Applying Experienced Self-Tuning PID Controllers to Position Control of Slider Crank Mechanisms

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Abstract

This paper proposes an experienced self-tuning PID control method to the position control of slider-crank mechanism. The mathematical model of a slider crank mechanism coupled with PM actuator is described, firstly. By the Hamilton principle and Lagrange multiplier method, the mathematical formula is derived. Secondly, according to the experience of engineers, the initial PID parameters under normal operating condition can be found out. By the same way, the best parameters of PID controller under full-load condition can be found, too. The proposed self-tuning PID controller will automatically tune its parameters under these ranges according to the position error and error derivation. Moreover, the PC-based controller is implemented to control the position of the motor mechanism coupling system. The simulation and experimental results will show the potential of the proposed controller.

1. Introduction

The slider crank mechanism is a basic structure in mechanical application. It is also widely used in practical application. For examples, fretsaws, petrol and diesel engines are the typical application of velocity control. Due to its mechanical coupling, the physical sense is not enough to derive its dynamic equations. Jasinski et al. [1], Zhu and Chen [2] and Badlani et al. [3] have solved the steady state solutions of a slider crank few years ago. According to reference [4], the response of slider crank is dependent on length, mass, damping, external piston force and frequency. Based on the viewpoints of the ratios, length and speeds of the crank to the connecting rod, the transient responses have been investigated [5]. Mostly, the slider crank mechanisms are actuated by the field oriented control PM synchronous [6-8]. The slider crank mechanism driven by PM synchronous is to transfer motion to translation motion. Hence, the computed torque based robust and adaptive controllers are designed to control the motor mechanism [8]. With Hamiliton principle and Lagrange multiplier method, model equations of the slider crank mechanism coupling with PM synchronous are formulated [8-11]. Observing these dynamic equations, highly coupling is existed in these nonlinear equations. Hence, the traditional control scheme usually is not suitable to this mechanism.

The PID method is the most popular controller up to now. Despite the progression of many control theories, the PID controller is still the majority of industrial processes [12-14]. Due to the easily understanding of the physical sense for parameters of PID controller, engineers used to apply it to practical objects. However, the PID controller is not robust to wide parameter varying and large external disturbance. Especially for the highly coupling nonlinear system of slider crank mechanisms, the PID controller is lack of adaptive capability. Usually, the parameters of PID controller are manually tuned under ideal condition, that is the operating point without load. However, these parameters are mostly not suitable for the condition with full load. To achieve practical requirement, engineers have to adjust the parameters under different operating conditions. However, the robustness is limited with a small range. A rule to overcome this disadvantage is called self-tuning rule. Many researches and reports of self-tuning PID have been published. The parameter tuning at any time instance is usually based on a structurally fixed mathematical model produced by on-line identification procedure [15-17]. Unfortunately, recent plants are mostly difficult to obtain their fixed mathematical models. However, experts easily obtain the most proper parameters for no-load and full load. This paper proposed an intelligent self-tuning PID controller. The tuning method is based on the skilled engineers' experience. Engineers will easily accept the straightforward design procedure. At the same time, the robustness will be extended.

This paper is arranged as follows. Firstly, the mathematical model of a slider crank mechanism coupled with a PM synchronous is derived. Second, the tuning method of the proposed controller will be discussed. Section 4 will show simulation results. In section 5, the PC-based controller is set up for experimental. The proposed controller is applied to position control of a slider crank. The experimental results are also presented in this section.

2. Slider Crank Actuated by a PM Synchronous

2.1 PM Synchronous

A model of a PM synchronous motor can be simplified to the following block diagram.

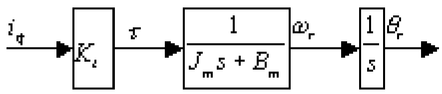


Fig 1 Block Diagram of a PM synchronous motor

Usually, the PM synchronous motor is coupled with a gear speed reducer with a gear ratio of n. Hence, the applied torque can be described as

$$\tau = n\left(K_t i_q - nJ_m\ddot{\theta}_r - nB_m\dot{\theta}_r\right) \quad (1)$$

where τ is the torque in the direction of ω_r, K_t is the torque constant, J_m and B_m are the inertia and viscous damping ration, respectively.

2.2 Slider Crank Mechanism

In this section, Hamilton's principle and Lagrange multiplier are used to derive the differential equation for the slider-crank mechanism. The slider crank mechanism system is shown in Fig. 2.

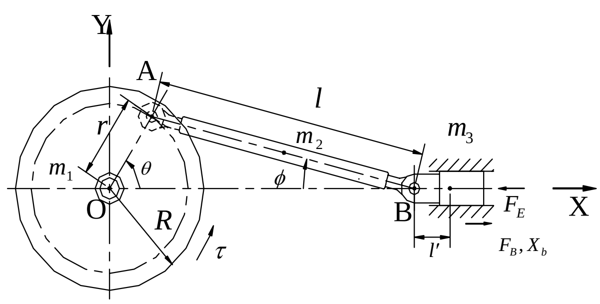


Fig 2 A Slider Crank Mechanism System

The slider-crank mechanism consists of three parts: crank, rod and slider. The holomonic constraint equation [12] is

$$\Phi(\Psi) = r\sin\theta - l\sin\phi = 0 \quad (2)$$

where $\Psi = [\theta \quad \phi]^T$.

The kinematic velocity is obtained by the first derivation of eq. (2), as

$$\Phi_\Psi \dot{\Psi} = r\dot{\theta}\cos\theta - l\dot{\phi}\cos\phi = 0 \tag{3}$$

The kinematic acceleration is obtained by the second derivatives of eq. (2), as

$$\Phi_\Psi \ddot{\Psi} = r\dot{\theta}^2 \sin\theta - l\dot{\phi}^2 \sin\phi = \lambda . \tag{4}$$

where λ is the Lagrange multiplier as

$$\lambda = r\dot{\theta}^2 \sin\theta - l\dot{\phi}^2 \sin\phi \tag{5}$$

The Lagrangian L, that is the total kinetic energy minus the potential energy, is

$$\begin{aligned} L = &\frac{1}{4}m_1 R^2\dot{\theta}^2 + \frac{1}{6}m_2 l^2\dot{\phi}^2 + \frac{1}{2}m_2 r^2\dot{\theta}^2\sin^2\theta + \frac{1}{2}m_2 rl\dot{\theta}\sin\theta\sin\phi \\ &+ \frac{1}{2}m_3 r^2\dot{\theta}^2\sin^2\theta + m_3 rl\dot{\theta}\dot{\phi}\sin\theta\sin\phi + \frac{1}{2}m_3 l^2\dot{\phi}^2\sin^2\phi - \frac{1}{2}m_2 gl\sin\phi \end{aligned} \tag{6}$$

The virtual work δW^A includes the applied torque τ with the virtual angle $\delta\theta$, the friction force F_B and the external force F_E with the virtual displacement δX_B ,that is

$$\delta W^A = \tau\,\delta\theta + (F_B - F_E)\delta X_B = \tau\,\delta\theta + F_{BE}(-r\sin\theta\delta\theta - l\sin\phi\delta\phi) \tag{7}$$

where $F_B = -\mu m_3 g\,\mathrm{sgn}(V_B)$ and V_B is the velocity of the slider B.

Rewrite eq. (6) in terms of the generalized coordinate Ψ, then

$$\delta W^A = -\delta\Psi^{\mathrm{T}}\mathbf{Q}^{\mathbf{A}} \tag{8}$$

where $\mathbf{Q}^A$ is the generalized force and given as

$$\mathbf{Q}^A = \begin{bmatrix} F_{BE} r\sin\theta - n(K_t i_q - J_m\ddot{\theta}_r - B_m\dot{\theta}_r) \\ F_{BE} l\sin\phi \end{bmatrix}$$

The generalized constraint force can be formed in term of Lagrange multiplier as

$$\mathbf{Q}^{\mathbf{C}} = \Phi_\Psi^T \lambda \tag{9}$$

where $\Phi_\Psi = [\, r\cos\theta \quad -l\cos\phi \,]$.

Thus, the virtual work by all constraint reaction forces is

$$\delta W^C = \delta\psi^{\boldsymbol{T}}\mathbf{Q}^{\boldsymbol{C}} \tag{10}$$

The general form of Hamilton's principle is

$$\begin{aligned} 0 &= \int_{t_1}^{t_2}\left[\delta L + \delta W^{\mathrm{A}} + \delta W^{\mathrm{C}}\right]dt \\ &= \int_{t_1}^{t_2}\delta\Psi^T\left[\frac{\partial L}{\partial\Psi} - \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\Psi}}\right) - \mathbf{Q}^{\mathbf{A}} + \mathbf{Q}^{\mathbf{C}}\right]dt, \end{aligned} \tag{11}$$

The eq. (11) must hold for all $\delta\Psi$ and $\delta\Psi(t_1) = \delta\Psi(t_2) = 0$. Thus, according to the Euler-Lagrange equations of motion, the dynamic equation can be obtained as

$$\mathbf{M}(\Psi)\ddot{\Psi} + \mathbf{N}(\Psi,\dot{\Psi}) - \mathbf{BU} - \mathbf{D}(\Psi) + \mathbf{\Phi}_\Psi^T\lambda = 0 \tag{12}$$

where

$$\mathbf{M}(\Psi) = \begin{bmatrix} -\frac{1}{2}m_1R^2 - (m_2+m_3)r^2\sin^2\theta - n^2 J_m & -\left(\frac{1}{2}m_2 - m_3\right)r\,l\sin\theta\sin\phi \\ -\left(\frac{1}{2}m_2 - m_3\right)r\,l\sin\theta\sin\phi & -\frac{1}{3}m_2 l^2 - m_3 l^2\sin^2\phi \end{bmatrix}$$

$$\mathbf{N}(\Psi,\dot{\Psi}) = \begin{bmatrix} -(m_2+m_3)r^2\dot{\theta}^2\sin\theta\cos\theta - \left(\frac{1}{2}m_2 + m_3\right)rl\dot{\phi}^2\sin\theta\cos\phi - n^2 B_m\dot{\theta} \\ -\left(\frac{1}{2}m_2 + m_3\right)rl\dot{\theta}^2\cos\theta\sin\phi - m_3 l^2\dot{\phi}^2\sin\phi\cos\phi - \frac{1}{2}m_2 gl\cos\phi \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} -nK_t \\ 0 \end{bmatrix} \;;\quad \mathbf{U} = \lfloor i_q \rfloor ;\quad \mathbf{D}(\Psi) = \begin{bmatrix} F_{BE} r\sin\theta \\ F_{BE} l\sin\phi \end{bmatrix}$$

where m_1, m_2 and m_3 are the mass of crank, rod and slider, respectively, and *r* and *l* are length of crank and rod, and θ and ϕ are angle of crank and rod. The translation position X_b can be obtained by transforming θ, that is

$$\begin{aligned} X_b &= r\cos\theta + l\cos\phi + l' \\ &= r\cos\theta + \left(l^2 - r^2\sin^2\theta\right)^{\frac{1}{2}} + l' \end{aligned} \tag{13}$$

3. Self-Tuning PID controller

In most researches, the parameter tuning is based on known mathematical models. Even the models unknown, the on-line identification or parameter estimation is proposed to establish an estimated model. However, the modeling error usually causes the unexpected conditions.

Fortunately, for any complex system, the experienced engineers can easily obtain the most proper parameters under no-load and full-load conditions. Take the proportional controller for example. Under operating conditions between no-load and full load, the proportional gain should be in the range of the gains of full-load and no-load conditions. Let the proportional gains are respectively $P_{\min}$ and $P_{\max}$ for no-load and full-load conditions. The proportional gain should be decreased from $P_{\max}$ to $P_{\min}$ along with the error and error derivation. According the common sense [19], if the proportional (P) controller's gain increases, then the rising time and steady state error will be reduced. Too large gain will make the great overshoot and extreme oscillation. Too small gain will make steady state error existed. While the absolutions of error and error derivation are large, the proportional gain should be working on largest number, that is $P_{\max}$. While the error and error derivation are small enough, the proportional gain should be $P_{\min}$ to reduce the steady state error. The relationship can be shown as figure 3. Hence, let the tuning rule is defined as

$$K_P(t) = K_P^0 + \Delta K_P\left(1 - e^{-\left(\frac{E^2(t)+\Delta E^2(t)}{\gamma}\right)}\right) \tag{14}$$

where $K_P^0 = P_{\min}$, $\Delta K_P = P_{\max} - P_{\min}$ and γ is the adjusting rate. E(t) is error function and Δ E(t) is variety of the error function.

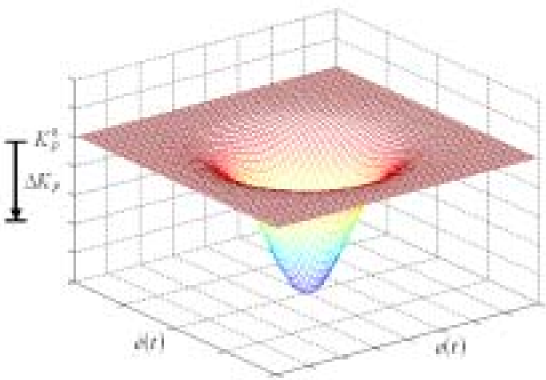


Fig 3 Tuning Rule of Proportional Gain

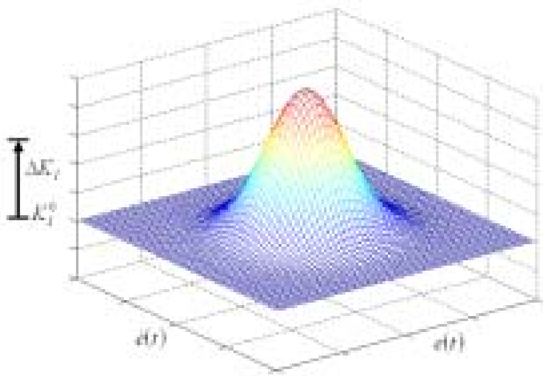


Fig 4 Tuning Rule of Integral Gain

By the same conception, the integral controller is helpful for steady state and hurtful for transient state. Hence, the integral (I) controller's gain should be increased along with the error and error derivation decreasing. The curve is shown as figure 4. Let the tuning rule is defined as

$$K_I(t) = K_I^0 - \Delta K_I \left(1 - e^{-\left(\frac{E^2(t)+\Delta E^2(t)}{\gamma}\right)}\right) \tag{15}$$

Let $K_I^0 = I_{\min}$, $\Delta K_I = I_{\max} - I_{\min}$ and $I_{\max}$ and $I_{\min}$ are the gains under no-load and full-load conditions respectively. E(t) is error function and ΔE(t) is variety of the error function.

Large differential controller can increase the speed of response. At the same time, large differential controller will cause large steady state error. Therefore, the differential (D) controller's gain should be decreased along with the error and error derivation decreasing. The tuning rule can be shown as following figure and equation.

$$K_D(t) = K_D^0 + \Delta K_D \left(1 - e^{-\left(\frac{E^2(t)+\Delta E^2(t)}{\gamma}\right)}\right) \tag{16}$$

Let $K_D^0 = D_{\min}$, $\Delta K_D = D_{\max} - D_{\min}$ and $D_{\max}$ and $D_{\min}$ are the gains under full-load and no-load conditions respectively. E(t) is error function and ΔE(t) is variety of the error function.

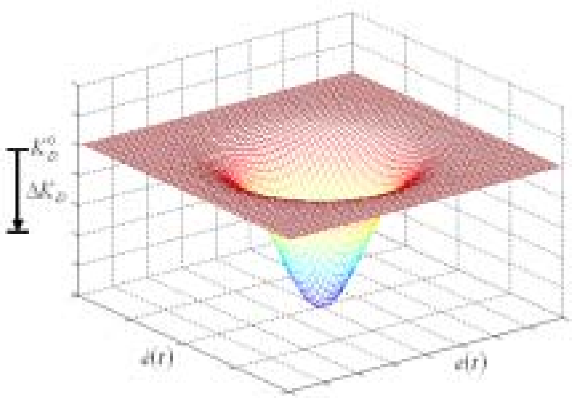


Fig 5 Tuning Rule of Differential Gain

Due to the proposed PID self-tuning method based on the engineer's experience, the experienced engineers will easily accept it.

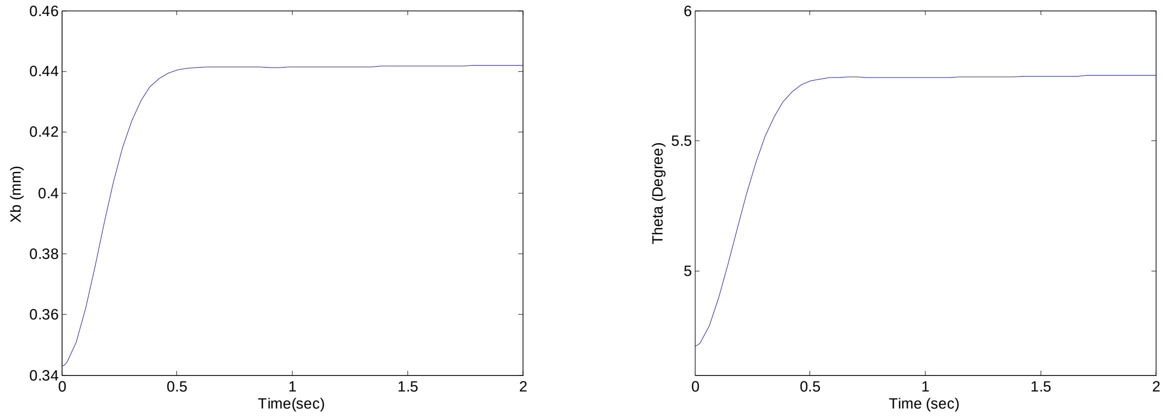
4. Simulation Results

In this section, numerical simulation results are used to demonstrate the potential of the proposed control rule. To demonstrate the performance, the PID self-tuning method is compared with a fixed PID control.

The actual slider crank mechanism dimensions are m_1=3.64, m_2=1.18, m_3=1.8, r =0.1, R=0.12, l =0.305, l'=0.055, K_t =0.6732, J_m =0.00062 and B_m =0.000153. The initial angle is $\theta(0) = \frac{1}{4}\pi$ rad. The objective is to control the desired translation position to 0.1m.

Figures 6-7 show the response for system with the fixed PID controller. The parameters of PID controller is manually setup for optimal performance under no-load condition, where K_P^{no}=3.5695, K_I^{no}=2.9267 and K_D^{no}=0.7727. The desired specifications are settling time t_s =0.5sec, rising time t_r =0.25 sec, maximum overshoot M_P<5% and steady state error

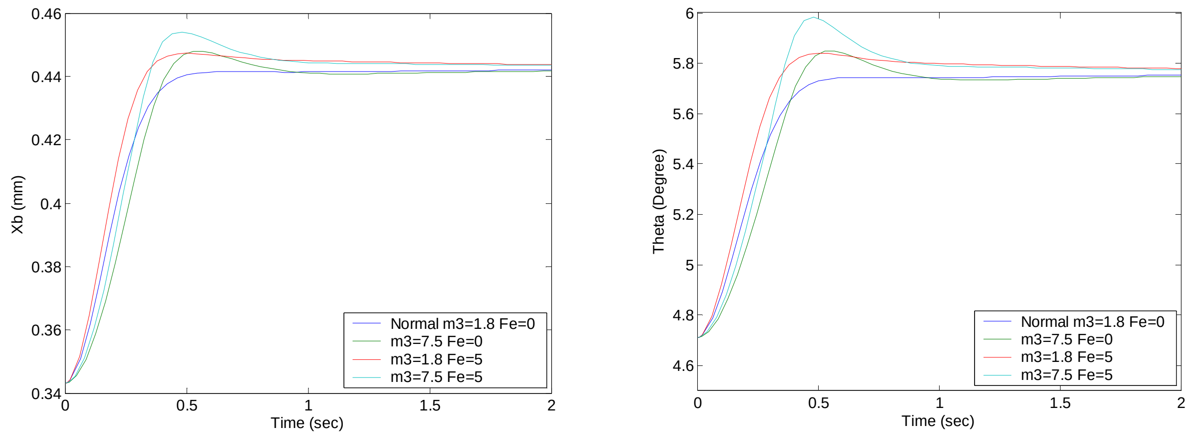
e_{ss} <1%. Figure 6 shows the responses of translation position and angular under no load condition. These curves show this PID controller can achieve the desired specification.



(a) Response of Translation Position (b) Response of Angular

Fig 6 Simulation Response of Fixed PID Controller (No-Load Condition)

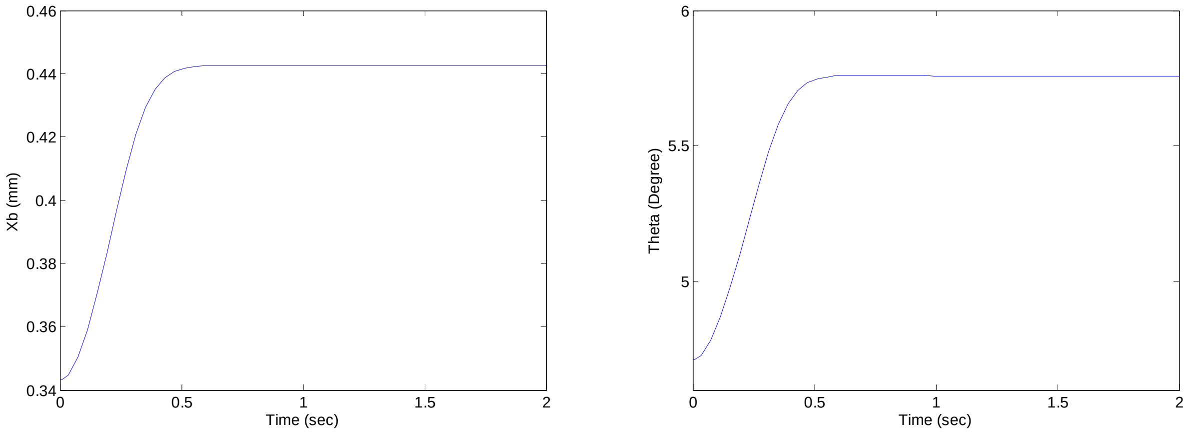
Figure 7 shows the responses under different operating conditions. The parameter variation is appeared by the mass of slider crank m_3 changed from 1.8Kg to 7.5Kg. The external force is changed from 0Nt to 5Nt. Obviously, the fixed PID obtained under no-load condition cannot achieve the robustness with parameter varying and load existed.



(a) Response of Translation Position (b) Response of Angular

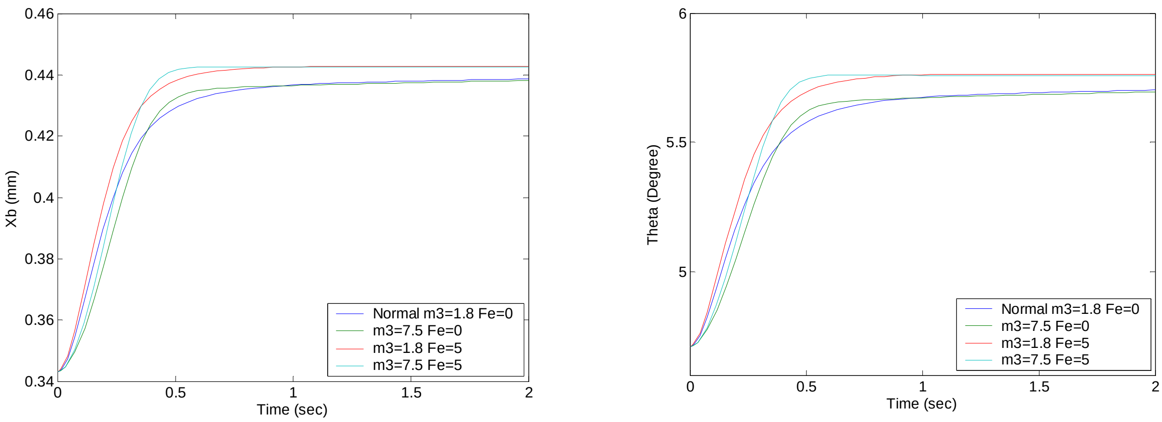
Fig 7 Simulation Response of Fixed PID Controller (No-Load Condition)

Let the parameters of PID controller is manually setup for optimization with full-load existed. The optimal parameters for PID controller with full-load are K_P^{full} =4.1942, K_I^{full} =1.2997 and K_D^{full} =1.1313. The response of the system with full-load is shown in figure 8. As see to this curve, the fixed PID controller can satisfy the request of parameter and external load. However, once the parameter varying and external load are removed, the responses will cause the large steady-state error and unexpected transient state shown in figure 9.



(a) Response of Translation Position (b) Response of Angular

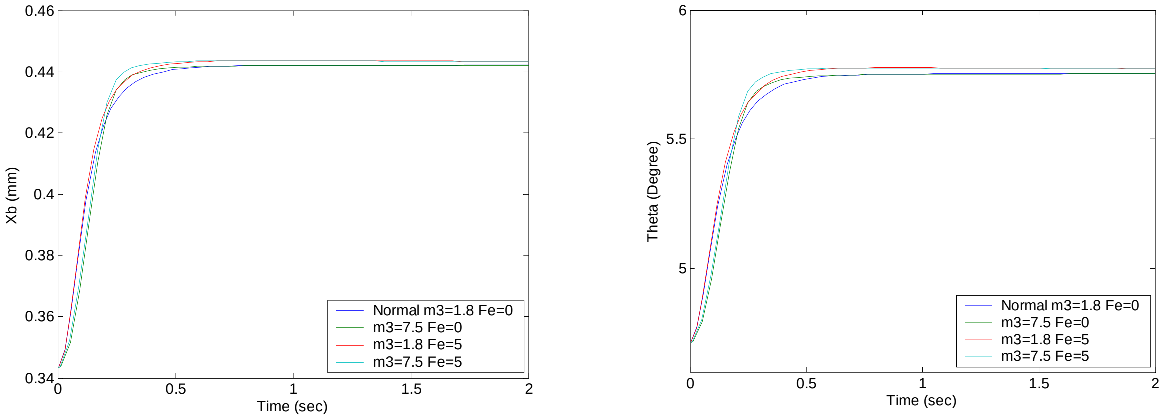
Fig 8 Simulation Response of Fixed PID Controller (Full-Load Condition)



(a) Response of Translation Position (b) Response of Angular

Fig 9 Simulation Response of Fixed PID Controller (Full-Load Condition)

The proposed self-tuning PID controller is based on these two optimal parameters. Let the normal parameters are based on no-load condition, that is $K_P^0 = K_P^{no}$ =3.5695, $K_I^0 = K_I^{no}$ =2.9267, and $K_D^0 = K_D^{no}$ =0.7727. The tuning ranges are selected as $\Delta K_P = K_P^{full} - K_P^{no}$ =0.6247, $\Delta K_I = K_I^{full} - K_I^{no}$ =1.6270 and $\Delta K_D = K_D^{full} - K_D^{no}$ =0.3586. Applying eqs (14)~(16), the responses of the proposed control rule are shown in figure 10. Obviously, the proposed self-tuning PID controller displays the robust characteristic. Under different operating conditions, it can still maintain the desired performance.



(a) Response of Translation Position (b) Response of Angular

Fig 10 Simulation Response of Self-Tuning PID Controller

5. Experimental Results

In order to demonstrate the proposed control rule, a PC-based experimental equipment is setup in this paper. The experimental instrument of slider crank is divided into three parts: actuator, slider crank and controller. The photographic is shown in figure 11. The first part consists of a PM Synchronous motor, driver. The driver is worked on 3-phase, 220 V and 60 Hz. The slider crank is coupled with the PM motor. The translation position is measured by a potentiometer. The output of potentiometer is 0~5V, which mapped to real translation position is 0~0.2m. The controller is based on a PC with Pentium-586 CPU. The data acquisition interface card (Advantech CO., PCL-1800) is installed in the ISA bus to handle the A/D and D/A process. The graphical software of Simulink is used to implement the proposed control rule. At the same time, the linear converts between the physical scale and voltage from sensors are also worked in this software. To carry out the parameter varying and external load is to add an external mass (7.4Kg) on slider.

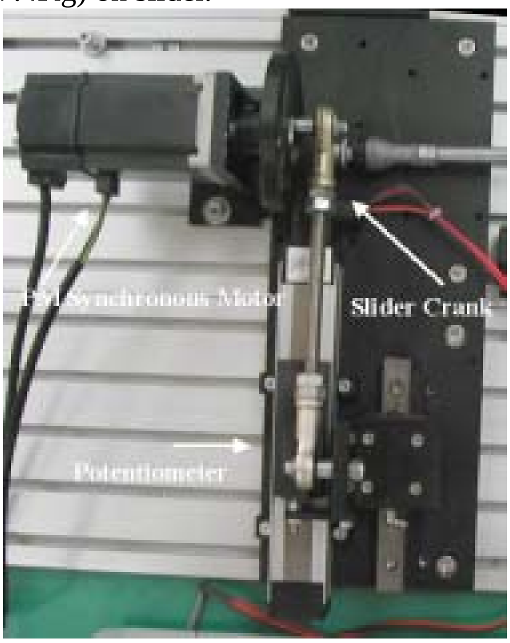
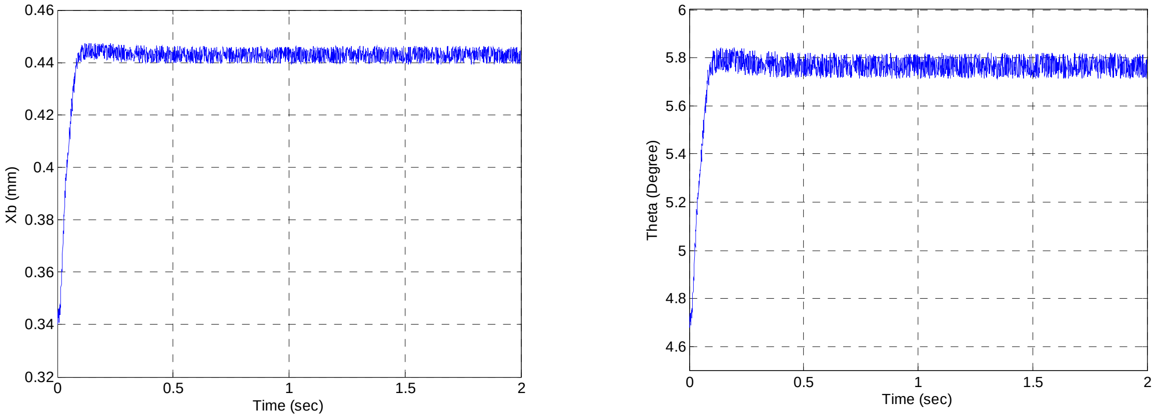


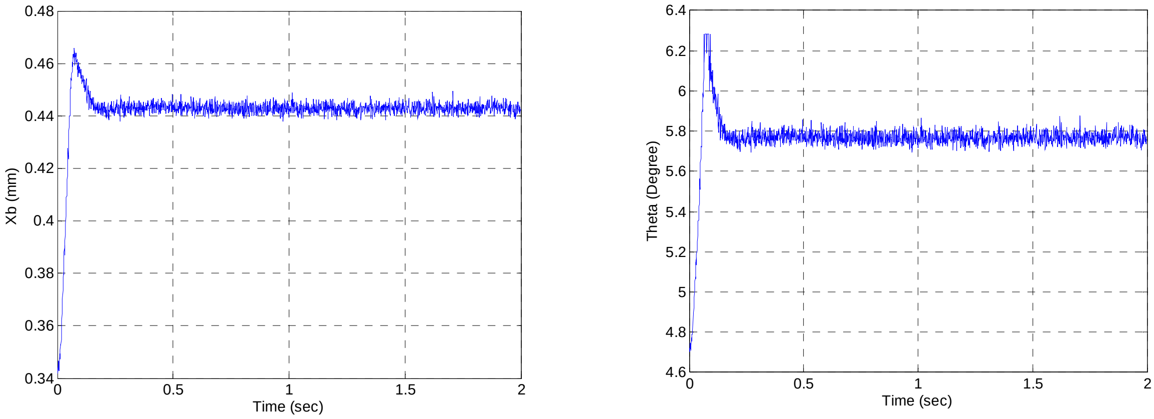
Fig 11 Experimental Instrument of Slider Crank

Based on the same requirement of simulation, the experimental results are shown in figures 12~17. Firstly, the fixed PID setup under no-load is experimented. Under no-load condition, the experimental result is shown in figure 12. However, the response shown in fig 13 has large overshoot when the load added. It shows the bad robustness of fixed PID controller.



(a) Response of Translation Position (b) Response of Angular

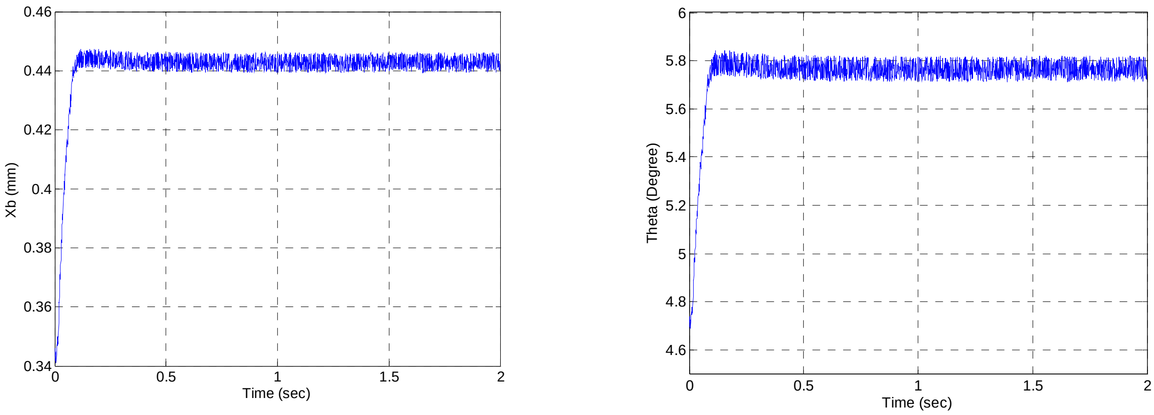
Fig 12 Experimental Response of Fixed PID Controller (No-Load Condition) with No-Load



(a)Response of Translation Position (b) Response of Angular

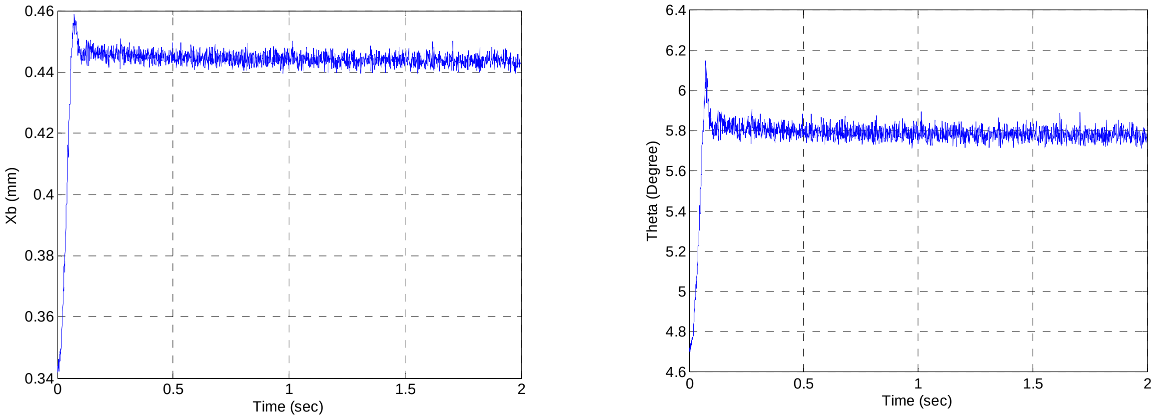
Fig 13 Experimental Response of Fixed PID Controller (No-Load Condition) With 7.5 Kg External Load

The second experiment is fixed PID controller whose parameters are obtained under full-load condition. Figure 14 shows the experimental result. It shows the manually chosen parameters can achieve desired requirement under full-load condition. However, figure 15 shows the same controller applied when load removed. Obviously, the fixed PID controller cannot overcome the large change in operating situation.



(a) Response of Translation Position (b)Response of Angular

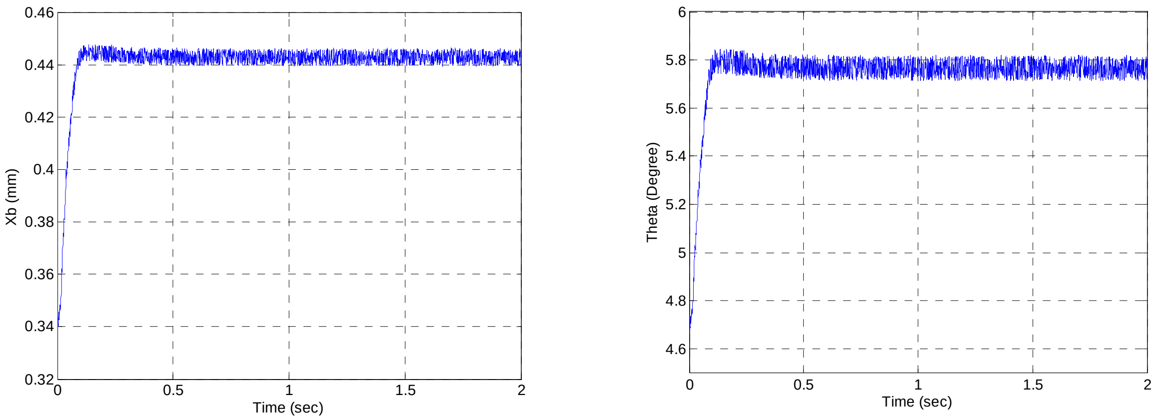
Fig 14 Experimental Response of Fixed PID Controller (Full-Load Condition) With 7.5 Kg External Load



(a) Response of Translation Position (b) Response of Angular

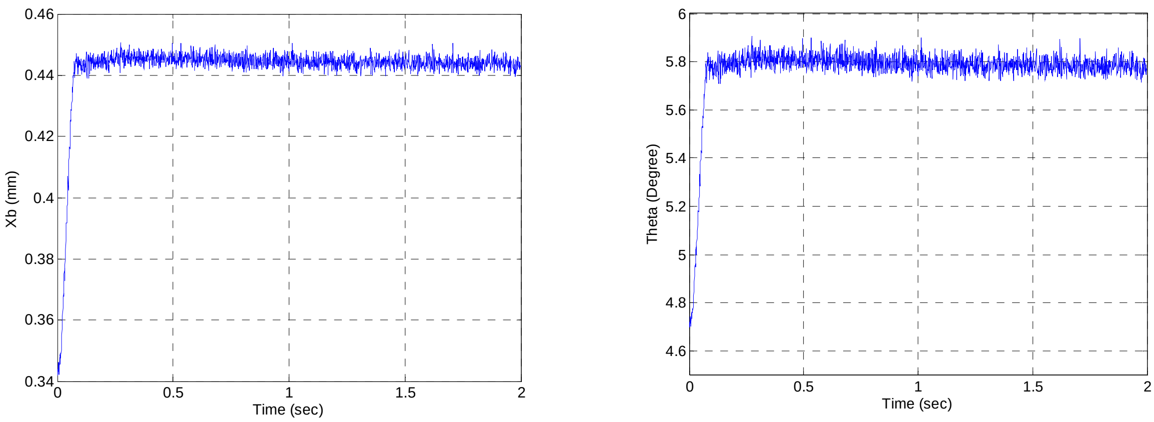
Fig 15 Experimental Response of Fixed PID Controller (Full-Load Condition) With No-Load

Finally, applying the proposed self-tuning PID controller to the same experimental instrument. The responses of translation position and angular with no-load existed are shown in figure 16. It can achieve the same performance with the fixed PID controller under no-load condition. The response with load existed is shown in figure 17. The dynamic response is almost the same as the result of no-load. It also approves that the experienced self-tuning PID controller has the robustness to parameter variation and external load changed.



(a) Response of Translation Position (b) Response of Angular

Fig 16 Experimental Response of Self-Tuning PID Controller With No-Load



(a) Response of Translation Position (b) Response of Angular

Fig 17 Experimental Response of Self-Tuning PID Controller With 7.5 Kg External Load

6. Conclusion

This paper proposed a self-tuning approach for PID controllers. Based on the experienced engineer, the nominal values and tuning ranges of PID parameters can be assigned. According to the common sense, an intelligent PID self-tuning rule is proposed in this paper. A PC-based controller is implemented and applied to the translation position control of a slider crank mechanism. Simulation and experimental results show that the proposed controller is more robust than the fixed PID controller.

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