

[1/2] REVIEW QUESTIONS

Exercises UNIT 5: INTRO TO ESTIMATION

NOT GRADED

5.1 Parameter or estimate? Say whether each of the boldface numbers is either a parameter or an estimate. [Hint: Before providing an opinion, clearly state the implied research question.]

(A) A study of survival in 1225 newly diagnosed breast cancer cases finds that survival varies greatly by stage of diagnosis. The average seven-year survival rates for Stage I breast cancer was 92%; the Stage II survival rate was 71%; the stage III survival rate was 39%; and the stage IV survival rate was 11%. ESTIMATES

(B) A review of divorce records for a county in Connecticut indicates that the marriages that end in divorce last an average of 72 months. DEPENDS ON USE; IF USED TO GENERALIZE BEYOND CONNECTICUT -> ESTIMATE

(C) For U.S. men, the average life expectancy is 76. For women, it's 81. [Life expectancy is based on data from the National Death Index; deaths are uniformly recorded in the U.S.] PARAMETERS

NOT GRADED

5.2 Sampling experiment. Let us conduct a small, artificial sampling experiment that will demonstrate what is meant by the sampling distribution for a mean (SDM). A tiny finite population consists of the following values: {1, 3, 5, 7, 9}. This population has  $\mu = 5$  and  $\sigma = 2.828$ .

DONE IN CLASS 3/23/16 - SEE ONLINE KEY FOR FULL RESPONSE

(A) Construct a stemplot of the population.

(B) List of all possible unique samples each of  $n = 2$  from this population. For example, the first sample can be {1, 3}. The second such sample can be {1, 5}. There 10 such unique samples.

(C) Calculate the mean of each of the 10 samples.

(D) Construct a stemplot of the 10 sample means. This is an experimental sampling distribution of the mean (SDM).

(E) Is the SDM more or less normal shaped than the population? This is a demonstration of the central limit theorem, which states that the SDM tend to be more normal than the population. This is particularly true with when the sample size is large. Although the sample size in this experiment is small, you should still a trend toward the normalization of the SDM.

(F) Calculate the mean of the SDM (i.e., the mean of the means). Is the mean of the SDM equal to, less than, or greater than the mean of the population? This is a demonstration of the unbiasedness of the x-bar as a reflection of  $\mu$ .

(G) Use your calculator to calculate the standard deviation of the SDM (i.e., the standard deviation of the means is analogous to the SEM). Report both the  $s$  and  $\sigma$  derived by your calculator. Are these standard deviations equal to, less than, or greater than  $\sigma$  in population (which is 2.828)? This demonstrates that the SEM is smaller than the  $\sigma$ .

NOT GRADED

5.3 Health survey. A survey takes a simple random sample of 500 people from a town of 55,000. On the average, it finds 2.30 health problems per person (standard deviation = 1.65). Say whether each of the following statements is true or false. Explain your reasoning in each instance.

(A) The standard deviation of the mean (i.e., the SEM) is 0.0738. TRUE.

(B) The 95% confidence interval for the average number of health problems in the sample is (2.16, 2.44). FALSE

(C) The 95% confidence interval for the average number of health problems in the town is (2.16, 2.44). TRUE.

(D) While the number of health problems in the population is not normally distributed, according to the central limit theorem (Exercise 5.2E) it reasonable to assume that the sampling distribution of the mean (SDM) will be normal. TRUE.

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5.4 Lab instrument. Measurements from a particular lab instrument has known standard deviation  $\sigma = 10$ . A lab assistant takes 4 measurements with this instrument and calculates the mean of the four samples.

(A) Explain the advantage using the average of the four measurements rather than each individual measurement as the true value for a sample. MEANS ARE MORE PRECISE REFLECTIONS OF  $\mu$  THAN INDIVIDUAL MEASUREMENTS.

(B) Calculate the standard error of the mean when  $n = 4$ . SEM =  $10/\sqrt{4} = 5$

(C) How many times must the assistant repeat the measurement to reduce the standard error of the mean to 2.5? [Hint: Rearrange the formula  $SEM = \sigma / \sqrt{n}$  to get  $n = (\sigma / SEM)^2$  to determine the required sample size].

$n = (10/2.5)^2 = 16$

$$SEM = \frac{7}{\sqrt{30}} = 1.278$$

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**5.5 Pharmacy survey.** A survey of 30 pharmacies found that the average cost of a month's supply of a particular drug was \$33. The standard deviation of the cost of the prescription is assumed to be \$7.

- (A) Based on the data, determine the mean price  $\mu$  with 90% confidence for the Rx from all pharmacies.  $33 \pm 1.64 \cdot 1.278 = 33 \pm 2.10 = (30.90, 35.10)$
- (B) Now determine  $\mu$  with 95% confidence.  $= 33 \pm 1.96 \cdot 1.278 = 33 \pm 2.50 = (30.50, 35.50)$
- (C) What is the margin of error for 90% confidence? What is the margin of error for 95% confidence?  $\$2.10$  &  $\$2.50$
- (D) A pharmacist reads that a 95% confidence interval for the average price is \$30.50 to \$35.50. Asked to explain the meaning of this, the pharmacist states "95% of all pharmacies sell the drug for between \$30.50 and \$35.50." Is the pharmacist correct? Explain your response. **WRONG. THE CI APPLIES TO  $\mu$ , NOT INDIVIDUAL VALUES.**

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**5.6 Laboratory scale.** A manufacturer of a laboratory scale with a digital readout claims the scale is accurate to 0.0015 of a gram. You read the fine print in the documentation that accompanies the scale and find that the manufacturer means to say that the weights will have standard deviation  $\sigma = 0.0015$  grams. You are willing to assume that the measurement error varies according to a normal pdf with a mean equal to the true weight  $\mu$  of an object. Duplicate measurements on an object produce weights of 24.31 and 24.34 grams. Estimate the true weight of the objects with 95% confidence.  $95\% \text{ CI for } \mu = 24.325 \pm 1.96 \cdot 0.00106 = 24.325 \pm 0.0021 = (24.323, 24.327)$

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**5.7 Hemoglobin survey, sample size.** Hemoglobin levels in 11-year old boys have a normal distribution with unknown mean  $\mu$  and  $\sigma = 1.209$  g/dl. How large a sample is needed to estimate  $\mu$  with 95% confidence and a margin of error of 0.5?  $n = \left(\frac{1.96 \cdot 1.209}{.5}\right)^2 = 22.46 \rightarrow 23$

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**5.8 Sugar consumption survey, sample size.** A nutritionist is willing to assume that the standard deviation of the weekly sugar consumption in children is 100 grams. How large a sample is needed to calculate a 95% confidence interval for  $\mu$  so that its margin of error is no greater than 10 grams?  $n = \left(\frac{1.96 \cdot 100}{10}\right)^2 = 384.16 \rightarrow 385$

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**5.9 Birth weights.** Assume the birth weights of newborns in a population of full-term infants varies normally with  $\sigma = 2$  pounds. Random samples of size  $n$  are selected from this population of birth weights of full term infants. Calculate 95% confidence intervals for  $\mu$  based on each of the following samples. State the margin of error in each instance.

	SEM	m	(LCL, UCL)
(A) $n = 81$ , sample mean = 6.2 pounds	.2222	0.436	(5.8, 6.6)
(B) $n = 36$ , sample mean = 7.0 pounds	.3333	0.653	(6.3, 7.7)
(C) $n = 9$ , sample mean = 5.8 pounds	.6667	1.307	(4.5, 7.1)

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**5.10 AIDS risk factor study.** A national survey of AIDS risk factors based on a random sample of 2673 heterosexual adults found the 170 individuals reported two or more sexual partners in the past 12 months. (Assume the responses are truthful.)

- (A) Describe the population to which inferences will be made. Describe the parameter to be estimated. **VS HETEROSEXUALS @ TIME OF SURVEY**
- (B) Calculate the point estimate for the prevalence of this risk factor.  $\hat{p} = \frac{170}{2673} = .0636$
- (C) What is the parameter of interest? Can the large sample method be used to calculate confidence intervals for the prevalence of this risk factor?  $n\hat{p}\hat{q} = 2673 \cdot .0636 \cdot (1 - .0636) = 159 \rightarrow \text{"OK"}$
- (D) Calculate a 95% CI for the prevalence of multiple sexual partners in this population.  $SE\hat{p} = \sqrt{\frac{.0636 \cdot (1 - .0636)}{2673}} = .00472$
- (E) Calculate a 90% CI for the prevalence.  $.0636 \pm 1.64 \cdot .00472 = .0636 \pm .0077 = (.056, .071)$   $= .0636 \pm .0093 = (.054, .073)$

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**5.11 Patient preference.** A doctor finds that 7 out of 8 of her patients preferred to have a "finger stick" blood sample compared to a venous blood draw. Can you use the large sample method to determine the proportion of patients preferring the finger-stick method with 95% confidence using this data? Justify your response with the "npq rule."  $n\hat{p}\hat{q} = 8 \cdot .875 \cdot (1 - .875) = 0.875 \rightarrow \text{NOT OK}$

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**5.12. Sample size for a proportion.** You are planning a study that wants to estimate a population proportion with 95% confidence. How many individuals do you need to study to achieve a margin of error of no greater than .06? A reasonable estimate for the population proportion is not available before the study is begun so you assume  $p^* = 0.50$ . How large a sample is needed to cut the margin of error to .03?

$$n = \frac{z^2 p^* q^*}{m^2} = \frac{1.96^2 \cdot .5 \cdot .5}{.06^2} = 266.78 \rightarrow 267$$

$$n = \frac{1.96^2 \cdot .5 \cdot .5}{.03^2} = 1067.11 \rightarrow 1068$$

+5 FREE POINTS IF HAND IN ON TIME

# 5: Introduction to Estimation

## Review Questions

1. Define "statistical inference." GENERALIZING FROM DATA TO LARGER PHENOMENON W/ CALCULATED CERTAINTY
2. Name the two primary forms of statistical inference. ESTIMATION; NHTS
3. Name the two forms of statistical estimation. POINT ESTIMATION; CONFIDENCE INTERVALS
4. Although parameters and estimators are related, they are *not* the same. List ways they differ.
5. What does it mean when we say that the sample mean is an unbiased estimator of  $\mu$ ? ESTIMATES FROM DATA ARE RANDOM VARIABLES; PARAMETERS NOT.
6. A particular random sample of  $n$  observations can be used to calculate a sample mean. We can determine the characteristics of the distribution of means derived by other samples of the same size taken from the same populations under identical conditions without actually taking additional samples. The theorem that postulates that the sampling distribution of the mean (SDM) will tend to be normal (Gaussian) is called the central limit theorem. This SDM will have a mean of  $\mu$  and standard deviation that is equal to  $SEM = \sigma/\sqrt{n}$  [formula].
7. The standard deviation of the SDM is called the SEM.
8. When the SDM is normal (Gaussian), 95% of the sample means will fall within 1.96 SEMs of  $\mu$ .
9. True or false? The square root law says that the SEM is inversely proportional to the square root of the of the sample size.
10. Match the definitions below with each of the following terms: alpha, statistical inference, parameter, confidence interval, sampling distribution of the mean, standard error of the mean
  - (a) A numeric constant that describes something about a pmf, pdf, or statistical population. PARAMETER
  - (b) The act of generalizing from the data to a natural phenomenon or population with calculated degree of certainty STATISTICAL INFERENCE
  - (c) The hypothetical frequency distribution of all possible sample means based on samples of size  $n$  taking from the same population under identical conditions. SAMPLING DISTRIBUTION OF MEAN (SDM)
  - (d) The chance researcher is willing to take in not capturing the parameter with an estimate.  $\alpha$
  - (e) The standard deviation of the sampling distribution of the mean. STANDARD ERROR OF MEAN (SEM)
  - (f) An interval that is calculated so that it has known probability of capturing the parameter. CONFIDENCE
11. What percentage of 95% confidence intervals for  $\mu$  will fail to capture  $\mu$ ? 5%
12. What percentage of (1-alpha)100% confidence intervals for  $\mu$  will fail to capture  $\mu$ ?  $\alpha \cdot 100\%$
13. The CI for the mean seeks to capture the POPULATION mean. [M/C: (a) sample (b) population].
14. A confidence interval is  $3 \pm 1.2$ . The 3 in this statement is the POINT ESTIMATE and the 1.2 is the MARGIN OF ERROR.
15.  $\bar{x}$  is the point estimator of  $\mu$ .
16.  $\hat{p}$  is the point estimator of  $p$ .
17.  $s$  is the point estimator of  $\sigma$ .
18. The formula for calculating confidence intervals for  $p$  presented in this CHAPTER should only be used in large samples as tested by this "rule."  $n\hat{p}\hat{q} > 5$  ← PRECISION OF  $\hat{p}$  AS ESTIMATE OF  $p$
19. What does SEP stand for? What does it measure? How do we estimate it (i.e., what's the formula)? STANDARD ERROR OF PROPORTION  $SEP = \sqrt{\frac{\hat{p}\hat{q}}{n}}$
20. The sample size requirements for estimating a mean with confidence depends on these things.  $m; \sigma; 1-\alpha$
21. The sample size requirements for estimating a proportion with confidence depend on these things.  $m; p^*; 1-\alpha$
22. Increasing the sample size will INCREASE the precision of the estimate by DECREASES the confidence interval length.
23. Requiring increased confidence will INCREASE the margin of error.

STANDARD ERROR OF PROPORTION