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# Paired difference analysis (summary)

## Exploration and description

1. Read the research question, verify sample is paired and outcome is quantitative.
2. If not already given, calculate within-pair differences (DELTAs).
3. Calculate $\bar{x}_d$, $s_d$, and $n_d$. If data are asymmetrical, report the 5-point summary.
4. Plot the DELTAs and explore the distribution's location, spread, and shape.

## Estimation

1. Read the research question, verify that the data are quantitative and based on paired samples. Confirm that the parameter of interest is $\mu_d$.
2. Calculate the point estimate $\bar{x}_d$ and the $SEM_d = \frac{s_d}{\sqrt{n}}$
3. Calculate the $(1-\alpha)100\%$ CI for $\mu_d = \bar{x}_d \pm t_{1-\frac{\alpha}{2},n-1} \cdot SEM_d$
4. Report the point estimate for the mean difference, direction of the difference (increase or decrease), and the CI. Round appropriately (approx. 3 significant digits). Include units of measure.
5. Sample size for limiting the margin of error was also covered in this chapter.

## NHST

1. Read the research question, verify that the data are quantitative and based on paired samples. Confirm that the parameter of interest is $\mu_d$. State $H_0$: $\mu_d = 0$.
2. Calculate $t_{\text{stat}} = \frac{\bar{x}_d - \text{expected mean difference when } H_0 \text{ true}}{\text{SEM}_d}$; df = $n - 1$
3. Determine $P$ value (app).
4. Report the point estimate, note direction of the difference (increase or decrease), and the $P$ value. Use plain language and round appropriately; be kind to your reader.

Sample size to limit $m$ was also considered earlier in this chapter.

## Illustration using OATBRAN data

**Exploration and description.** This cross over trial looked at within-pair differences on oatbran and corn flake diet. There were $n$ = 14 paired observations. **Data, procedures, and calculations are shown throughout in this chapter.** Here, we present only the interpretation of key results. Differences (CORNFLK – OATBRAN, mmol/l) are fairly symmetrical (stemplot below) with $\bar{x}_d$= 0.362, median approx. 0.35, and $s_d$ = 0.406.

```
-0 | 42
 0 | 011334
 0 | 6677
x1
```

**Estimation.** The oat bran diet *decreased* cholesterol by an *average* of 0.362 mmol/l, 95% CI for $\mu_d$ = (0.129 - 0.597) mmol/l.

**NHST.** The decline of 0.362 mmol/l was statistically significant ($P$ = .0053).

## *Estimation of a proportion (step-by-step summary)*

**Step 1. Review the research question and identify the parameter.** Read the research question. Verify that we have a single sample that addresses a binomial proportion ($p$).

**Step 2. Point estimate.** Calculate the sample proportion ($\hat{p}$) as the point estimate of the parameter.

**Step 3. Confidence interval.** Determine whether the $z$ (normal approximation) formula can be used with the "$npq$ rule." If so, determine the $z$ percentile for the given level of confidence (table) and the standard error of the proportion $SEP = \sqrt{\frac{\hat{p}\hat{q}}{n}}$. Apply the formula $\hat{p} \pm (z_{1-\alpha/2})(SEP)$.

**Step 4. Interpret the results.** In plain language report what proportion and the variable it address. Report the confidence interval; being clear about what population is being addressed. Reported results should be rounds as appropriate to the reader.

Illustration

Of 2673 people surveyed, 170 have risk factor X. We want to determine the population prevalence of the risk factor with 95% confidence.

**Step 1.** Prevalence is the proportion of individuals with a binary trait. Therefore we wish to estimate parameter $p$.

**Step 2.** $\hat{p} = 170 / 2673 = .06360 = 6.4\%$.

**Step 3.** $n\hat{p}\hat{q} = 2673(.0636)(1-.0636) = 159$ → z method OK.

$$SEP = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{(.0636)(1-.0636)}{2673}} = .00472$$

The 95% CI for $p = \hat{p} \pm (z_{1-\alpha/2})(SEP) = 0.636 \pm 1.96 \cdot .00472 = .0636 \pm .0093 = (.0543, .0729)$ $= (5.4\%, 7.3\%)$

**Step 4.** The prevalence in the sample was 6.4%. The prevalence in the population is between 5.4% and 7.3% with 95% confidence.

# The Exact Binomial Test

The exact binomial test is suited to test a binomial proportion from a single sample. A step-by-step analysis of the exact binomial test is presented.

**Step 1. Review the research question and identify the null hypothesis.** Read the research question. Verify that we have a single sample that addresses a binomial proportion. Identify the value of binomial parameter $p$ when there is truly "no difference." Write the null hypothesis in this form:

$$H_0: p = \text{the value of } p \text{ if } H_0 \text{ is true}$$

Calculate the sample proportion $(\hat{p})$ to see how much it differs from the value proposed by the null hypothesis.

**Step 2. In lieu of a test statistic, determine the binomial pmf that applies under $H_0$.** Since this test is based on exact probabilities, there is no test statistic *per se*. Instead, list the pmf that applies when $H_0$ is true.

$$\text{When } H_0 \text{ is true, } X\sim\text{b}(n, p)$$

where $n$ is the sample size and $p$ is the assume value of $p$ when $H_0$ is true.

**Step 3: Determine the *P* value.** The $P$ value is the probability of observing the data or data more extreme. When we are looking for an increase in the number of successes, $P$ value = $\Pr(X \geq x)$ where $x$ is the observed number of successes. When we are looking for an increase in the number of successes $P$ value = $\Pr(X \leq x)$.

**Step 4: Interpret results in narrative form.** Note the sample proportion, direction of the observed difference (increase or decrease), and P value. When the $P$ value is small (say, less than .10), the evidence against the null hypothesis cannot easily be explained by chance ("statistical significance").

## Illustration

Suppose a treatment has an expected success rate of 0.25. We observe successful treatment in 2 out of 3 patients. Is this observation worthy of note, i.e., is it statistically significant?

Step 1. $H_0$: $p = .25$. Note that $\hat{p} = \frac{2}{3} = .6667$.
Step 2. Under $H_0$, $X\sim\text{b}(3,.25)$

| $X$ | $\Pr(X = x)$ |
|---|---|
| 0 | 0.4219 |
| 1 | 0.4219 |
| 2 | 0.1406 |
| 3 | 0.0156 |

Step 3. $P = \Pr(X \geq 2) = \Pr(X = 2)$ or $\Pr(X=3) = 0.1406 + 0.0156 = 0.1562$

Step 4: The difference between the observed proportion (2 of 3) and the expected proportion under the null hypothesis is explicable by chance, i.e., not statistically significant ($P = .1562$).