DUE 3/21

4: Probability Review Questions and Exercises

KEY

Review Questions

1. Define "probability." EXPECTED RELATIVE FREQUENCY IN LONG RUN
2. What is a random variable? A VAR THAT TAKES ON VALUES DEPENDING ON CHANCE
3. What are the two distinct types of random variables? DISCRETE, CONTINUOUS
4. What is a discrete random variable? AN RV THAT TAKES ON A FINITE SET OF POSSIBLE OUTCOMES
5. What is a continuous random variable? AN RV WITH AN INFINITE CONTINUUM OF POSSIBLE OUTCOMES
6. What is a binomial random variable? THE COUNT OF SUCCESSES IN n INDEPENDENT BOURNOULLI TRIAL EACH WITH SAME p
7. What does pmf stand for? PROBABILITY MASS FUNCTION
8. What is a pmf? ASSIGNS A PROBABILITY FOR EVERY POSSIBLE OUTCOME FOR A DISCRETE RANDOM VAR
9. Binomial pmfs have two parameters. Name these. n & p
10. What does the symbol *X* represent in this statement *X*~b(*n*,*p*)? What does the symbol ~ represent? What does *n* represent? What does *p* represent? X = THE RV; n & p = PARAMETERS; "DISTRIBUTED AS"
11. What is a Bernoulli trial? A SINGLE TRIAL WITH ONLY TWO POSSIBLE OUTCOMES
12. What does *X* represent in the statement Pr(*X* = *x*)? What does *x* represent? RANDOM VAR X; VALUE x
13. How do you read the statement? "Pr(*X* = *x*)" THE PROBABILITY RANDOM VAR TAKES ON A VALUE OF x.
14. How do you read the statement? "*X*~b(*n*,*p*)" RANDOM VAR X IS DISTRIBUTED AS A BINOMIAL WITH PARAMETERS n & p.
15. Pr(*X* ≤ *x*) represents the CUMULATIVE probability of *x*.
16. Probabilities can be no less than 0 and no more than 1.
17. The probability of all possible outcomes for a random variable sum exactly to 1.
18. What does it mean to say that two events are disjoint? MUTUALLY EXCLUSIVE
19. If two events are disjoint, then Pr(A U B) = Pr(A) + Pr(B).
20. What is the complement of an event? EVENT NOT HAPPENING
21. Pr(Ā) = 1 – Pr(A)
22. What does AUC stand for? AREA UNDER CURVE
23. The AUC in a pmf and pdf graph is equal to the PROBABILITY of those occurrences.
24. Probabilities for discrete random variables are assigned via pmfs. Probabilities for continuous random variables are assigned via pdf's.
25. What does pdf stand for? PROBABILITY DENSITY FUNCTION
26. Normal pdfs have two parameters. Name these. μ & σ
27. Normal pdfs are centered on μ.
28. The spread of a normal pdf is controlled by its σ
29. What is an inflection point? CHANGE IN SLOPE
30. Normal curves have inflection points that are one σ above and below μ.
31. How many different normal pdfs are there? ∞
32. 68% of the AUC for a normal curve is within μ ± σ; 95% is within μ ± 2σ; 99.7% is within μ ± 3σ.
33. The total AUC OF A pdf CURVE sums exactly to 1.
34. The AUC between any two points on a normal pdf corresponds to the PROBABILITY of values within that range.
35. The AUC to the left of a point on a normal pdf corresponds to the CUMULATIVE probability of that value.

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Exercises

4~~3~~.1 Tay-Sachs. Tay-Sachs is a metabolic disorder that is inherited as an autosomal recessive trait. Both recessive alleles are necessary for expression of the disease. Therefore, when each parent is a carrier, there is a 1 in 4 chance of transmitting the genetic disorder to each offspring.

(A) Let X represent the number of offspring affected in 4 consecutive conceptions from Tay-Sachs carrier parents. Is X a binomial random variable? Explain your response. Use notation to express the pmf for random variable X. $X \sim b(4, .25)$

(B) Complete this table for the binomial pmf for this variable. (Use the app.)

x	Pr(X = x)	Pr(X ≤ x)
0	.3164	.3164
1	.4219	.7383
2	.2109	.9492
3	.0469	.9961
4	.0039	1.0000

(C) What is the probability the couple has no offspring with the Tay-Sachs trait in the 4 pregnancies? .3164

(D) What is the probability the couple has at least one offspring with the Tay-Sachs trait? 1-.3164=.6836

ONE OR MORE

4~~3~~.2 Breast cancer. The lifetime risk of developing breast cancer in women is 1 in 10. Let X represent the number of women in a group of 21 women who ultimately develop breast cancer.

(A) Use notation to express this binomial pmf. $X \sim b(21, .1)$

(B) Build the first part of the pmf by filling in this table.

x	Pr(X = x)	Pr(X ≤ x)
0	.1094	.1094
1	.2553	.3647
2	.2837	.6484
3	.1996	.8480
4	.0998	.9478
5	.0377	.9856
6	.0112	.9967
7	.0027	.9994
8	.0005	.9999
9	.0001	1.0000
10	0.0000	1.0000
etc....	etc.	etc.
21	0.0000	1.0000

(C) What is the probability of no (0) cases among the 21 women? .1094

(D) What is the probability of seeing 5 or fewer cases? .9856

4 ~~3~~.3 **Gestation.** Assume that gestation in humans is normally distributed with a mean of 280 days after the last menstrual period with a standard deviation of 13 days.

(A) Create a sketch of the normal curve depicting this pdf. Mark the horizontal axis with values (days) that are one standard deviation above and below the mean. Locate the points of inflection on the curve as accurately as possible. SEE BELOW

(B) Use the 68-95-99.7 rule to determine the middle 68% of gestation periods. What percent of values will fall below this range? $\mu \pm \sigma = 280 \pm 13 = (267, 293)$ $Pr(X \leq 267) \approx 16\%$ BY 68-95-99.7 RULE

(C) Use the 68-95-99.7 rule to determine the middle 95% of gestation periods. What percent of values will fall above this range? $\mu \pm 2\sigma = 280 \pm 2 \cdot 13 = (254, 306)$ → 2.5% BY 68-95-99.7 RULE

4 ~~3~~.4 **Tall man.** A man who is 6'7" tall (79 inches) is actually only 13% taller than a man who is 5'10" (70 inches). So why does the man who is 6'7" appear so tall? Let us assume that men's height is normally distributed with a mean of 70 inches and standard deviation of 3 inches.

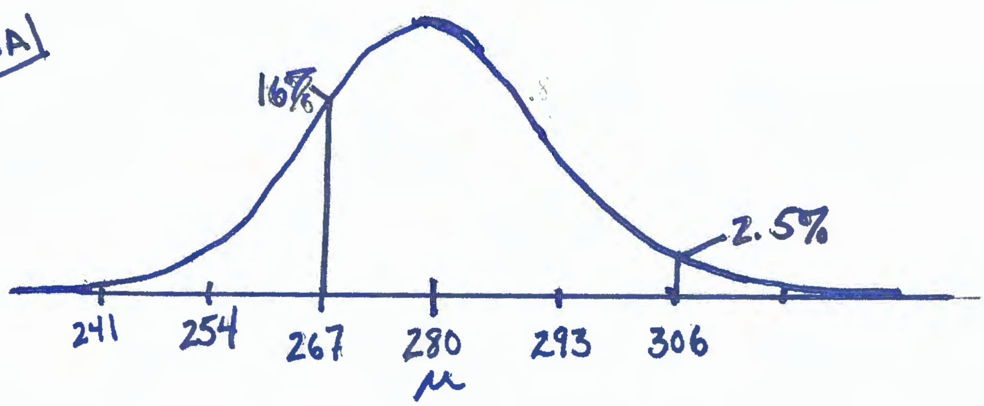
(A) Use notation to express the pdf of men's height. $X \sim N(70, 3)$

(B) If selecting a man at random from the population, what is the probability of encountering a man that is 5'10" or taller? (Use the normal pdf app.) $Pr(X \geq 70) = .5$

(C) What is the probability of encountering a man that is 6'7" or taller? $Pr(X \geq 79) = .0013$

(D) How does rarity affect our perception of height? RARE EVENTS ARE QUITE SUPRISING!

4.3A)



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~~3~~4.5 Warriors. The probability that the 2016 Warriors basketball team wins a game on any given night is 90%. Consider 10 consecutive games. (Use the app in each instance.) $X \sim b(10, .9)$

(A) In playing the 10 games, what is the probability the warriors win all 10? $Pr(X=10) = .3487$

(B) What is the probability they lose 1 or more? $Pr(X \leq 9) = .6513$

(C) What is the probability they lose exactly 1 game (i.e., win 9 out of 10)? $Pr(X=9) = .3874$

~~3~~4.6 Linda's omelet. Linda hears a story on National Public Radio stating that one in six eggs in the United States are contaminated with *Salmonella*. If Salmonella contamination occurs randomly within and between egg cartons and Linda makes a three-egg omelet, what is the probability that her omelet will contain at least one Salmonella-contaminated egg? (Hint: Use proper notation to denote the binomial pmf describing the number of contaminated eggs in the omelet and then determine Pr(X ≥ 1).) $X \sim b(3, 1/6)$ ↑ .1667

$Pr(X \geq 1) = 1 - Pr(X=0) = 1 - .5786 = .4214$

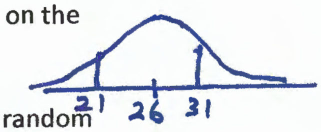
~~3~~4.7 Low IQ. A death row inmate in the state of Illinois has a Wechsler adult intelligence score of 51. Recall that Wechsler adult intelligence scores are normally distributed with a mean of 100 and standard deviation of 15. What percentage of adults have score of less than or equal to 51? (Use the app.)

$X \sim N(100, 15)$ $Pr(X \leq 51) = .0005$

~~3~~4.8 BMI. Body mass index is weight in kilograms divided by height in meters squared. This provides a height adjusted index of body weight by which to judge whether an individual is underweight, overweight, or obese. Suppose a community survey finds that the mean BMIs of women between the ages of 40 and 49 is approximately normally distributed with a mean of 26 and standard deviation of 5.

$X \sim N(26, 5)$

(A) Draw the pdf curve. Include the landmarks for μ (26) and μ ± σ (with the values for μ ± σ) on the horizontal axis of the curve.



(B) Below are the standards for weight status based on BMI cutoffs. If I selected someone at random from this population, what is probability they'd be overweight? (Use the app in each instance.)

BMI	Weight Status
Below 18.5	Underweight
18.5 – 24.9	Normal or Healthy Weight
25.0 – 29.9	Overweight
30.0 and Above	Obese

$Pr(X \geq 25) = .5793$

SOME STUDENTS INTERPRETED THIS TO MEAN OVERWEIGHT BUT NOT OBESE

$Pr(25 < X < \cancel{30}\ 29.9) = .3616$

(C) What is the probability a person selected from this population is obese? $Pr(X \geq 30) = .2119$

(D) What is the probability they are underweight? $Pr(X \leq 18.5) = .0668$

(E) What is the probability they are in the normal or healthy range?

$Pr(18.5 \leq X \leq 24.9) = .3461$