Chapter 19

Two Sample Problems

Conditions

- Two populations, independent samples
- Population means and standard deviations not known
  Population 1: μ_1, σ_1
  Population 2: μ_2, σ_2
- Use *t* procedures in inferential methods
- In theory, both populations Normal, but in practice, not necessary when *n* large -- see Ch 18

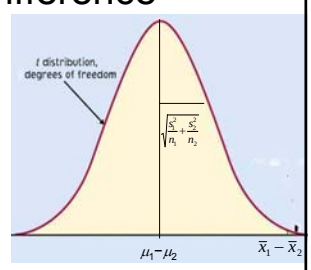
Before Using *t* Procedures

- Explore data with techniques from Ch 1 (e.g., side-by-side stemplots)
- Summarize data with techniques from Ch 2 (e.g., calculate x-bar, *s*, quartiles)
- Inference (CIs, H_0 tests) directed toward $\mu_1 - \mu_2$ based on sampling distribution of $\text{xbar}_1 - \text{xbar}_2$

Sampling Distribution of Mean Difference

- Figure shows sampling distribution of $\text{xbar}_1 - \text{xbar}_2$
- **Standard error** of $\text{xbar}_1 - \text{xbar}_2$ is:

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$



Two-Sample *t*-test

- H_0: $\mu_1 = \mu_2$ versus
  H_a: $\mu_1 \neq \mu_2$ (two-sided) or
  H_a: $\mu_1 > \mu_2$ (one-sided, right) or
  H_a: $\mu_1 < \mu_2$ (one-sided, left)
- Test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE_{\bar{x}_1 - \bar{x}_2}} \quad \text{where } SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

- Conservative estimate for **df =** the smaller of $(n_1 - 1)$ or $(n_2 - 1)$

Example: "Smoking Moms"

- Does parental smoking damage kid's lungs?
- Forced Vital Capacity (FVC) = physiologic measure of lung capacity (volume of air exhaled in 6 seconds)
- Measure VFC in exposed and nonexposed children

Group	xbar	*s*	*n*
Exposed	75.5	9.3	30
Non-exp	88.2	15.1	30

Hypothesis Test (Smoking Moms)

Hypotheses

H_0: $\mu_1 = \mu_2$ versus
H_a: $\mu_1 < \mu_2$ (one-sided) or
H_a: $\mu_1 \neq \mu_2$ (two-sided)

Test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{75.5 - 88.2}{\sqrt{\frac{9.3^2}{30} + \frac{15.1^2}{30}}} = \frac{-12.7}{3.238} = -3.922$$

$$df = n - 1 = 30 - 1 = 29$$

P-value (Smoking Moms)

- Use absolute value of *t* statistic (3.922)
- Table C, row for *df* = 29
- Largest *t** in that row = 3.659 (*P* = 0.0005)
- Therefore:
  P < 0.0005 (one-sided)
  P < 0.001 (two-sided)
- Highly significant evidence against H_0

C Level Conf. Int. for $\mu_1 - \mu_2$

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \cdot SE_{\bar{x}_1 - \bar{x}_2}$$

where

$$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

and

$$df = \text{the smaller of } (n_1 - 1) \text{ or } (n_1 - 1)$$

CI for "Smoking Mom's" Example

95% confidence for example, use *t** = 2.045

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{9.3^2}{30} + \frac{15.1^2}{30}} = 3.238$$

$$\begin{aligned}(\bar{x}_1 - \bar{x}_2) \pm (t^*)(SE) &= (75.5 - 88.2) \pm (2.045)(3.238) \\ &= -12.7 \pm 6.6 \\ &= (-19.3, -6.1)\end{aligned}$$

CI for "Smoking Mom's" Example

90% confidence for example, use *t** = 1.699

$$\begin{aligned}&(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \\ &= (75.5 - 88.2) \pm (1.699)\sqrt{\frac{9.3^2}{30} + \frac{15.1^2}{30}} \\ &= -12.7 \pm 5.5 \\ &= (-18.2, -7.2)\end{aligned}$$

Which type of test? One-sample? paired samples? two samples?

- **Comparing vitamin content of bread, immediately after baking versus 3 days later (the same loaves are used on day one and 3 days later).**
- **Comparing vitamin content of bread, immediately after baking versus 3 days later (tests made on independent loaves).**
- **Average fuel efficiency for 2005 vehicles is 21 miles per gallon. Is average fuel efficiency higher in the new generation "green vehicles?"**
- **Is blood pressure altered by use of an oral contraceptive? Comparing a group of women not using an oral contraceptive with a group taking it.**
- **Review insurance records for dollar amount paid after fire damage in houses equipped with a fire extinguisher versus houses without one. Was there a difference in the average dollar amount paid?**