

## Worksheet 14: Diagonalization of symmetric matrices

**Example 0.118.** The following matrices are all symmetric:

$$\begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix}, \quad \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

**Example 0.119.** Diagonalize the following symmetric matrices:

$$\begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

**Example 0.120.** Diagonalize the following symmetric matrices through **orthogonal matrices**:

$$\begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

**Example 0.121.** Determine the positive definiteness of each of the following symmetric matrices by finding their eigenvalues:

$$\begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}, \quad \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$$