynamically active).
- Positive Heat Flux ($H_S > 0$)
  - Vertical turbulent energy increases
  - Associated with positive buoyancy; unstable air
- Negative Heat Flux ($H_S < 0$)
  - Vertical turbulent energy decreases
  - Associated with negative buoyancy; stable air
- Quantity $B \equiv (g/\theta)(H_s/\rho_a c_p)$ called “buoyancy flux”
- Let $H_{S,0}$ be the surface heat flux (H_S in Lecture 2).
- Let $B_0 = (g/\theta)(H_{s,0}/\rho_a c_p)$ therefore be the surface buoyancy flux.
- B_0 is “new” scaling variable for stratified conditions.
  - Used to extend neutral SL layer theory to stratified conditions.
  - Modified theory called “Monin-Obukhov” (MO) theory

Monin-Obukhov Length, L

$$L \equiv -\frac{u_*^3}{kB_0} = -\frac{u_*^3\theta}{kg(H_s / \rho_a c_p)}$$

Use to define a non-dimensional stability parameter $\zeta \equiv z/L$

$\zeta > 0$ (stable)
$\zeta < 0$ (unstable)
$\zeta = 0$ (neutral)

$$\zeta = z / L = -\frac{kzg(H_{s,0} / \rho_a c_p)}{u_*^3\theta}$$

Physical Meaning of z/L

z/|L| “large” (> 1)
Buoyancy effects start to affect turbulence and profiles (or even dominate)
L > 0 (stable, buoyant suppression of turbulence and mixing)
L < 0 (unstable, buoyant enhancement of turbulence and mixing)

z ↑ z =|L| i.e. |z/L| = 1

z/|L| “small” (<< 1)
Buoyancy effects relatively small
Mechanical turbulence dominates (i.e. due to wind shear)

surface (z = 0)

Wind Speed Profile in Stratified Surface Layer (1)

Applying stability parameter ζ = z/L …

$$\frac{\mathrm{d}\overline{\mathrm{U}}}{\mathrm{dz}} = \frac{u_*}{kz}\phi_m(\zeta)$$

where ϕ_m is a "stability function" of z/L.

Following standard equations determined from theory and field measurements

$$\phi_m(\zeta) = 1 + \beta_1\zeta$$

$\zeta > 0$ (stable)
(typical value $\beta_1 = 4.7$)

$$\phi_m(\zeta) = (1 - \gamma_1\zeta)^{-1/4}$$

$\zeta < 0$ (unstable)
(typical value $\gamma_1 = 15$)

See also Arya equations 11.6 and 11.7

Wind Speed Profile in Stratified Surface Layer (2)

Upon vertically integrating dU/dz from previous slide, mean wind speed profile is a "modified" version of log-law as follows …

$$\overline{\mathrm{U}}(z) = (u_* / k)\,[\ln(z / z_{0,m}) - \psi_{\mathrm{m}}(z / L)]$$

where ψ_m is a function of z/L determined from vertically integrating the $\varphi_m(z/L)$ functions on the previous slide. Exact form of ψ_m is different in stable ($z/L > 0$) and unstable ($z/L < 0$) conditions, since φ_m functions are different in stable and unstable conditions.

See Arya Section 11.3 (equations 11.12 – 11.14) for further details, and for exact form of ψ_m functions.

Scalar Profiles in Stratified Surface Layer (1)

For potential temperature, for example …

Applying stability parameter ζ = z/L …

$$\frac{\mathrm{d}\overline{\theta}}{\mathrm{dz}} = \frac{\theta_*}{kz}\phi_h(\zeta)$$

where ϕ_h is a "stability function" of z/L.

Following standard equations determined from theory and field measurements

$$\phi_h(\zeta) = 1 + \beta_2\zeta$$

$\zeta > 0$ (stable)
(typical value $\beta_2 = \beta_1 = 4.7$)

$$\phi_h(\zeta) = (1 - \gamma_2\zeta)^{-1/2}$$

$\zeta < 0$ (unstable)
(typical value $\gamma_2 = 9$)

See Arya equations 11.6 and 11.7

Scalar Profiles in Stratified Surface Layer (2)

For potential temperature, for example

Upon vertically integrating dθ/dz from previous slide, mean potential temperature profile is a "modified" version of log-law as follows …

$$\overline{\theta}(z) = \theta_0 + (\theta_* / k)\,[\ln(z / z_{0,h}) - \psi_{\mathrm{h}}(z / L)]$$

where ψ_h is a function of z/L determined from vertically integrating the φ_h(z/L) functions on the previous slide. Exact form of ψ_h is different in stable (z/L > 0) and unstable (z/L < 0) conditions, since φ_h functions are different in stable and unstable conditions.

See Arya Section 11.3 (equations 11.12 – 11.14) for further details, and for exact form of ψ_h functions.

Drag Coefficients

(Modified to Account for Stability via MO Theory)

Redoing derivation for neutral expressions (previous slides), except this time accounting for ψ functions …

$$C_M = \frac{k^2}{[\ln(z_r / z_{0,m}) - \psi_m(z_r / L)]^2}$$

$$C_H = \frac{k^2}{[\ln(z_r / z_{0,m}) - \psi_m(z_r / L)][\ln(z_r / z_{0,h}) - \psi_h(z_r / L)]}$$

- where z_r is the reference (measurement) height
- $C_H = C_Q = C_A$
- see Arya Equations 11.17

Calculating wind speed at one height given wind speed at another …

(extended for MO theory)

$$\frac{\overline{\mathrm{U}}(\mathrm{z}_1)}{\overline{\mathrm{U}}(\mathrm{z}_2)} = \frac{\ln(\mathrm{z}_1/\mathrm{z}_{0,\mathrm{m}}) - \psi_m(z_1/L)}{\ln(\mathrm{z}_2/\mathrm{z}_{0,\mathrm{m}}) - \psi_m(z_2/L)}$$

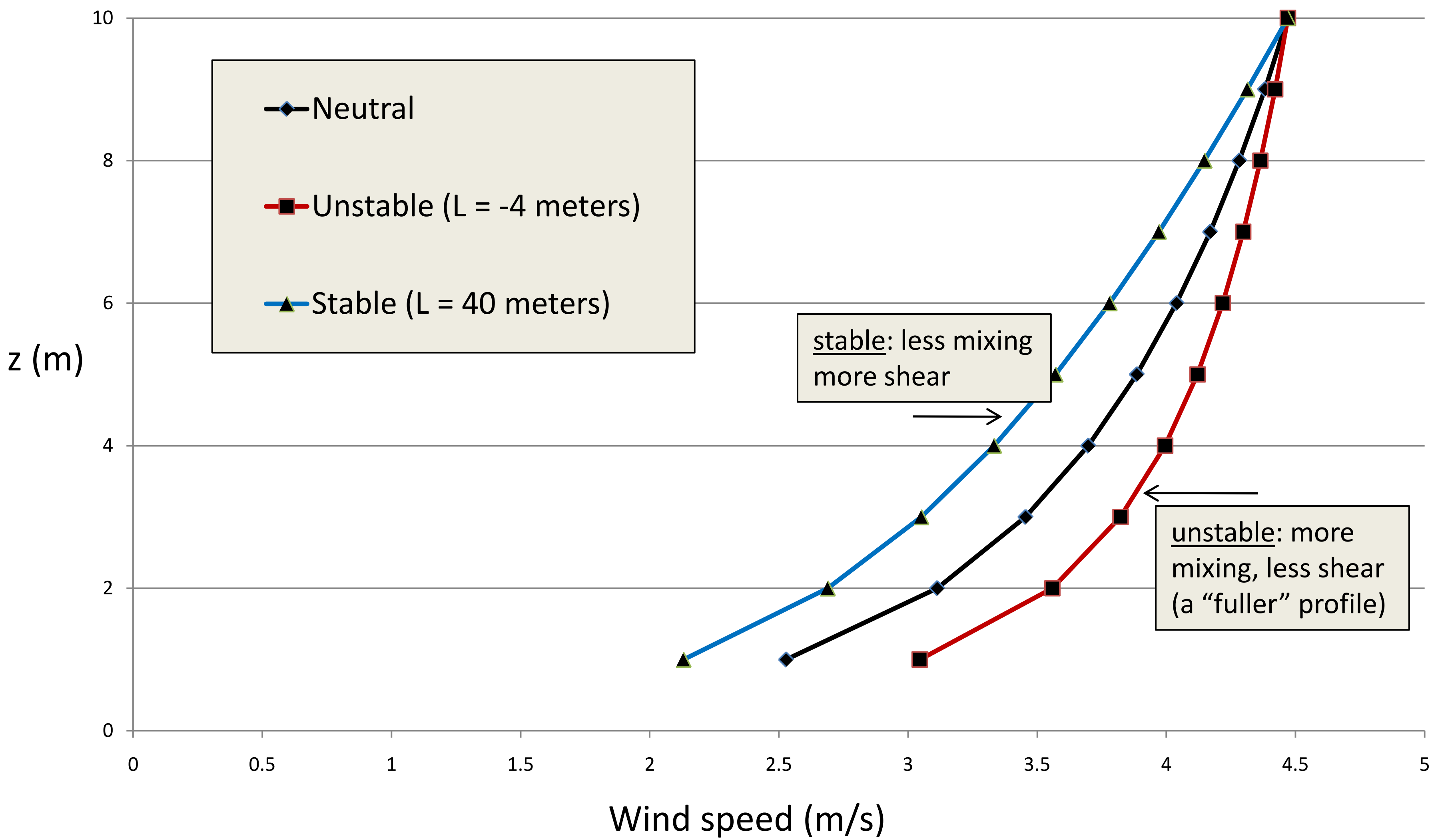
$$\frac{\overline{\mathrm{U}}(z_1) = \cancel{(u_*/k)}[\ln(z_1/z_{0,m}) - \psi_{\mathrm{m}}(z_1/L)]}{\overline{\mathrm{U}}(z_2) = \cancel{(u_*/k)}[\ln(z_2/z_{0,m}) - \psi_{\mathrm{m}}(z_2/L)]}$$

or with zero-plane displacement

$$\frac{\overline{\mathrm{U}}(\mathrm{z}_1)}{\overline{\mathrm{U}}(\mathrm{z}_2)} = \frac{\ln[(\mathrm{z}_1 - \mathrm{D})/\mathrm{z}_{0,\mathrm{m}}] - \psi_m[(z_1 - D)/L]}{\ln[(\mathrm{z}_2 - \mathrm{D})/\mathrm{z}_{0,\mathrm{m}}] - \psi_m[(z_2 - D)/L]}$$

Wind Speed Profiles Computed for Neutral, Stable and Unstable Stratification using log-law (neutral) and MO Theory (stable and unstable)

(z_{0m} = 0.05, 10-meter wind speed = 4.5 m/s = 10 mph, specified values for L)



Wind Speed Profiles Computed for Neutral, Stable and Unstable Stratification using log-law (neutral) and MO Theory (stable and unstable)

(z_{0m} = 0.05, 10-meter wind speed = 4.5 m/s = 10 mph, specified values for L)

SAME PROFILE AS PREVIOUS SLIDE, BUT NOW PLOTTED UP TO Z = 50 METERS