

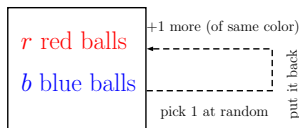
Worksheet 3: Conditional probability & independence

Example 0.26 (Toss two fair dice). Let $B = \{\text{Sum}=10\}$. Find $P(B)$.

Example 0.27 (cont'd). What if we are given that the two numbers are identical (event A)?

Example 0.28 (cont'd). Find $P(A | B)$. Is it equal to $P(B | A)$?

Example 0.29 (Polya's urn scheme). Suppose an urn initially has r red balls and b blue balls. A ball is drawn at random and its color noted. The it together with an extra ball of the same color (as the drawn ball) is put back into the urn. Now select a second ball at random. What is the probability that the two drawn balls are both red?



Example 0.30. Three cards are dealt from the top of a well-shuffled deck of 52 cards. What is the probability that they are all hearts?

Example 0.31 (Toss a single die). The sample space is $S = \{1, 2, 3, 4, 5, 6\}$. Which of the following are partitions of the sample space?

- $E_1 = \{1\}, \dots, E_6 = \{6\}$
- $A = \{1, 3, 5\}, A^c = \{2, 4, 6\}$
- $A = \{1, 2, 3\}, B = \{4, 5\}, C = \{6\}$
- $A = \{1, 3, 5\}, B = \{2, 4\}$
- $A = \{1, 3, 5\}, B = \{2, 4\}, C = \{5, 6\}$

Example 0.32 (Polya's urn scheme, cont'd). What is the probability that **the second drawn ball is red**?

Example 0.33 (Polya's urn scheme, cont'd). Find the probability that **the first drawn ball was red given that the second ball drawn is red**.

Example 0.34. Suppose that 65% of the defendants are truly guilty. Suppose also that juries vote a guilty person innocent with probability 0.2 whereas the probability that a jury votes an innocent person guilty is 0.1. Find the probability that a defendant is convicted. What percentage of convicted defendants are truly guilty?

Example 0.35. A card is selected at random from an ordinary deck of 52. If A denotes the event that the selected card is an ace, and B a spade. Are A, B independent?

Example 0.36. Suppose we draw two cards from an ordinary deck of 52, with replacement. Find the probability that both are diamonds. What if the two cards are drawn without replacement instead?

Example 0.37. Suppose toss 2 fair dice. Let $E = \{\text{Sum}=6\}$ and $F = \{\text{First}=4\}$. Determine if E, F are independent.

Example 0.38 (n coin tosses). Consider the experiment of tossing a coin n times independently. Suppose the probability of the coin landing on heads is p . Find the probability that (1) only the first k tosses are heads (2) at least one toss is a head.

Solution: Let

$$H_i = \{i\text{th toss is a head}\}, \quad i = 1, \dots, n.$$

Then H_1, \dots, H_n are (mutually) independent.