DEPARTMENT OF ECONOMICS  
SAN JOSE STATE UNIVERSITY  
MASTER'S COMPREHENSIVE EXAMINATION

DECEMBER 2, 2022  
6:00 P.M. TO 9:30 P.M.  
PROCTOR: MUKHERJEE

**INSTRUCTIONS:**

1. Answer ONLY the specified number of questions from the options provided in each section. Do not answer more than the required number of questions. Each section takes one hour.

2. Your answers must be on the paper provided. No more than one answer per page. Do not answer two questions on the same sheet of paper.

3. If you use more than one sheet of paper for a question, write "Page 1 of 2" and "Page 2 of 2."

4. Write ONLY on one side of each sheet. Use only pen. Answers in pencil will be disqualified.

5. Write **------ END -----** at the end of each answer.

6. Write your exam identification number in the upper right-hand corner of each sheet of paper.

7. Write the question number in the upper right-hand corner of each sheet of paper.

**Section 1: Microeconomic Theory—Answer Any Two Questions.**

1A. (Econ 201) Grace's preferences are described by the utility function $U(x_1, x_2) = \alpha x_1 + \beta ln\, x_2$ . Her income is $I$ and prices of both good are $p_1$ and $p_2$, respectively. $\alpha$ and $\beta$ are positive constants.

a. Find her uncompensated demand functions for $x_1^*$ and $x_2^*$ using the Lagrangian method.
b. Calculate the compensated demand functions for $x_1$and $x_2$.

1B. (Econ 201) The market (inverse) demand function for a homogeneous good is $P(Q) = 10 - Q$. There are three firms: firm 1 and 2 each have a total cost of $C_i(q_i) = 4q_i$ for $i \in \{1, 2\}$ and firm 3 has a total cost of $C_3(q_3) = 2q_3$. The three firms compete by setting their quantities of production, and the price of the good is determined by the market demand function given the total quantity. Calculate the Nash equilibrium in this game and the corresponding market price.

**1C**. (Econ 104) A firm has $L$ units of labor at its disposal. Its output are three different commodities. Producing $x$, $y$, and $z$ units of these commodities requires $\alpha x^2$, $\beta y^2$, and $\gamma z^2$ units of labor, respectively.

a. Consider the maximization problem:

$$\max\ ax + by + cz \quad \text{subject to} \quad \alpha x^2 + \beta y^2 + \gamma z^2 = L.$$

Put a = 2, b = c = 1, $\alpha = 1$, $\beta = 1/4$, and $\gamma = 1/2$, and solve the problem in this case. (Hint: Your answers should be in terms of L.)

b. What happens to the maximum value of $2x + y + z$, when $L$ increases from 100 to 101? Find both the exact change in the maximum value and the appropriate linear approximation based on the interpretation of the Lagrange multiplier.

(over)