Basic Biostatistics

Statistics for Public Health Practice

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Chapter 12: Comparing Independent Means

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In Chapter 12:

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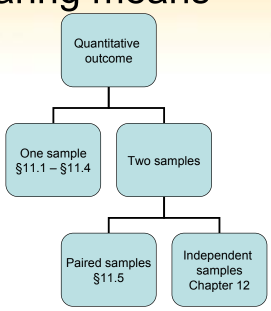
Types of Samples when comparing means

- **Single sample**. One group; no concurrent control group
- **Paired samples.** Data points in samples uniquely matched
- **Two independent samples**. Separate groups (no matching or pairing)



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Comparing Two Populations or Two Treatments

- "Comparative studies"
- One of the most common statistical situations
- Can arise
  - Experimentally (e.g., randomized trial)
  - Non-experimentally (various study designs)

"Two Groups" by Pieter Bruegel the Elder (c. 1525 – 1569)

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Example: Cholesterol & Personality Type

Do cholesterol levels (mg/dl) differ by personality type?

Type A personality men (n = 20)
233, 291, 312, 250, 246, 197, 268, 224, 239, 239, 254, 276, 234, 181, 248, 252, 202, 218, 212, 325

Type B personality men (n = 20)
344, 185, 263, 246, 224, 212, 188, 250, 148, 169, 226, 175, 242, 252, 153, 183, 137, 202, 194, 213

* Data set WCGS.sav documented on p. 49 in the text.

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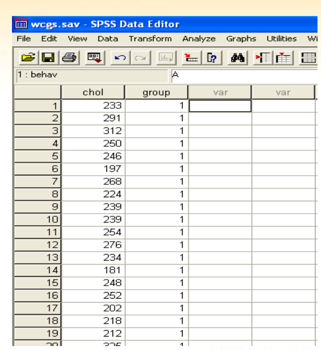
Data Table

- One column for the response variable (chol)
- One column for the explanatory variable (group)



	chol	group	var	var
1	233	1		
2	291	1		
3	312	1		
4	250	1		
5	246	1		
6	197	1		
7	268	1		
8	224	1		
9	239	1		
10	239	1		
11	254	1		
12	276	1		
13	234	1		
14	181	1		
15	248	1		
16	252	1		
17	202	1		
18	218	1		
19	212	1		

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EDA

- Explore and compare the distributions before inferences
- EDA techniques
  - Side-by-side stemplots (right)
  - Side-by-side boxplots (next slide)

```
Group 1 |  | Group 2
-------------------
        |1t|3
        |1f|45
        |1s|67
      98|1.|8889
     110|2*|011
   33332|2t|22
   55544|2f|4455
      76|2s|6
       9|2.|
      21|3*|
        |3t|
        |3f|4
       (×100)
```

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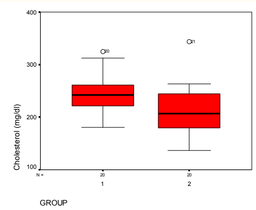
Side-by-Side Boxplots



- Location: group 1 > group 2
- Spreads: comparable
- Shapes: fairly symmetrical (although outside values are of some concern)

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Notation

Parameters (population)

Group 1	N_1	μ_1	σ_1
Group 2	N_2	μ_2	σ_2

Statistics (sample)

Group 1	n_1	$\bar{x}_1$	s_1
Group 2	n_2	$\bar{x}_2$	s_2

$\bar{x}_1 - \bar{x}_2$ is the point estimator of $\mu_1 - \mu_2$

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Example: Summary Statistics

Group	*n*	mean	std dev
1	20	245.05	36.64
2	20	210.30	48.34

Use computer or calculator to help calculate summary statistics

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Standard Error of Mean Difference

How precisely does $\bar{x}_1 - \bar{x}_2$ estimate $\mu_1 - \mu_2$?

$$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

For the data example:

$$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{36.638^2}{20} + \frac{48.340^2}{20}} = 13.563$$

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Degrees of Freedom

Two ways to estimate **degrees of freedom**:

- df_{Welch} [formula on p. 244, but use computer]
- $df_{\text{conserv.}}$ = the smaller of $(n_1 - 1)$ or $(n_2 - 1)$

For the illustrative data:

df_{Welch} = 35.4 (via SPSS)

$df_{\text{conserv.}}$ = smaller of $(n_1 - 1)$ or $(n_2 - 1)$

= 20 – 1 = 19

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(1 – α)100% CI for μ1–μ2

$$(\bar{x}_1 - \bar{x}_2) \pm (t_{df,1-\frac{\alpha}{2}})(SE_{\bar{x}_1-\bar{x}_2})$$

Note consistency with other CI formulas:

(point estimate) ± (z or t)(SE)

Also note:
point estimate ± margin of error

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Comparison of CI Formulas

$$(\text{Point estimate}) \pm (t_{df,1-\frac{\alpha}{2}})(SE)$$

Type of sample	Point estimate	*df* for *t**	*SE*
Single	$\bar{x}$	$n-1$	$\frac{s}{\sqrt{n}}$
Paired	$\bar{x}_d$	$n-1$	$\frac{s_d}{\sqrt{n}}$
Independent	$\bar{x}_1 - \bar{x}_2$	smaller of n_1-1 or n_2-1	$\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$

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Example

$\bar{x}_1 - \bar{x}_2 = 245.05 - 210.30 = 34.75$

$SE_{\bar{x}_1-\bar{x}_2} = 13.563$ (prior slide)

$df_{\text{conserv}} = 20 - 1 = 19$

For 95% confidence ($\alpha = .05$) use $t_{19,1-\frac{.05}{2}} = t_{19,.975} = 2.093$ From Table C

$$\text{95\% CI for } \mu_1 - \mu_2 = (\bar{x}_1 - \bar{x}_2) \pm (t_{19,.975})(SE_{\bar{x}_1-\bar{x}_2})$$
$$= 34.75 \pm (2.093)(13.563)$$
$$= 34.75 \pm 28.38$$
$$= (6.4 \text{ to } 63.1) \text{ mg/dL}$$

Likely mean difference in mean cholesterol, Type A and Type B *populations*

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Interpretive Notes

- The CI is aiming for $\mu_1 - \mu_2$ (NOT $\text{xbar}_1 - \text{xbar}_2$)
- Recall the meaning of "confidence," i.e., based on a repeated sampling *model*
- Recall from prior chapters: the CI can be used to address a null hypotheses (covered next)

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Hypothesis Test

- Objective: To assess the "significance" of the observed difference between two sample means
- Note: widely different means can arise just by chance
- The null hypothesis starts with an assumption of no difference:
  H_0: $\mu_1 - \mu_2 = 0$; equivalently H_0: $\mu_1 = \mu_2$
- The alternative hypothesis can be
  H_a: $\mu_1 \neq \mu_2$ (two-sided) or
  H_a: $\mu_1 > \mu_2$ ("right-sided") or
  H_a: $\mu_1 < \mu_2$ ("left-sided")

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Two-Sample *t* Statistic

$$t_{\text{stat}} = \frac{(\bar{x}_1 - \bar{x}_2)}{SE_{\bar{x}_1-\bar{x}_2}} \text{ where } SE_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

df_{Welch} or df_{conserv} (see previous slide)

$$t_{\text{stat}} = \frac{(\bar{x}_1 - \bar{x}_2)}{SE_{\bar{x}_1-\bar{x}_2}} = \frac{34.75}{13.563} = 2.56$$

$$df_{\text{conserv}} = 20 - 1 = 19$$

Note: Statisticians use two different two-sample *t* statistics. This one does *not* assume equal variance.

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P-value

- Use Table C or an Applet to determine the tail area beyond the observed $|t_{stat}|$ (one-tailed *P*-value).
- Double this for the two-tailed *P*
- Two-tailed *P*-value for t_{stat} of 2.56 with 19 *df* is .019

StaTable for Java (Version 1.0.2)

Distribution: Continuous Student's t

t	2.56
df	19.0
Left-tail	0.990426
Two-tail	0.019148

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SPSS Output

T-Test

Group Statistics

	GROUP	N	Mean	Std. Deviation	Std. Error Mean
Cholesterol (mg/dl)	1	20	245.05	36.638	8.193
	2	20	210.30	48.340	10.809

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means					95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Cholesterol (mg/dl)	Equal variances assumed	1.293	.263	2.562	38	.014	34.75	13.563	7.293	62.20
	Equal variances not assumed			2.562	35.413	.015	34.75	13.563	7.227	62.27

Equal variance *t* procedure (§12.4)

Equal Variance Not Assumed
Preferred method (§12.3)

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Interpretation

- Recall: smaller-and-smaller P-values provide stronger-and-stronger evidence against H_0.
- Recall these conventions:
- .10 < *P* < 1.0: evidence against H_0 is not significant
- .05 < *P* ≤ .10: evidence against H_0 is marginally signif.
- .01 < *P* ≤ .05: evidence against H_0 is significant
- *P* ≤ .01: evidence against H_0 is highly significant
- The current two-tailed *P*-value of .019 provides good evidence against H_0; the observed difference is thus significant.

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Summary of independent *t* test

A. H_0: $\mu_1 - \mu_2 = 0$ vs. H_a: $\mu_1 - \mu_2 \neq 0$

B. Test statistic

$$t_{\text{stat}} = \frac{(\bar{x}_1 - \bar{x}_2)}{SE_{\bar{x}_1 - \bar{x}_2}} \text{ where } SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$df_{\text{Welch}} \text{ or } df_{\text{conserv}}$$

C. One-tailed *P* = Pr(T ≥ $|t_{stat}|$)
Two-tailed *P* = 2 × one-tailed *P*

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Hypothesis Test with the CI

- H_0: $\mu_1 - \mu_2 = 0$ vs. H_a: $\mu_1 - \mu_2 \neq 0$
- Can be tested at the α-level of significance with the (1 – α)100% CI
- For the illustrative example:
  - The 95% CI for $\mu_1 - \mu_2$ is (6.4 to 63.1). This excludes $\mu_1 - \mu_2 = 0$. Therefore, data are significant at α = .05
  - The 99% CI for $\mu_1 - \mu_2$ is (-2.2 to 71.7). This includes $\mu_1 - \mu_2 = 0$. Therefore, data are not significant at α = .01

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§12.5 Sample Size & Power

- Sample size and power for two-sample problems about mean can be address from these perspectives
- Sample size required for a CI to achieve margin of error *m* (pp. 250 – 1)
- d need to achieve a given margin of error
- Sample size to achieve given power (pp. 251 – 2)
- Minimal detectable difference (pp. 252 – 3)
- Power of a given test (pp. 252 - 3)

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