San José State University

Math 263: Stochastic Processes

Welcome to the first class!

Dr. Guangliang Chen

Agenda

1. Introductions

2. Course overview

3. Course policies

Introductions

Instructor: Guangliang Chen, Associate Prof. of Statistics & Data Science, 2014-present

Education:

- BS Math, Univ. of Sci.& Tech. of China, Hefei, 2003
- PhD Applied Math, University of Minnesota, July 2009

Employment history:

- Duke University: Visiting Assistant Professor, 2009-2013
- Claremont McKenna College: Visiting Assist. Prof., 2013-2014

Now it is your turn

Please tell us

- **your name**,
- **program of study**,
- **academic year**, and
- **anything else about you that you want us to know**.

Overview of Math 263

- A graduate course covering the following topics
  - *Discrete time Markov chains*,
  - *Poisson processes*,
  - *Continuous time Markov processes*,
  - *Renewal theory*,
  - *Brownian motion and Gaussian processes*
- **Prerequisites**: Math 39 and 163 (each with a grade of B or better)

What is a stochastic process?

The word **stochastic** is jargon for **random**.

A stochastic process is a system which evolves in time while undergoing chance fluctuations.

We can describe such a system by defining a family of random variables, $\{X_t\}$, where X_t measures, at time t, the aspect of the system which is of interest.

For example, X_t might be the number of the tth generation offspring produced by an individual of certain species, or the number of customers in a queue at time t.

Def 0.1. Formally, a stochastic process is a collection of random variables

$$\{X_t\}_{t\in T} = \{X_t, t \in T\} = \{X(t)\}_{t\in T} = \{X(t), t \in T\},$$

that are

- indexed by elements of a set $T \subseteq \mathbb{R}$,
- defined on a common sample space Ω, and
- take values in the same set $S \subseteq \mathbb{R}$.

That is,

$$X_t \; : \; \Omega \mapsto S, \quad \text{for any } t \in T$$

In the above definition,

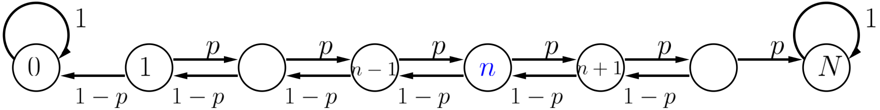
- T: index set, often interpreted as being time,
- S: state space (each element of S is called a state).

Remark. Both T and S can be discrete, or continuous:

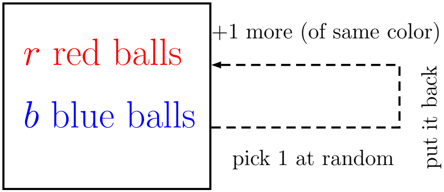
- The process is called a discrete-time process when T is a discrete set such as $\mathbb{Z}, \mathbb{Z}^+ = \mathbb{N}, \mathbb{Z}_0^+ = \mathbb{N} \cup \{0\}$, or a continuous-time process when T is an interval such as $\mathbb{R}^+ = (0, \infty)$ or $\mathbb{R}_0^+ = [0, \infty)$.
- The process is said to be integer-valued if $S = \mathbb{Z}$, or real-valued if $S = \mathbb{R}$.

A number of classical probability problems can be modeled as stochastic processes.

Example 0.1 (The Gambler's Ruin Problem)**.** Consider a gambler with n dollars in his pocket who repeatedly places 1 dollar bets. At each play, he either wins 1 dollar with probability p, or loses 1 dollar (with probability $1-p$). He will exit the game when he either is broke or has reached N dollars. Let X_t be the number of dollars the gambler has after t bets. Then $\{X_t\}$ is a discrete-time, integer-valued stochastic process.



Example 0.2 (Polya's Urn problem)**.** An urn contains r red and b blue balls. One ball is drawn randomly from the urn and its color observed; it is then returned in the urn, and an additional ball of the same color is added to the urn, and the selection process is repeated. Let X_t be the fraction of red balls in the url after t steps. Then $\{X_t\}$ is a discrete-time, real-valued stochastic process.



.

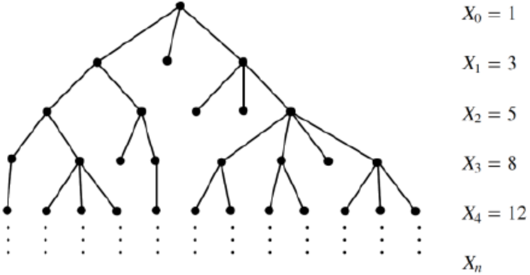
Stochastic processes have applications in even broader situations.

Example 0.3 (Branching Process)**.** Consider a population consisting of individuals able to produce offspring of the same kind. Suppose that each individual will, by the end of its lifetime, have produced j new offspring with probability $p_j, j \geq 0$, independently of the numbers produced by other individuals.

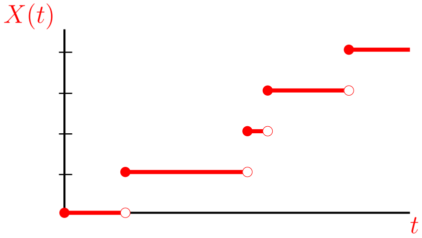
- The number of individuals initially present, denoted by X_0, is called the size of the zeroth generation.
- All offspring of the zeroth generation constitute the first generation and their number is denoted by X_1.

- In general, let X_n denote the size of the nth generation.

It follows that $\{X_n, n = 0, 1, \ldots\}$ is a stochastic process with state space $S = Z_0^+$.



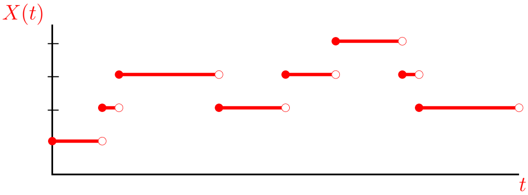
Example 0.4 (Poisson Process). Suppose that we have an infinite supply of light bulbs whose lifetimes are independent and identically distributed. Suppose also that we use a single light bulb at a time, and when it fails we immediately replace it with a new one. Under these conditions, $\{X(t), t \geq 0\}$ is a stochastic process where $X(t)$ represents the number of light bulbs that have failed by time t.



Poisson processes can be used to solve the following probability problem (we will see how to do it later in the course).

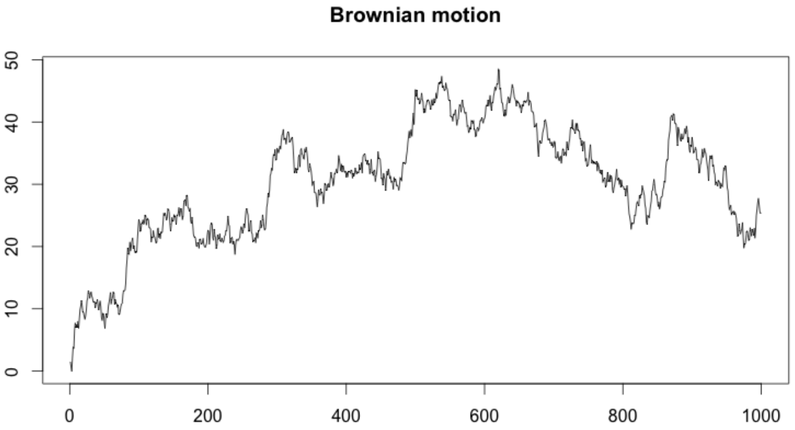
Example 0.5 (The Coupon Collecting Problem)**.** There are m different types of coupons. Each time a person collects a coupon (independently of the ones previously obtained), it is a type j coupon with (fixed) probability p_j (with $p_j > 0$ and $\sum p_j = 1$). Let N denote the total number of coupons one needs to collect in order to have a complete collection of at least one of each type. Find $\mathrm{E}[N]$.

Example 0.6 (Birth and death process)**.** Consider a system such as a country or a restaurant and let $X(t)$ be the number of people in the system at time t. Suppose that whenever there are n people in the system, then (i) new arrivals enter the system at an exponential rate λ_n, and (ii) people leave the system at an exponential rate μ_n. The above continuous-time stochastic process $\{X(t), t \geq 0\}$ is called a birth and death process, with arrival (or birth) rate $\{\lambda_n\}_{n=0}^{\infty}$ and departure (or death) rates $\{\mu_n\}_{n=1}^{\infty}$.

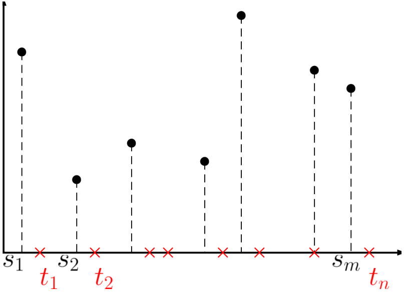


Example 0.7 (Brownian motion, or Wiener process)**.** A continuous time stochastic process $\{X_t\}_{t\geq 0}$ is called a Brownian motion if it satisfies the following conditions:

- $X_0 = 0$;
- $\{X_t\}_{t\geq 0}$ has independent and stationary increments;
- For every $t > 0$, $X_t \sim N(0, \sigma^2 t)$.


Brownian motion
0
10
20
30
40
50
0
200
400
600
800
1000

Example 0.8 (Gaussian processes regression)**.** Assume a set of observations $(s_i, x_i), i = 1, \ldots, m$ from some unknown function, predict the values of the function in new locations $t_i, i = 1, \ldots, n$.



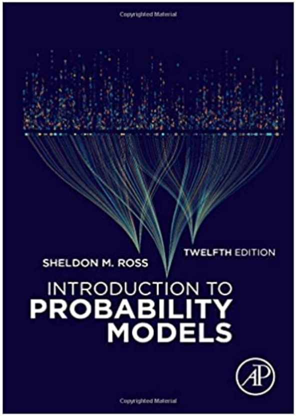
We will model the values of the function in both the old and new locations as jointly multivariate normal and determine the conditional distribution of $X(t_1), \ldots, X(t_n)$ given $X(s_1) = x_1, \ldots, X(s_m) = x_m$.

Textbook

Introduction to Probability Models, Sheldon M. Ross, Academic Press, 12th edition (March 9, 2019).

We will cover Chapters 4 - 7 and 10 of the textbook.

Older editions of the book are fine for reading, but homework will be assigned based on the 12th ed.



Learning management system

I will use **Canvas** in various ways:

- Post homework assignments and tests
- Record homework and test scores
- Make announcements (e.g. reminders, clarifications, deadline changes)

Make sure to check your Canvas settings to receive timely notifications. Also, check if your email address in record is still good.

Piazza

This term we will be using Piazza for class discussion. The system is highly catered to getting you help fast and efficiently from classmates and the instructor.

Rather than emailing questions to me, I encourage you to post your questions on Piazza. If you have any problems or feedback for the developers, email team@piazza.com.

Find our class signup link at:
`https://piazza.com/sjsu/spring2023/math263`

Course webpage

I am maintaining a course webpage[1] for posting the following information:

- Course syllabus
- Lecture slides (and other learning resources)

Please visit the webpage before each class to download the latest slides (try refreshing your browser if you don't see them).

[1] https://www.sjsu.edu/faculty/guangliang.chen/Math263.html

Course requirements

- **Homework (20%)**: Assigned regularly via Canvas.
- **Midterm 1 (25%)**: March 8, Wednesday, regular class time.
- **Midterm 2 (25%)**: April 19, Wednesday, regular class time.
- **Final exam (30%)**: May 23, Tuesday, 12:15 - 2:30pm.

Homework

Students may collaborate on homework but must independently write their own solutions (according to their own understanding).

You may write your work on paper or a tablet. Once completed, submit a legible, electronic copy to Canvas (as a single file attachment).

You must submit homework on time to receive full credit (late submissions within 24 hours of the due time can still be accepted but will automatically lose 10% of the total grade).

Your lowest homework score will be dropped.

What is allowed

Collaboration is encouraged on homework for the learning part. This includes (no acknowledgment needed in the following cases):

- Discuss homework questions together;
- Come up with a strategy/solution together;
- Help each other with certain question or step;
- Compare answers with each other

However, you must write your own solution independently (as homework is supposed to be individual work, not group work).

What is considered cheating

A few examples of cheating in completing a homework assignment:

- Copy other people's work partly or fully
- Use other people's work for your own submission (even with acknowledgment)
- Give your work to other people for copying or submission
- Copy solution found on the internet (even with acknowledgment).[2]

[2] However, you can study it and after you fully understand it, rewrite it independently using your own language.

Tests

The course has two midterms, each covering a distinct part of the course content, and a comprehensive final exam.

The three exams are all open book and open notes, but during each exam you are not allowed to communicate with other people or use the internet to search for answers.

No make-up exam will be given if you miss a midterm exam, unless you have a legitimate excuse such as illness or other personal emergencies and can provide documented proof.

Some reminders about homework and tests

You must show all steps to earn full credit:

- It is your entire work (in terms of *correctness*, *completeness*, and *clarity*) that is graded.
- Correct answers with no or poorly written supporting steps will be given very little credit.

Please **write legibly** (unrecognizable work will receive no credit).

Please submit a single file (in picture or pdf format).

Academic dishonesty

Students who are suspected of cheating in completing an assignment (homework or exam) will be referred to the Student Conduct and Ethical Development office, and depending on the severity of the conduct, will receive a zero on the assignment or even a grade of F in the course.

Grade cutoffs

...will be determined by combining the following **percentages**:

- A+: 95%, A: 91%, A-: 88%
- B+: 85%, B: 78%, B-: 75%
- C+: 72%, C: 68%, C-: 65%
- D+: 63%, D: 60%, D-: 58%
- F: 57% or less

and **the actual distribution of the class** at the end of the semester.

Your responsibilities in learning

My duty as an instructor is to disseminate knowledge while helping you learn. **The ultimate responsibility of learning is upon the student, not the instructor**. Thus, you should make efforts to

- Attend all classes
- Participate in classroom discussions
- Read the textbook before and after class
- Take time to think through the concepts
- Do your homework
- ASK whenever you don't understand something!!!

Special accommodations

If you anticipate needing any special accommodation during the semester (e.g., you have a disability registered with SJSU's Accessible Education Center), please let me know as soon as possible.

Instructor availability

- **Office hours**: Monday 10:30 - 11:30am, Tuesday 12:30 - 2pm, and by appointment (Thursday).
- **Piazza**: `piazza.com/sjsu/spring2023/math263`.
- **Email**: `guangliang.chen@sjsu.edu`. I check my emails frequently, but you should allow a turnaround time of up to 24 hours (on weekdays) or 48 hours (during weekends).

Student feedback

I strive to teach in the best ways to facilitate your learning. To achieve this goal, it is very helpful for me to receive timely feedback from you.

You can choose to

- talk to me in person, or
- send me an email, or,
- submit your feedback anonymously through `http://goo.gl/forms/f0wUD5aZSK`.

Questions?